



Mode mismatch and sideband imbalance

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- Understanding of FFT results using modal model
 - Effect of mode mismatch due to thermal lens in ITMs
- Effect of locked arm
- Effect of PRM cavity
- Simple IFO
 - $\text{ROC}(\text{ITM}_x) = \text{ROC}(\text{ITM}_y)$, $\text{ROC}(\text{ETM}_x) = \text{ROC}(\text{ETM}_y)$
 - No BS curvature
 - ...

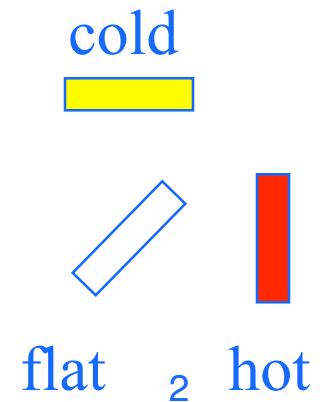
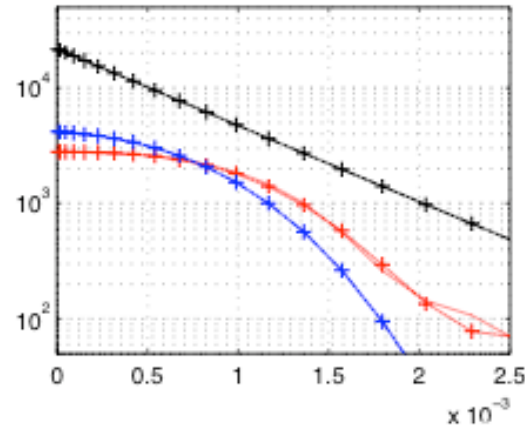
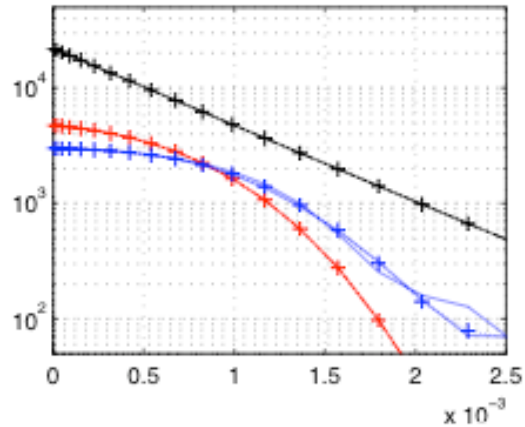
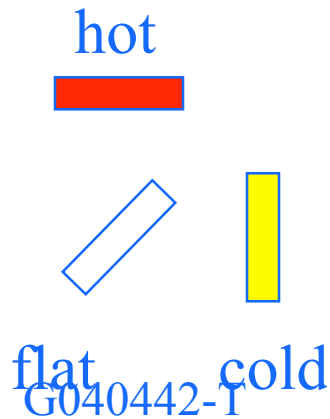
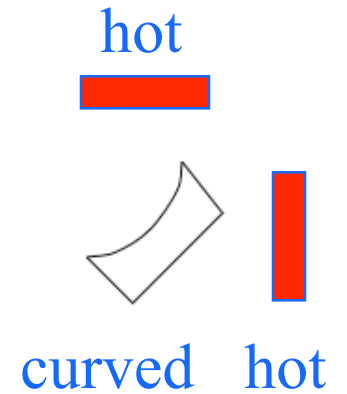
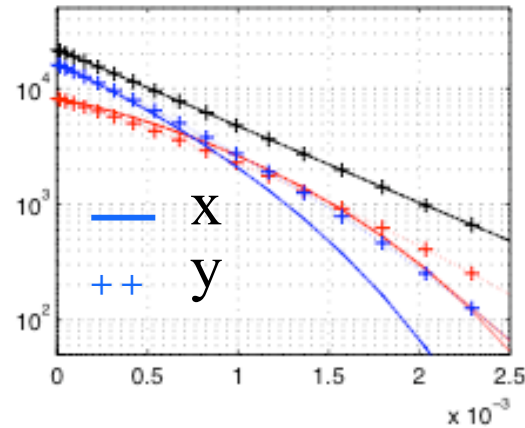
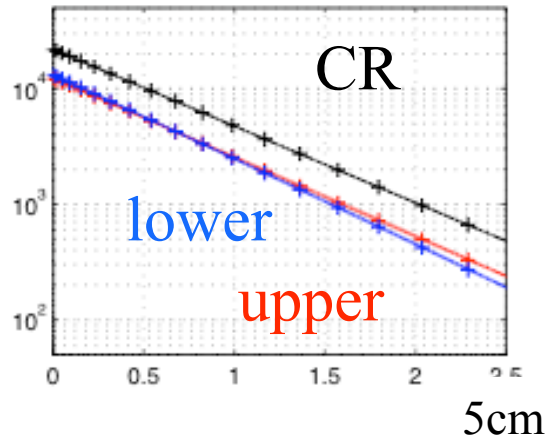
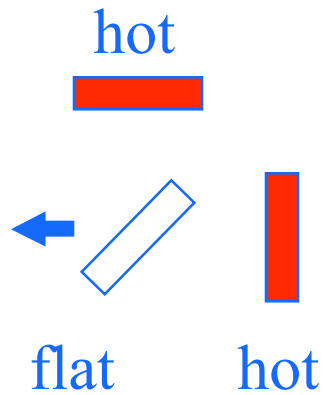


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Power only
thermal

Gaussianity of CR & SBs

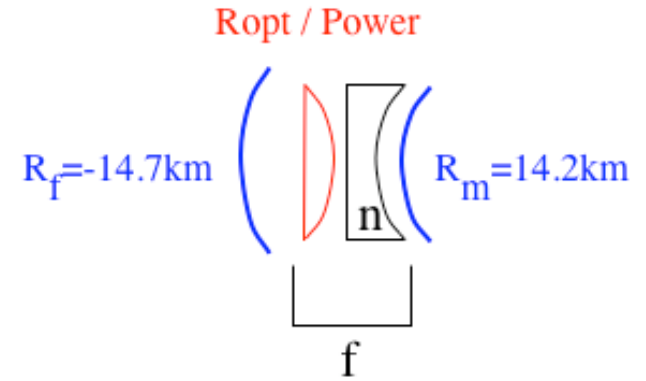
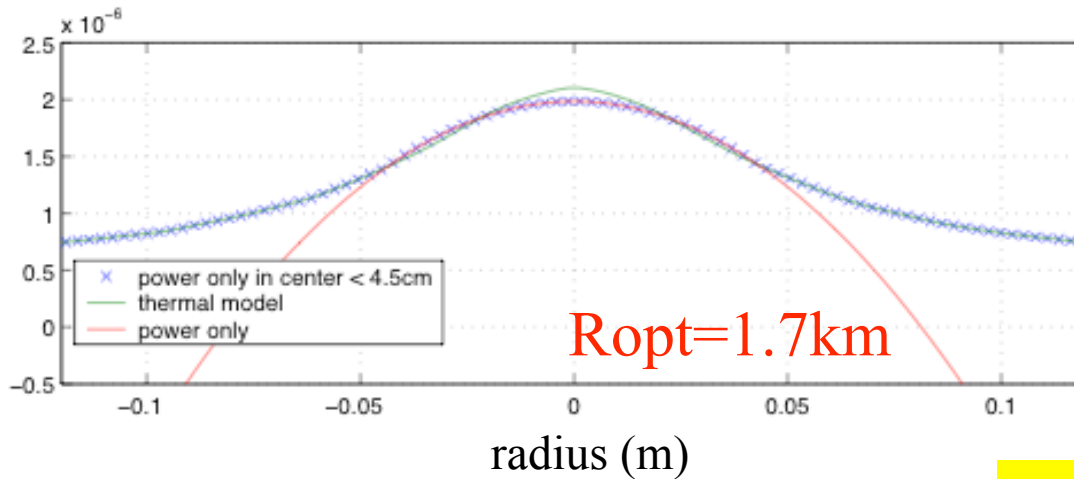
Power on Symmetric port : $\log(\text{power})$ vs x^2



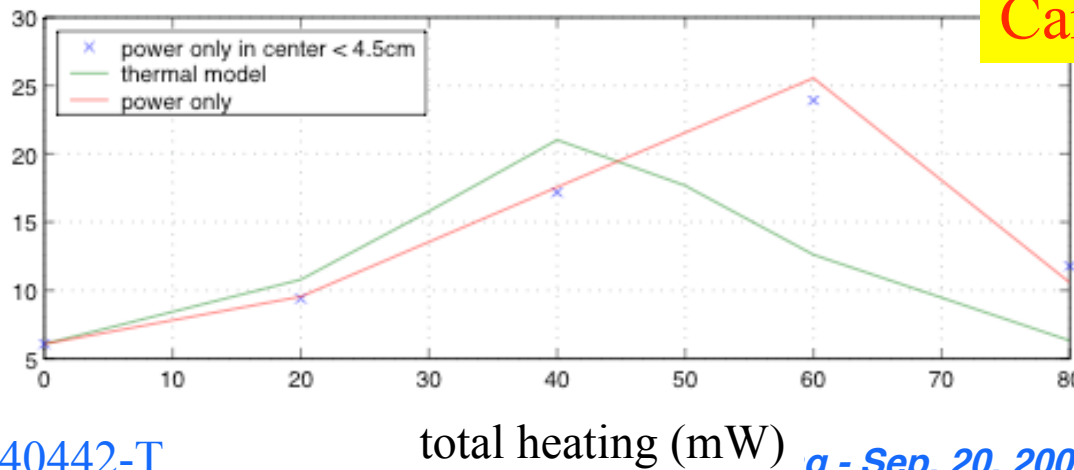
Thermal lensing in FFT

- Phil W. calculated based on MIT model -

Optical thickness @ 1w



Sideband recycling gain



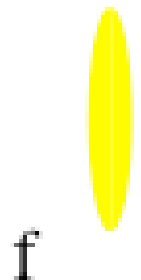
Carrier gain is constant

$$\frac{1}{R_f(HR)} = \frac{1}{R_f(AR)} - \frac{1}{f}$$

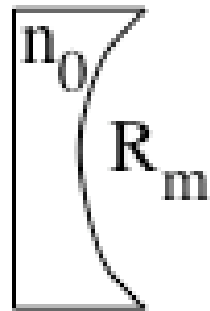
Power = 58mW

Simple thermal lensing

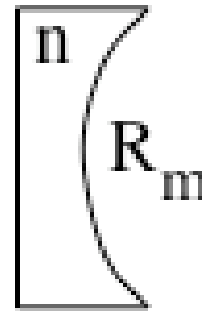
thermal lens



mirror1



mirror2



$$n = n_0 - \frac{R_m}{f}$$

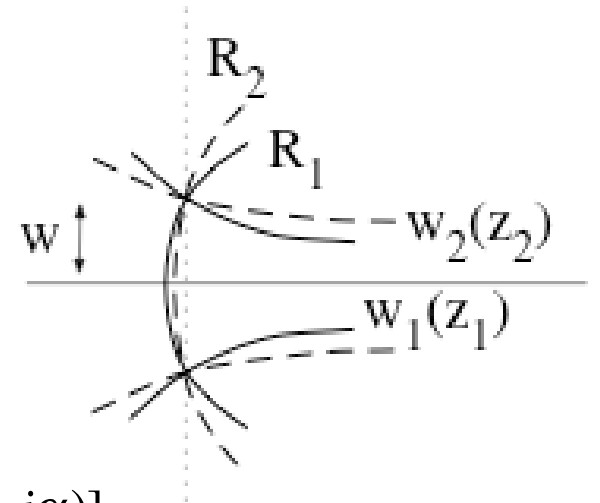
$$f = \frac{1.7 \text{ km}}{P_{abs} \text{ (W)}}$$

$$n = n_0 - 8.2 \cdot P_{abs} \text{ (W)} = 1.45 - 8.2 \cdot P_{abs} \text{ (W)}$$

$$n_h = n(60 \text{ mW}) = 0.96$$

Curvature mismatch with same beam size

$$TEM_{m_2 n_2}(x, y : w^x, R_2^x, w^y, R_2^y) = \sum_{m_1, n_1} M_{m_2, m_1}(\alpha_x) \cdot M_{n_2, n_1}(\alpha_y) \cdot TEM_{m_1 n_1}(x, y : w^x, R_1^x, w^y, R_1^y)$$



$$M_{m_2 m_1}(\alpha) = \int dx \cdot u_{m_2}(x : w, R_2) \cdot u_{m_1}(x : w, R_1)^*$$

$$= \frac{1}{\sqrt{\pi 2^{m_1+m_2} m_1! m_2!}} \int d\xi H_{m_1}(\xi) H_{m_2}(\xi) \text{Exp}[-\xi^2 (1 + i\alpha)]$$

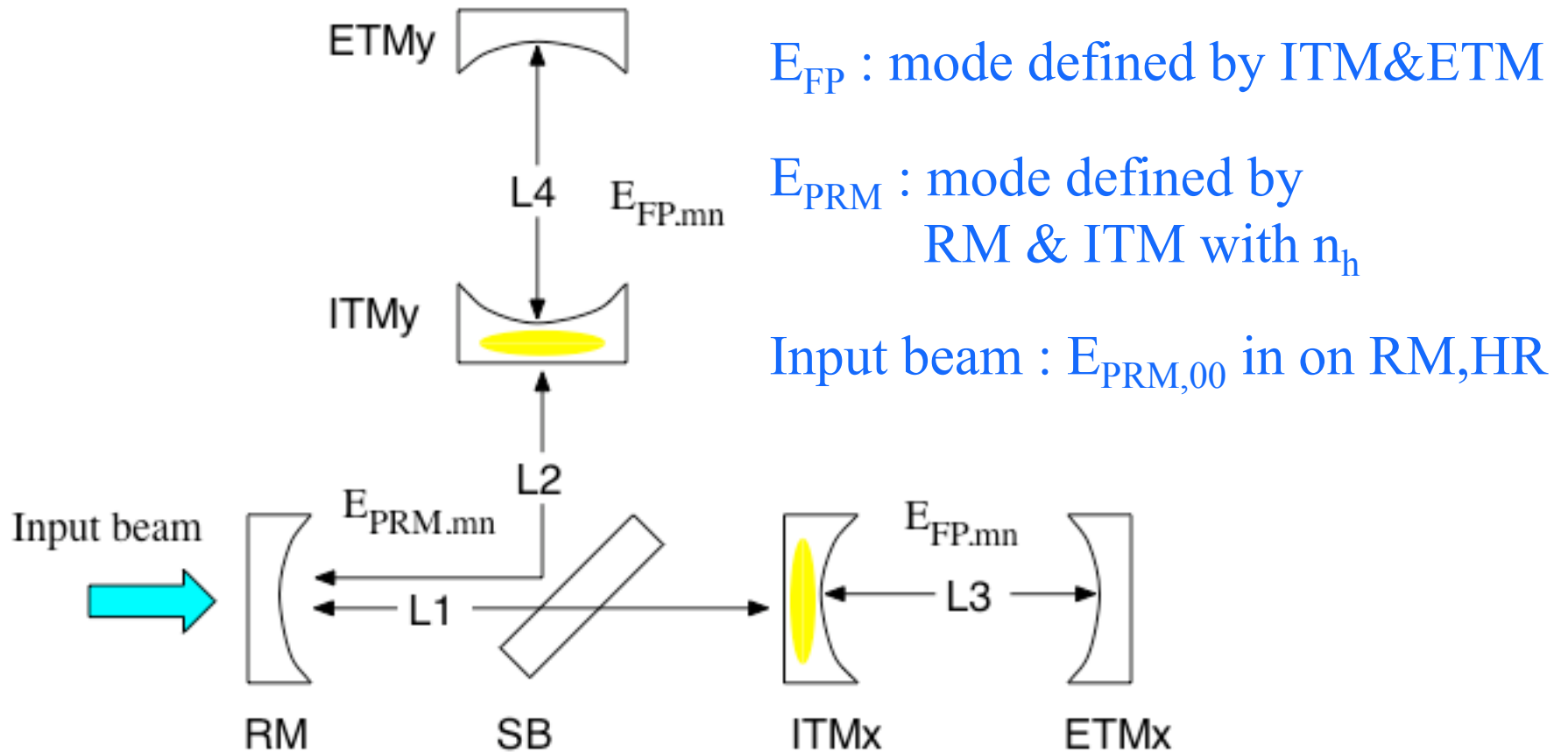
$$\alpha = \frac{k \cdot w^2}{4} \left(\frac{1}{R_2} - \frac{1}{R_1} \right) = \frac{z}{2z_0} \left(\frac{R_1}{R_2} - 1 \right)$$

$$M_{00}(\alpha) = \frac{1}{\sqrt{1 + i\alpha}}, \quad M_{02}(\alpha) = \frac{-i\alpha}{\sqrt{2} (1 + i\alpha)^{3/2}}$$



Two Hermite Gaussian mode bases

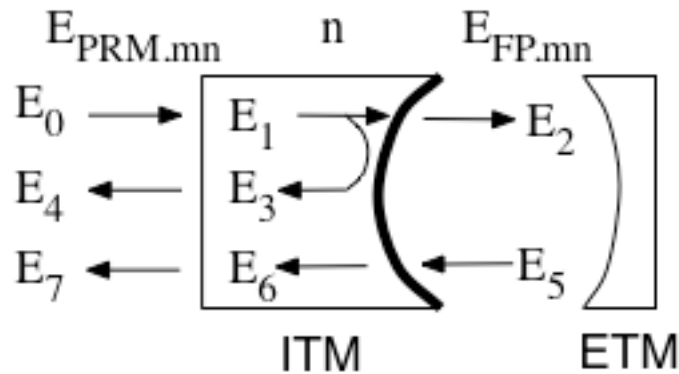
- thermal state independent -





Reflection by mismatched FP

- n of ITM is not n_h , the optimal -



Field	ROC(ITM)/ ROC(Field)
E0	$-n_h$
E1	$-1+n-n_h$
E2	-1
E3	$1+n-n_h$
E4	$2n-n_h$
E5	1
E6	1
E7	n

$$\alpha(E_1 \rightarrow E_2) = \alpha / 2$$

$$\alpha(E_6 \rightarrow E_7) = \alpha / 2$$

$$\alpha(E_0 \rightarrow E_4) = \alpha$$

$$\alpha = \frac{z_{FP}}{z_{0 FP}} (n - n_h) = 0.266(n - 0.96)$$



Reflected field

- CR~00, SB~00+02/20 -

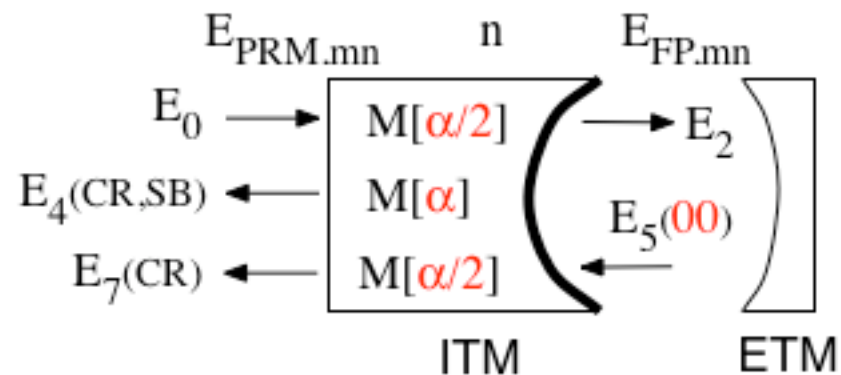
$$r_{ITM} \rightarrow 1, \frac{-r_{ETM} \cdot t_{ITM}^2}{1 - r_{ITM} r_{ETM}} \rightarrow -2$$

$$E_{SB} = \frac{1}{1 + i\alpha} E_{00} - \frac{i\alpha / \sqrt{2}}{(1 + i\alpha)^3} (E_{02} + E_{20}) + O(\alpha^2)$$

$$E_{CR} = \frac{1}{1 + i\alpha} E_{00} - \frac{i\alpha / \sqrt{2}}{(1 + i\alpha)^3} (E_{02} + E_{20}) + O(\alpha^2) \Leftarrow E_4$$

$$-2 \frac{1}{1 + i\alpha / 2} \left(\frac{1}{1 + i\alpha / 2} E_{00} - \frac{i\alpha / 2 / \sqrt{2}}{(1 + i\alpha / 2)^3} (E_{02} + E_{20}) + O(\alpha^2) \right) \Leftarrow E_7$$

$$= -\frac{1}{1 + i\alpha} E_{00} + O(\alpha^2)$$





FFT length lock

- when arm is locked to CR00, CR is clean -

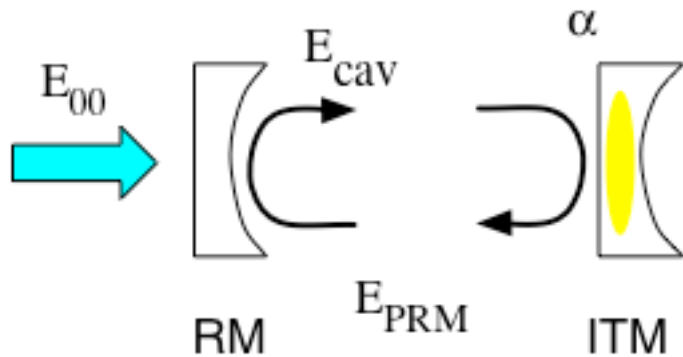
$$E_{CR}^{ref} \approx -\frac{1}{1+i\alpha} E_{PRM,00} \approx -Exp(-i\alpha) E_{PRM,00}$$

n(x)-n(y)	0.96-0.96	0.96-1.10	1.10-0.96	1.00-1.00	1.10-1.10	1.20-1.20
2dL(RM - ITMx) / k	0	0	$-\alpha(1.1)$	$-\alpha(1.0)$	$-\alpha(1.1)$	$-\alpha(1.2)$
2dL(RM - ITMy) / k	0	$-\alpha(1.1)$	0	$-\alpha(1.0)$	$-\alpha(1.1)$	$-\alpha(1.2)$
P(02/20)/P(00)	7.3e-7	10e-7	10e-7	7.5e-7	14e-7	34e-7

Power(PRM) = 46.5, Power(Arm) = 3270 for all cases

Field in mode mismatched FP

$$E_{cav} = \frac{t_{RM} \cdot E_{in}}{(1 - R)(1 + C_0 \cdot \alpha^2)} (E_{PRM,00} - i \cdot \alpha \cdot C_2 \cdot (E_{PRM,02} + E_{PRM,20})) + O(\alpha^3)$$



$$R = R_0 \cdot \text{Exp}[i\phi_{CR,00} + i\phi], \quad R_0 = r_{RM} \cdot r_{ITM}$$

$$\phi_{CR,00} = -2k_{CR}L + 2\eta - \arctan(\alpha)$$

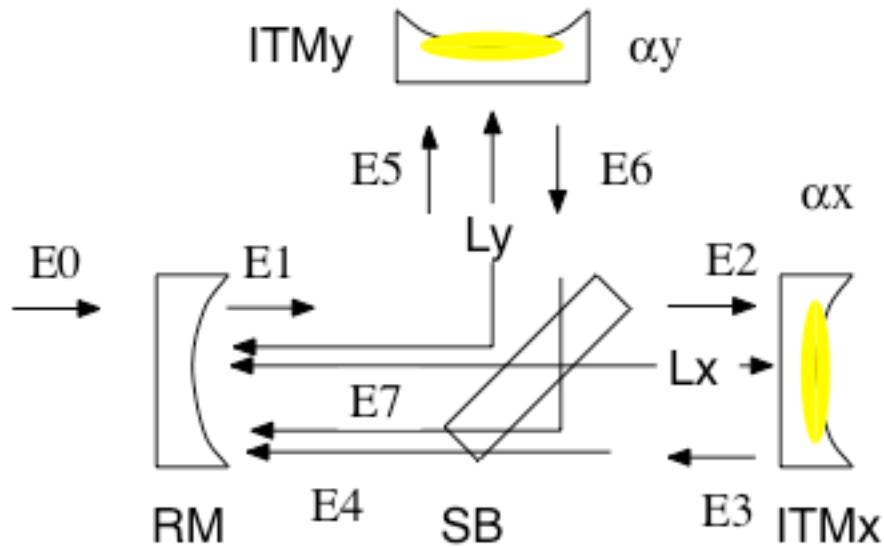
$$\phi_{mix} = -\frac{1}{2} \cot(2\eta) \cdot \alpha^2$$

$$\phi = -2k_{SB}L + \phi_{mix}$$

$$C_0 = \frac{(1 - i \cdot \cot(2\eta)) \cdot R}{2(1 - \text{Exp}(i4\eta)R)}$$

$$C_2 = \frac{\text{Exp}(i \cdot 2\eta)}{\sqrt{2}(1 - \text{Exp}(i \cdot 4\eta)R)}$$

Simple Michelson with curvature mismatch



nx-ny	FFT			Modal Model (n+m<=4)		
	Upper SB	Lower SB	Up/Low	Upper	Lower	Up/Low
0.96-0.96	25.8	25.8	1	25.9	25.9	1
1.1-0.96	24.2	22.2	1.09	20.2	20.6	0.98
1.2-0.96	20.8	18.4	1.13	15.1	14.1	1.07

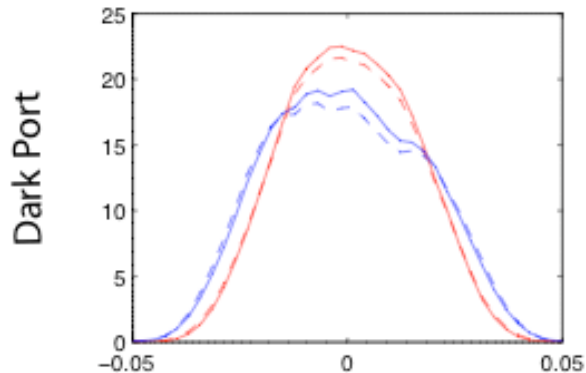


FFT vs LSC lock

$n(\text{ITMx}) - n(\text{ITMy})$

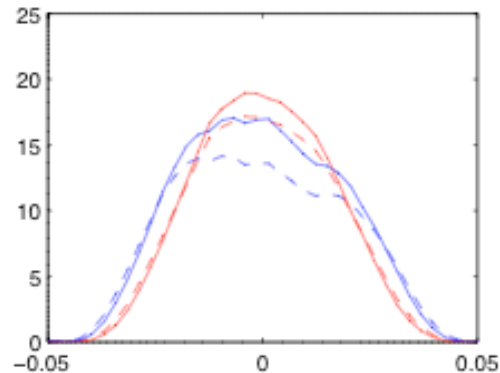
0.96-0.96

Symmetric Heating



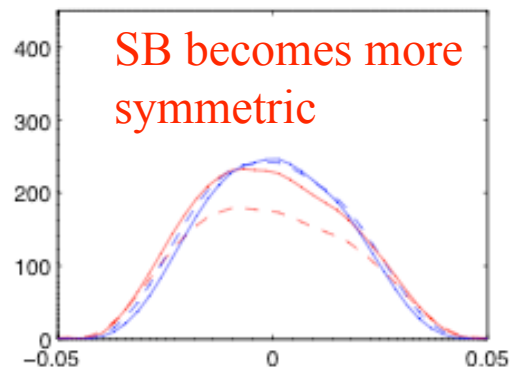
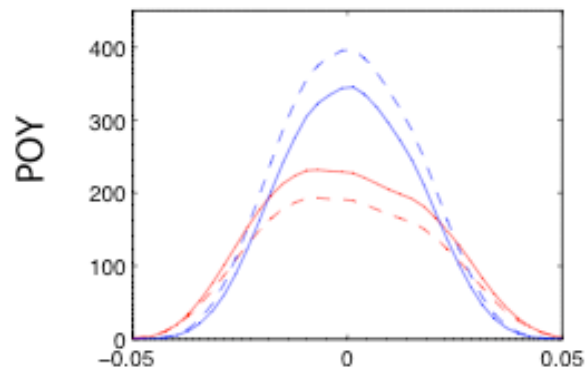
1.10-0.96

Differential Heating
ITMx cooler than ITMy



— lower SB — upper SB

- - - - FFT lock — LSC lock



symmetric
differential

	FFT	LSC
θ_{CR}	0.3	-1.9
θ_{SB+}	-0.6	-2.3
θ_{SB-}	7.2	5.1
Spob	-0.57i	-0.57i
θ_{CR}	0.2	-8
θ_{SB+}	4.9	-1.2
θ_{SB-}	11.8	5.1
Spob	-0.48i	-0.50i