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Ambient Magnetic Field Coupling  
to Quadruple Suspension Stainless Steel Parts

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Dennis Coyne, Norna Robertson

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of the LIGO Laboratory.

**California Institute of Technology**  
**LIGO Project – MS 18-34**  
**1200 E. California Blvd.**  
**Pasadena, CA 91125**  
Phone (626) 395-2129  
Fax (626) 304-9834  
E-mail: [info@ligo.caltech.edu](mailto:info@ligo.caltech.edu)

**Massachusetts Institute of Technology**  
**LIGO Project – NW22-295**  
**185 Albany St**  
**Cambridge, MA 02139**  
Phone (617) 253-4824  
Fax (617) 253-7014  
E-mail: [info@ligo.mit.edu](mailto:info@ligo.mit.edu)

**LIGO Hanford Observatory**  
**P.O. Box 1970**  
**Mail Stop S9-02**  
**Richland WA 99352**  
Phone 509-372-8106  
Fax 509-372-8137

**LIGO Livingston Observatory**  
**P.O. Box 940**  
**Livingston, LA 70754**  
Phone 225-686-3100  
Fax 225-686-7189

<http://www.ligo.caltech.edu/>

## 1 Introduction

Residual magnetic flux density and ferromagnetism in the stainless steel members that are part the upper stages of the quadruple suspension will cause a noise coupling with the ambient magnetic field and field gradient. The residual magnetic flux density and ferromagnetism are a consequence of work history of the material (rolling, machining, etc.). Unless the material is brought above the Curie temperature (above  $\sim 800\text{C}$ ) these residual magnetic properties will remain. In addition, the maraging steel, cantilevered, spring blades are also ferromagnetic and will force couple to the ambient magnetic field. The purpose of this note is to make approximate estimates of the magnitude of these noise coupling terms.

## 2 Ambient magnetic field

Since the transfer functions of force-to-displacement for forces applied on the Penultimate Mass (PM) and the Upper Intermediate Mass (UIM) to displacement on the Test Mass (TM) roll off as  $f^4$  or faster, we only need to consider the coupling at 10 Hz (the lowest measurement band frequency). The ambient magnetic field within the LIGO vacuum chambers is estimated<sup>1</sup> to be conservatively  $10^{-11} \text{ T}/\sqrt{\text{Hz}}$  at 10 Hz.

## 3 Allowable force

The stainless steel and maraging steel parts are not located on the TM or PM stage for main isolation chain of the quadruple pendulum; The lowest stage containing these parts is the UIM. The transfer function<sup>2</sup>, at 10 Hz, for force applied to the UIM to displacement at the TM is  $8.9 \times 10^{-10} \text{ m/N}$ . Based on the technical displacement noise limit of  $10^{-20} \text{ m}/\sqrt{\text{Hz}}$  for a TM at 10 Hz, the maximum force coupling to the UIM is  $1.1 \times 10^{-11} \text{ N}/\sqrt{\text{Hz}}$ .

Stainless steel parts are also part of the PM stage in the reaction chain. Force coupling to the reaction chain PM will result in motion of the Reaction Mass (RM) (aka Compensation Plate (CP) for the Input Test Mass quadruple suspension). The RM can couple to the TM through the Electro-Static Drive (ESD) actuator. The transfer function, at 10 Hz, for force applied to the PM to displacement at the TM is  $2.8 \times 10^{-8} \text{ m/N}$ . The transfer function from reaction chain PM to RM will be similar. The maximum allowed motion of the RM which when coupled through the ESD gives motion<sup>3</sup> at the technical displacement noise limit of  $10^{-20} \text{ m}/\sqrt{\text{Hz}}$  at 10 Hz at the TM, is  $3.9 \times 10^{-15} \text{ m}/\sqrt{\text{Hz}}$ . Hence the maximum allowed force coupling to the PM of the reaction chain is  $1.4 \times 10^{-7} \text{ N}/\sqrt{\text{Hz}}$ . Thus a much larger force coupling can be tolerated at the reaction chain PM than at the UIM, and we will use the UIM number of  $1.1 \times 10^{-11} \text{ N}/\sqrt{\text{Hz}}$  as the force limit for further analysis.

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<sup>1</sup> P. Fritschel, "Considerations regarding magnet strengths for the Advanced LIGO test mass quadruple suspensions", [LIGO-T050271-00](#)

<sup>2</sup> Ibid.

<sup>3</sup> K. Strain, "Reaction mass coupling at ETM/ITMs", [LIGO-T060043-00](#)

## 4 Force coupling due to ferromagnetism

The force,  $F$  (N), on a material with finite magnetic permeability,  $\mu_s$ , and surface area,  $S$  ( $m^2$ ), in the presence of a uniform magnetic flux density,  $B$  (T), is<sup>4</sup>:

$$F = \left( \frac{SB^2}{2} \right) \left( \frac{1}{\mu_0} - \frac{1}{\mu_s} \right)$$

where  $\mu_0 = 4 \pi 10^{-7}$  is vacuum permeability. For a worst case assumption of high permeability, i.e.  $\mu_s \gg \mu_0$ , then

$$F = \frac{SB^2}{2\mu_0}$$

For the parts in question,  $S < 1 m^2$ , so with  $B = 10^{-11} T/\sqrt{Hz}$  at 10 Hz, the force coupling is  $4 \times 10^{-17} N/\sqrt{Hz}$  at 10 Hz, or considerably less than the allowable force limit.

## 5 Force coupling due to residual magnetic flux density

Gauss meter measurements<sup>5</sup> of the magnetization of the quad suspension stainless steel parts suggest an average external magnetic flux density of  $< 0.5$  Gauss =  $0.5 \times 10^{-4}$  T.

The force,  $F$ , in an ambient magnetic field,  $B_a$ , on a magnetized body can be determined by attributing a magnetic dipole moment,  $p$ , to the body and then determining the force through<sup>6</sup>:

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<sup>4</sup> for example, J. Brauer, Magnetic Actuators and Sensors, John Wiley & Sons Inc., cr 2006, section 5.3, equation 5.12.

<sup>5</sup> Vern Sandberg and John Worden used a Bell gauss meter (a Hall effect probe) to sample 4 serialized parts (D060379-H) on 28-Jul-2009. The parts were selected after a quick field/permeability scan, with a bar magnet mounted on a gimbal, indicated that two of the parts had hot spots and two of the parts seemed to be uniform in response. They cleared an area on a plastic pallet and confirmed that they had only the earth's background field present (0.5 -1 gauss) This background field was zeroed out with the meter. Using both a flat probe and an axial probe they found very localized "hot spots" on two samples and very little on the other two samples. The "hot spot" was an area of less than a few millimeters square.

serial number 069 - max field at surface - 7.5 gauss

serial number 079 - max field at surface - 0.4 gauss

serial number 124 - max field at surface - 5.5 gauss

serial number 113 - max field at surface - 0.25 gauss

There were not too many sign reversals when scanning over the parts with the probe. The external, near-surface, magnetic flux density associated with a permanent magnetic dipole on the scale of the entire part is estimated to be less than 0.5 Gauss.

John suspects that they are detecting different levels of cold work in the parts - either due to machining or the mill rolling process, or the parts have a different heat history. The raw material can have significantly different properties depending on the location -- for example; the center of bar stock may have different properties than the surface due to cooling rates during mill processing.

<sup>6</sup> for example, H. Knoepfel, Magnetic Fields: A comprehensive Theoretical Treatise for Practical Use, John Wiley & Sons, Inc., cr 2000, section 6.2, equation 6.2-15.

$$F = \nabla(\vec{p} \cdot \vec{B}_a)$$

This force is predominantly in the direction of the prevailing field gradient and is maximized when  $p$  is aligned with  $B_a$ . Making the conservative assumption that  $p$  is aligned with  $B_a$ , then

$$F = p \frac{dB_a}{dx} \approx p \frac{B_a}{\ell}$$

where  $\ell$  is the length scale over which the field is varying around the suspension. It has been suggested<sup>7</sup> that  $\ell$  is  $\sim 0.1$  m.

The effective magnetic dipole moment of a magnetized part can be estimated by assuming a spherical permanent magnet with uniform magnetization<sup>8</sup>,  $M$ :

$$p = \frac{4\pi}{3} a^3 M$$

where  $a$  is the radius of the sphere. For the quad suspension parts  $a$  is approximately 0.1 m. The internal magnetization,  $M$ , can be calculated from the maximum external magnetic flux density,  $B^e$ , as follows<sup>9</sup>:

$$M = \frac{3B^e}{2\mu_0}$$

Consequently, the magnetic dipole moment is:

$$p = \frac{2\pi a^3 B^e}{\mu_0}$$

and the force is:

$$F = \left( \frac{2\pi a^3 B^e}{\mu_0} \right) \left( \frac{B_a}{\ell} \right)$$

There are apparently two sources of permanent magnetic dipole. Small scale magnetic dipoles associated with “hot spots” where the stainless steel has been work hardened and a magnetic dipole on the spatial scale of the part. For the “hot spot” dipoles:

$$B_a = 10^{-11} \quad T / \sqrt{Hz} \quad \text{at } 10 \text{ Hz}$$

$$\ell = 0.1 \quad m$$

$$B^e = 8 \quad G = 8 \cdot 10^{-4} \quad T$$

$$a = 0.002 \quad m$$

$$F = 3 \cdot 10^{-15} \quad N / \sqrt{Hz} \quad \text{at } 10 \text{ Hz}$$

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<sup>7</sup> P. Fritschel, [LIGO-T050271-00](#).

<sup>8</sup> H.Knoepfel, Magnet Fields: A Comprehensive Theoretical Treatise for Practical Use, John Wiley & Sons, Inc., cr 2000, section 2.1, equation 2.1-69.

<sup>9</sup> Ibid, equation 2.1-71.

For the overall magnetic dipole on the scale of the part:

$$B_a = 10^{-11} \text{ T} / \sqrt{\text{Hz}} \text{ at } 10 \text{ Hz}$$

$$\ell = 0.1 \text{ m}$$

$$B^e < 0.5 \quad G = 0.5 \cdot 10^{-4} \text{ T}$$

$$a = 0.1 \text{ m}$$

$$F < 2.5 \cdot 10^{-11} \text{ N} / \sqrt{\text{Hz}} \text{ at } 10 \text{ Hz}$$

While the force coupling due to the localized dipoles is well under the allowable force, the force coupling associated with the overall magnetization of the part is perhaps as much as twice the allowed force limit. Since the ambient field gradient used in the calculation is a conservative estimate<sup>10</sup>, the overall part magnetic dipole moment is a conservative estimate and the force limit is the technical noise limit (set at one tenth the fundamental noise limit), this slightly high force coupling is acceptable.

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<sup>10</sup> The ambient field gradient used in the calculation is 100 pT m<sup>-1</sup>/rt Hz. In [LIGO-T050087-00](#) Robert Schofield refers to typical ambient field gradients in LIGO of magnitude ~10 pT m<sup>-1</sup>. If in fact he meant 10 pT m<sup>-1</sup>/rt Hz this would be 10 times smaller than we have used in the calculation, confirming that we are being conservative in our estimate.