## Name

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## What Time is It?

Einstein's Theory of Relativity is centered on the postulate that the speed of light will be the same no matter how it is measured. We will call the speed of light $\mathbf{c}$.

That means that if you are standing still and turn on a flashlight, you will measure the speed of light to be c. Now, if you are standing on a train moving at a constant speed, you will also measure the speed of light out of the flashlight to be $\mathbf{c}$. But, if you move past a friend who can see your flashlight on the train, what is the speed of light the friend measures? Will they measure the speed of light to be $\mathbf{c}$ or $\mathbf{c}$ plus the speed of the train?

It turns out that even though it seems that the friend would measure the speed of light to be $\mathbf{c}$ plus the speed of the train, they would still measure the speed of light to be $\mathbf{c}$ !

Because of this, some interesting things happen when light is used to measure time...

## The Light-Clock

Consider a light-clock made of two mirrors separated by a distance $\mathbf{d}$. We will measure time by the interval it takes light to travel from one mirror to the other:

${ }^{*} \mathrm{t}_{0}$ : This is the time measured by all stationary (at rest) clocks.

Now, let us consider a light-clock that is traveling with a speed $\mathbf{v}$ moving to the right across the paper:

2. How long does it take for the 'tick' trip? Represent this time as $\mathbf{t}$.
(Hint: the distance light travels between mirrors is not simply $\mathbf{d}$ anymore. The mirrors have moved a distance $\mathbf{x}$ to the right from the time light leaves the bottom mirror to the time it reaches the top mirror [the vertical distance between the mirrors is still $\mathbf{d}]$. Use the relation in the last hint to find $\mathbf{x}$ (in terms of $\mathbf{v}$ and $\mathbf{t}$ ) and then use the Pythagorean theorem to find the distance of the 'tick' trip.)

[^0]3. What is the relationship between $\mathbf{t}_{\mathbf{0}}$ and $\mathbf{t}$ ?
(Hint: solve the equation found in question 1 for $\mathbf{d}$ and substitute this into the relationship for $\mathbf{t}$ [found in question 2].)
4. Simplify this relationship and solve for $\mathbf{t}$ :

What you have just derived is the equation for time dilation. This means that you (as an observer at rest) will always measure the passage of time to be constant $\left(\mathbf{t}_{\mathbf{0}}\right)$ but if you observe a clock moving by you with speed $\mathbf{v}$, you will measure the moving clock ticking slower ( $\mathbf{t}$ ) than yours.

Let's use the time dilation equation (found in question 4) to see this effect...
5. Imagine you observe a clock traveling at half the speed of light $(\mathbf{v}=0.5 \mathbf{c})$. Solve for $\mathbf{t}$ in terms of $\mathbf{t}_{\mathbf{0}}$ (this is how many more $\mathbf{t}_{\mathbf{0}}$ ticks there are between $\mathbf{t}$ ticks):
6. Now, imagine that you observe a clock traveling at $99 \%$ the speed of light $(\mathbf{v}=0.99 \mathbf{c})$. Now how much slower are the clocks at rest ticking compared to yours?
7. What happens to the time clocks traveling at the speed of light $(\mathbf{v}=\mathbf{c})$ keep?

## Does time dilation really happen?

This phenomenon of time dilation really happens in our everyday world. However, the speed of light is very large $-299,792,458$ meters per second or 186,282 miles per second! That means that when you are traveling in a car going 70 miles per hour on the interstate, you are only traveling at 0.000000104 c. Because you are still moving so slow compared to the speed of light, you will not notice the very, very small difference between a clock in the car and a clock beside the road (the effect is too small for a calculator to register).

Also, nothing that has mass can ever travel the speed of light and nothing (including light) can ever travel faster than $\mathbf{c}$. Because of this, $\mathbf{c}$ is sometimes referred to as the 'universal speed limit.'

Here is something to think about... If the measurement of time is not constant between clocks moving at different velocities, is the measurement of length constant? (Answer: No! This is called the Lorenz contraction and has a familiar form to the time dilation equation we just derived. With the Lorenz contraction, objects seem to be compressed only in a direction of motion.)


[^0]:    * t: This is the time as measured by a moving clock as observed by an observer at rest.

