LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY

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Astigmatism by the stable Michelson cavity

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1 Abstract

The astigmatism induced by the stable Michelson cavity is discussed quantitatively using a numerical simulation tool SIS(1). Because of the finite angle of incidence (AOI) at the PR2/PR3/SR2/SR3 mirrors, astigmatism of resonating fields is induced and this leads to the loss of the signal.

The astigmatism induced in the signal recycling cavity (SRC) and the power recycling cavity (PRC) are different because of the resonant condition of the field, i.e., the CR injected from the PRM is resonating in PRC, while the CR signal sideband injected from the arm into SRC is anti resonant in SRC.

It is concluded that the loss is negligible when AOI is around 1°, and that the advLIGO performance will not be compromised by the astigmatism introduced by the current design. With AOI larger than 2°, the loss becomes significant.

2 Introduction

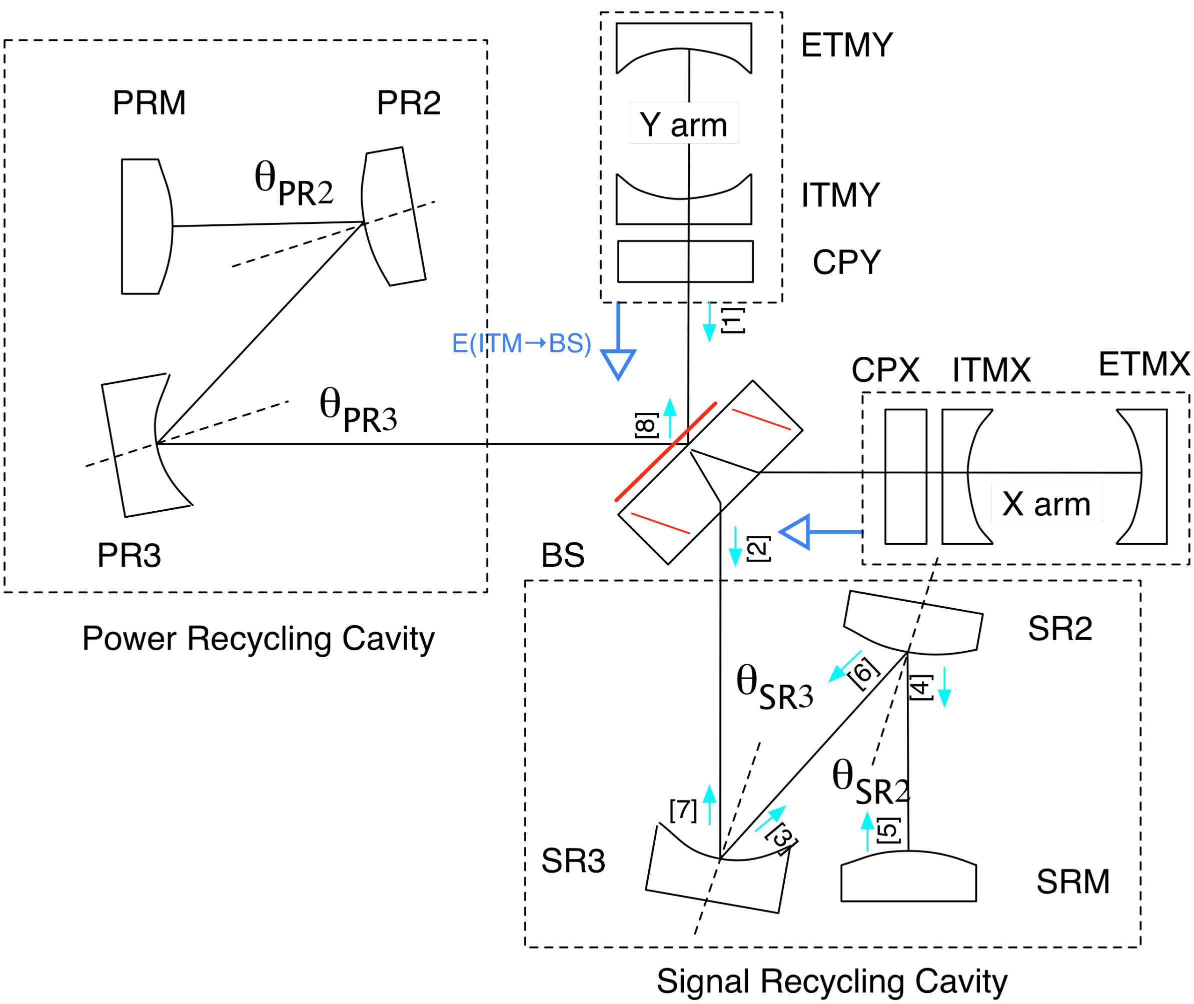


Figure 1 Optical Layout of advanced LIGO cavities

2.1 Basic

The optical layout of the advanced LIGO cavities is shown in Figure 1. The fields in the Michelson cavity have small astigmatism due to the finite size of the BS mirror. The beam size and the optics apertures were chosen so that the loss due to the BS and PR3/SR3 aperture and thickness is acceptable.

Due to the finite AOI at focusing mirrors in the recycling cavities, PR2/PR3/SR2/SR3, further astigmatism is induced and the effect becomes larger as AOI becomes larger. This astigmatism is studied quantitatively.

2.2 Convention of configuration

In this document, x and y are the axes in the plane perpendicular to the beam propagation direction, x axis is in the plane of the cavity and y axis points upward.

The parameters of cavities are given in Sec.6. Fields or powers in a cavity with θ_{XR2} = A° and θ_{XR3} = B° will be show with "(A,B)" appended to the quantity name, like E(A,B). Distributions in the plane defined by the cavity, E(A,B)(x,y=0) or along the vertical direction, E(A,B)(x=0,y), are shown as E(A,B)(x) or E(A,B)(y) for short. When two distributions along x and along y are compared or when the choice is irrelevant, r is used as the common name, e.g., r is used as the axis label when plotting a ration of E(A,B)(x) and E(A,B)(y), or P(r) is used when P(x) and P(y) are essentially the same.

2.3 Cavity configuration and resonant condition

Current version of SIS cannot simulation the full advanced LIGO configuration. In this study, two coupled cavities are studied, one formed by the power recycling cavity and the x arm (PRC-X), and the other is the one formed by the signal recycling cavity and y arm (SRC-Y). The BS is placed between the recycling cavity and the long arm, and the geometrical effect of the BS is included in the simulation. For both cases, the transmittance of BS is set to be 1.

Stationary fields in three different conditions are studied, PRC locked, SRC locked and SRC detuned.

2.3.1 PRC locked

The CR field in PRC-X configuration is calculated by injecting a field to PRM whose mode matches with the arm mode when there is no AOI. The x-arm and PRC are locked using error signals similar to the actual experiment. See SIS reference about the detail process.

The stationary fields calculated this way mainly determines the loss of the power in the long arm due to the mode mismatch between the PRC and X-arm mode. The mode in PRC is determined by three factors, intrinsic resonant mode of PRC, fields inject from the x-arm and the source field injected to PRM from MC.

With the current design of the recycling gain and arm finesse, the field from the PRM is 1/100 of the field from the arm, and the stationary field in PRC is determined by the balance of the pure Gaussian field injected from the arm and the PRC cavity mode.

2.3.2 SRC locked

The signal sideband in the SRC is calculated in two steps. First, the stationary field is calculated in the same way as for PRC locked. After cavities are locked and resonating fields are calculated, ETM is shaken to induce audio sideband. With all mirror positions fixed, the stationary field of this signal sideband is calculated. In other words, this is a simulation of a coupled cavity with the source field injected from the HR side of ETM.

2.3.3 SRC detuned

For the broadband configuration, SRC is set to be anti resonant for the signal sideband. To simulate fields in this configuration, one additional step is taken in the calculation of the SRC locked case. After cavities are locked and resonating fields are calculated, the position of SRM is shifted by λ/4. With those mirror positions, the stationary fields of the signal sideband is calculated by shaking ETM.

When the signal extraction scheme is changed, the beam profile of stationary fields in SRC will be something between SRC locked and SRC detuned calculation.

2.4 Higher order mode fraction and Beam parameters

SIS uses FFT methods to calculate the stationary fields. After stationary fields are calculated, mode expansion of the field is used to calculate the higher order mode (HOM) fraction and the signal TEM00 mode loss.

Another effective parameters calculated are the beam size and curvature. Each field is fit by a two dimensional Gaussian to find effective values along x and y directions. The region for the fit is chosen such that the power and phase changes are consistent with a simple wave form:

$$E(x,y) = A \cdot \exp(-(\frac{\bar{x}^2}{w_x^2} + \frac{\bar{y}^2}{w_y^2})) \cdot \exp(-i \cdot k \cdot (\frac{\bar{x}^2}{2R_x} + \frac{\bar{y}^2}{2R_y}))$$

$$\bar{x} = x - x_0$$

$$\bar{y} = y - y_0$$

This is for the simplification of the analysis, and may not be accurate.

3 Overview

In this section, the general characteristics are shown using fields in a coupled cavity formed by the SRC and the y arm. An arbitrarily chosen angle 2° is used.

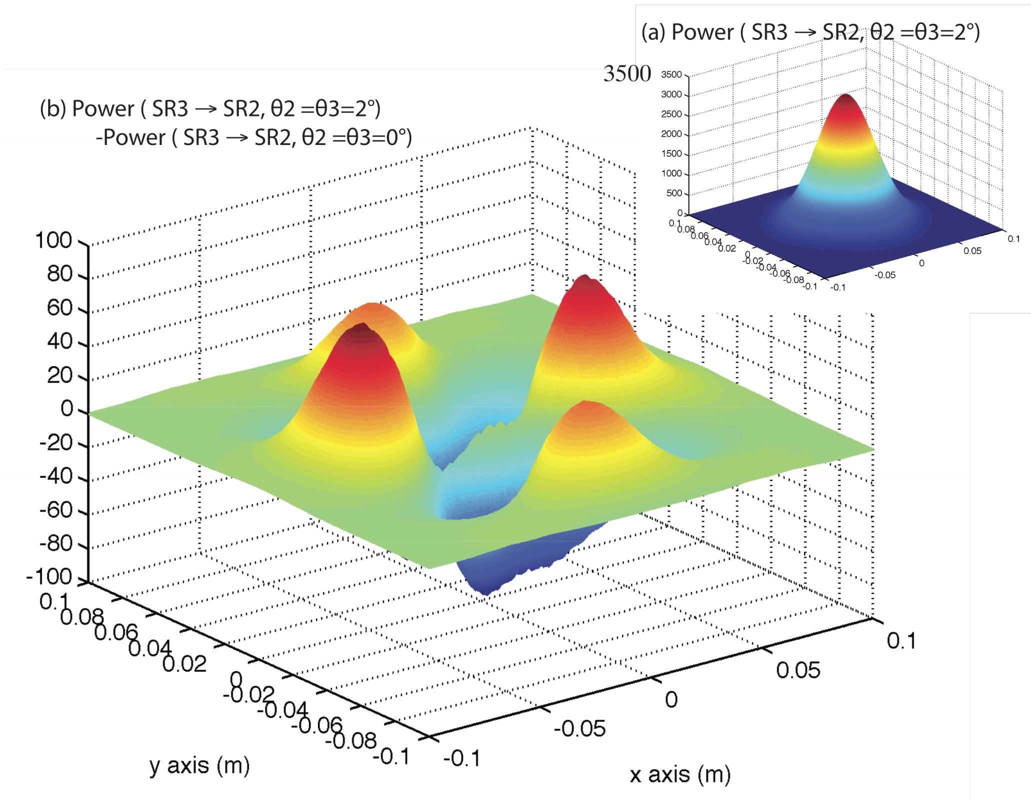


Figure 2 Power profile of E(R3->R2) with astigmatism

Figure 2 shows the power distribution of the field from SR3 to SR2 (green arrow in Figure 1). Fig.(a) is the power distribution with (θ_{PR2}, θ_{PR3}) = (2°, 2°). Fig.(b) shows the difference of the power distribution in a cavity with finite AOIs, i.e., Fig.(a), and that in a cavity with normal incidence, i.e., (θ_{PR2}, θ_{PR3}) = (0°, 0°), whose astigmatism is much smaller. This part is the loss of the signal.

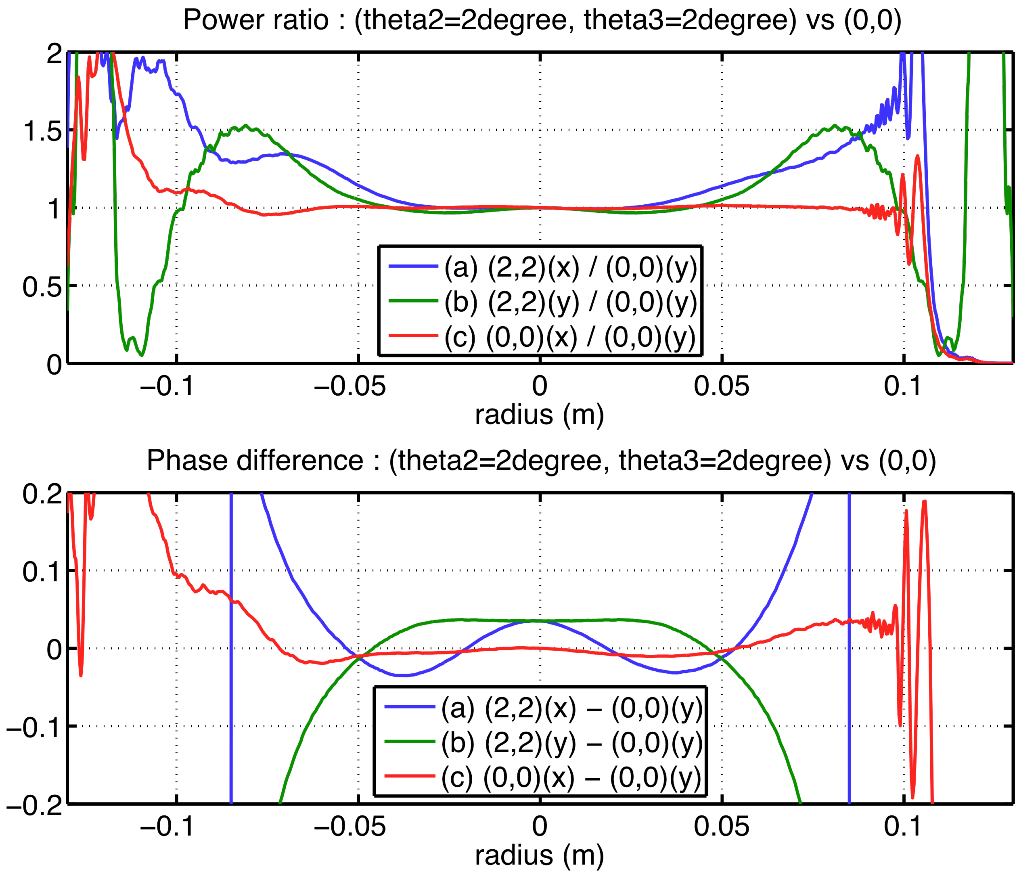


Figure 3 Power and phase distortion by astigmatism

Figure 3 shows the astigmatism of the field, SR3 to SR2, quantitatively. Because the BS is rotated around the vertical axis, field(0,0)(y) is least astigmatic, so other distributions are compared to (0,0)(y).

The top plot is the ratio of power distributions. If the power distribution is of a perfect Gaussian shape, the ratio is given as

$$P_1(r)/P_2(r) = A \cdot \exp(-2(\frac{1}{w_1^2} - \frac{1}{w_2^2})r^2)$$

The red line compares the distribution along the x axis in a cavity with zero AOI, normalized by that along y axis. If there is no astigmatism at all, the red line should be flat. The deviation at radius larger than 10cm is due to the finite BS size, and the difference in the positive x and negative x comes from the asymmetry of the effective intersection on the HR surface (red line on the BS surface in Figure 1).

The blue line, the power distribution in the plane of a cavity with finite AOI, normalized by the reference distribution, Power(0,0)(y), becomes larger as the radius goes larger, so the beam size in the cavity plane is larger with finite AOI.

The green line, the same distribution vertical to the cavity plane, is almost flat up to 5cm, goes up after that. So the effective beam size at the center is almost the same as the reference beam size, but becomes larger as the radius becomes larger.

The bottom plot in Figure 3 shows the phase of the field, relative to the reference field, (0,0)(y). The difference of phases of two fields with effective beam curvature of R_1 and R_2 is give by

$$\phi_1 - \phi_2 = -\frac{k}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \cdot r^2 = \frac{k}{2} \cdot \frac{|R_2| - |R_1|}{R_1 R_2} \cdot r^2$$

The curvature of this field E(R3→R2) is negative, because it is converging. From the radius dependence of the blue line, the curvature of this field, in a cavity with finite AOI, has larger curvature in the cavity plane than the reference case, i.e., |R((2,2)(x))| > |R((0,0)(y))|, in the central region, while the relation reverses in the outer region. The curvature in the vertical direction, R((2,2)(y)) is almost the same as the reference curvature, R((0,0)(y)), in the central region, but the absolute value of the curvature becomes larger in the outer region.

4 Quantitative analysis

The beam size on SRM is 5.4cm, and the effects of the astigmatism seen in Figure 3 affects the signal. In this section, the effect and the beam profile are discussed quantitatively.

Figure 4 shows various phases of the field, SR3 to SR2, under different conditions. These are phases after subtracting the phase of (0,0)(y), i.e., the curvature of the field suffering least astigmatism.

The blue dashed line, marked as "P(0,0)*cos locked", is the phase in a cavity with ROCs of SR3 and SR2 are multiplied by cos(2°) and AOI is zero. The red dashed line is the one with ROCs divided by cos(2°). The cavity is locked with these ROCs. The net effect making the curvature smaller (larger) by multiplying (dividing by) cos(2°) is to make the beam curvature smaller (larger). This is consistent with analytics calculation.

The blue and red solid lines are the same ones in the phase plot in Figure 3, i.e., beam phase in the x and y plane with finite AOI. When a field is reflected by a mirror with curvature of R at finite AOI, θ, the effective mirror curvature in the sagital plane is R*cos(θ) and that in the tangential plane is R/cos(θ). Naively speaking, solid lines and dashed lines are expected to match, but they are quite different.

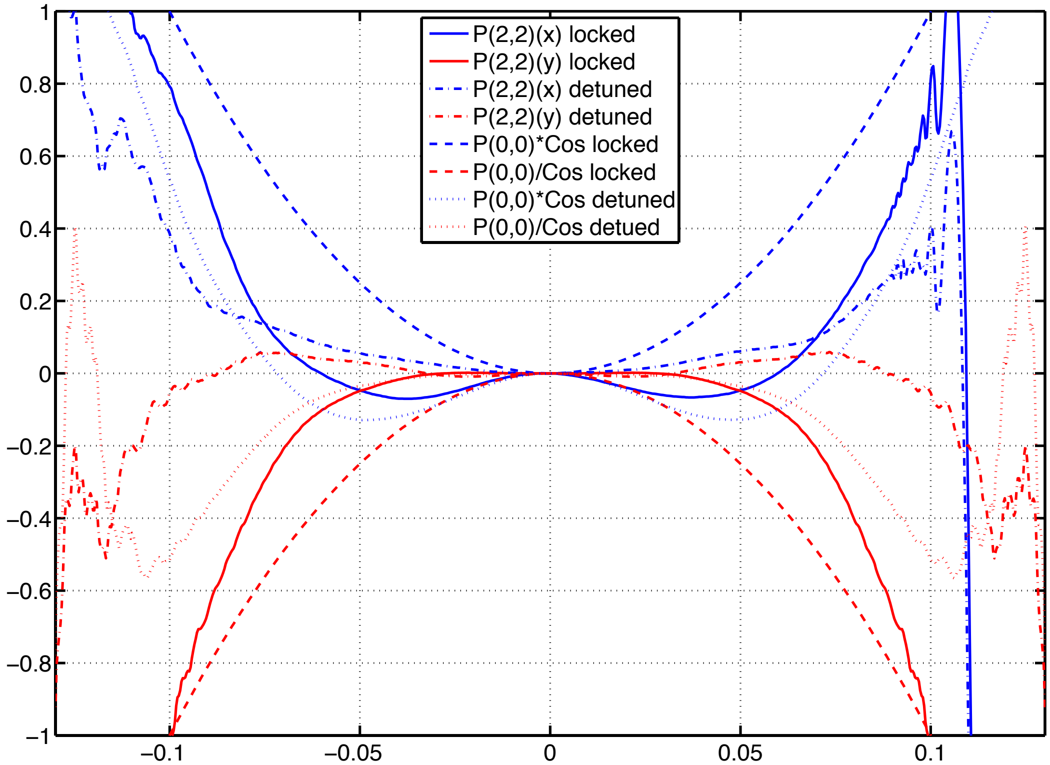
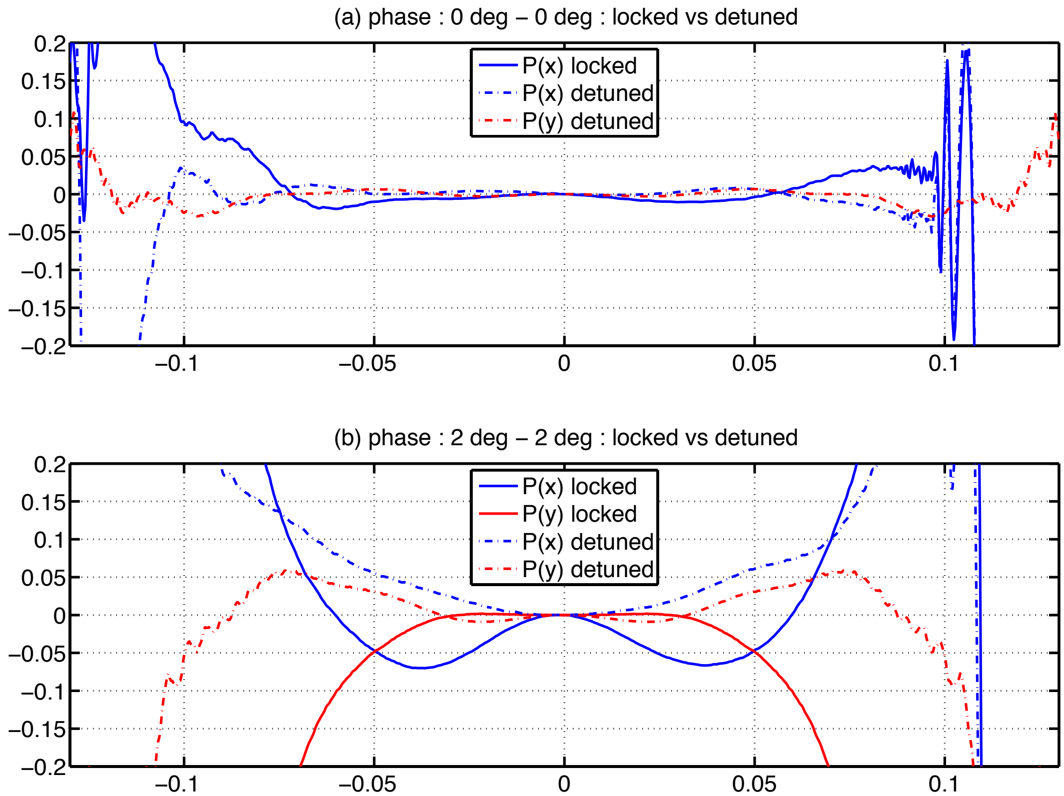
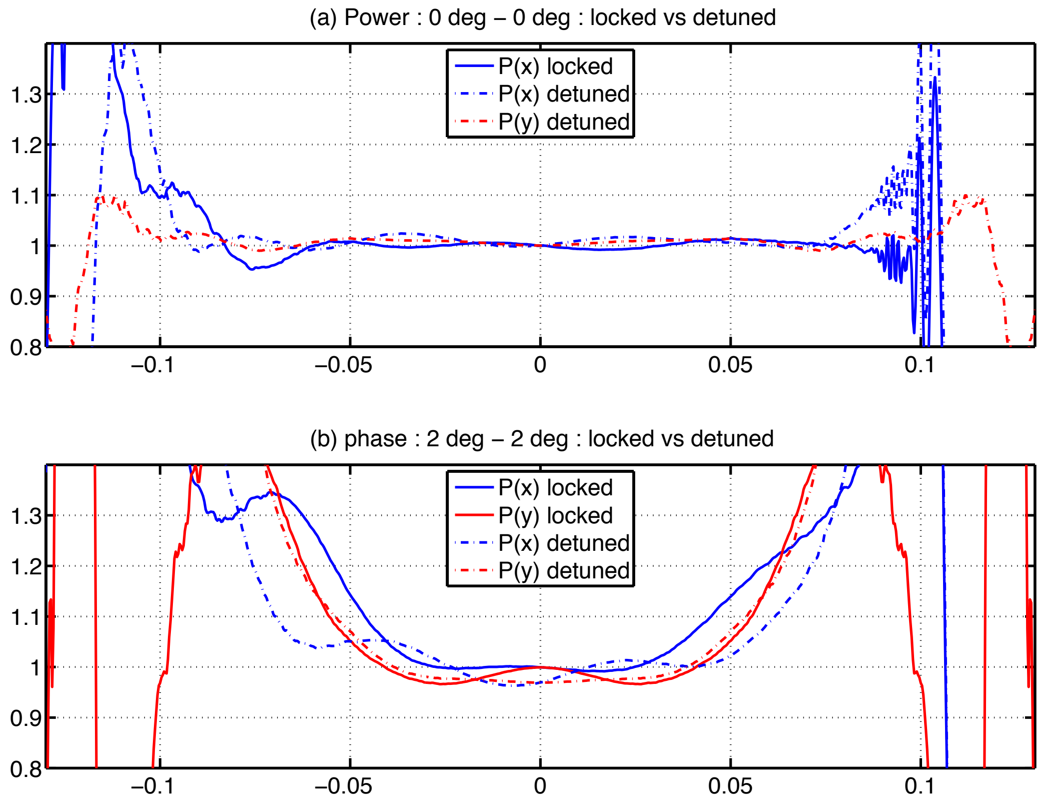
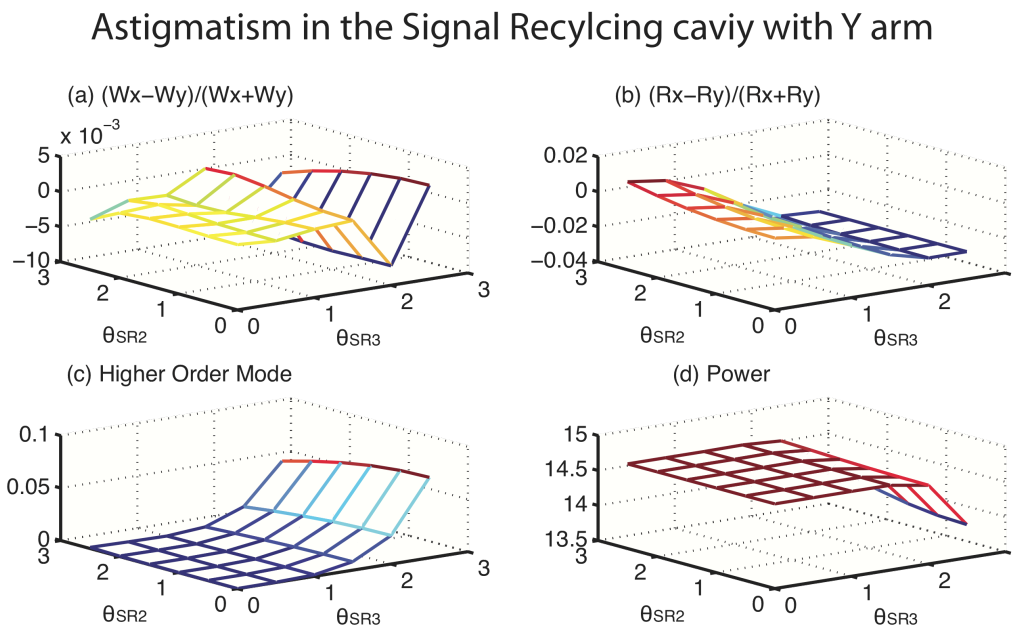
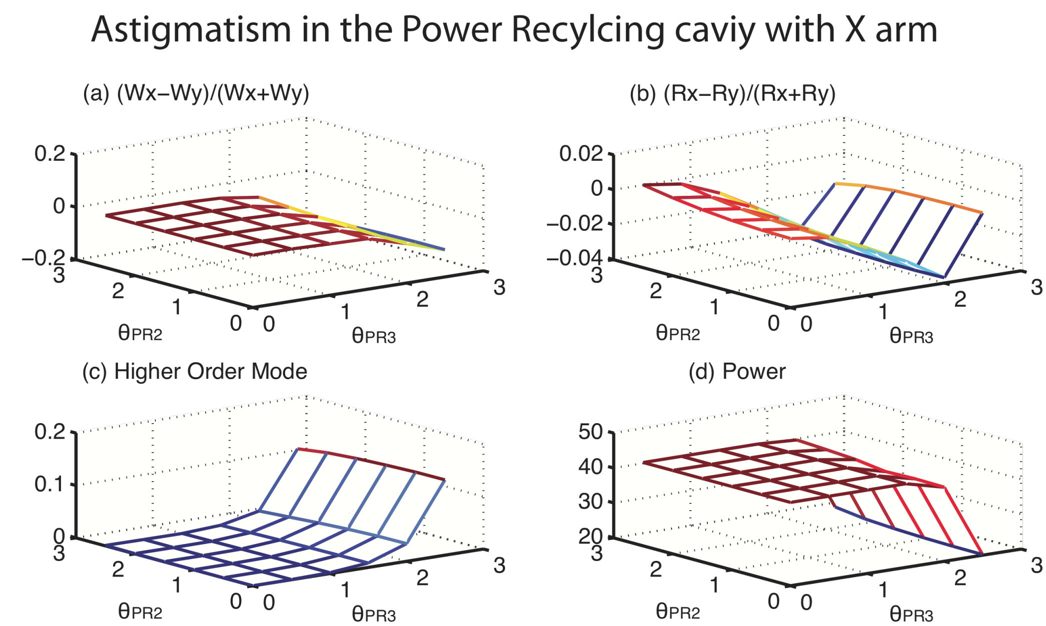


Figure 4 Comparison of phases of field SRM3 to SRM2

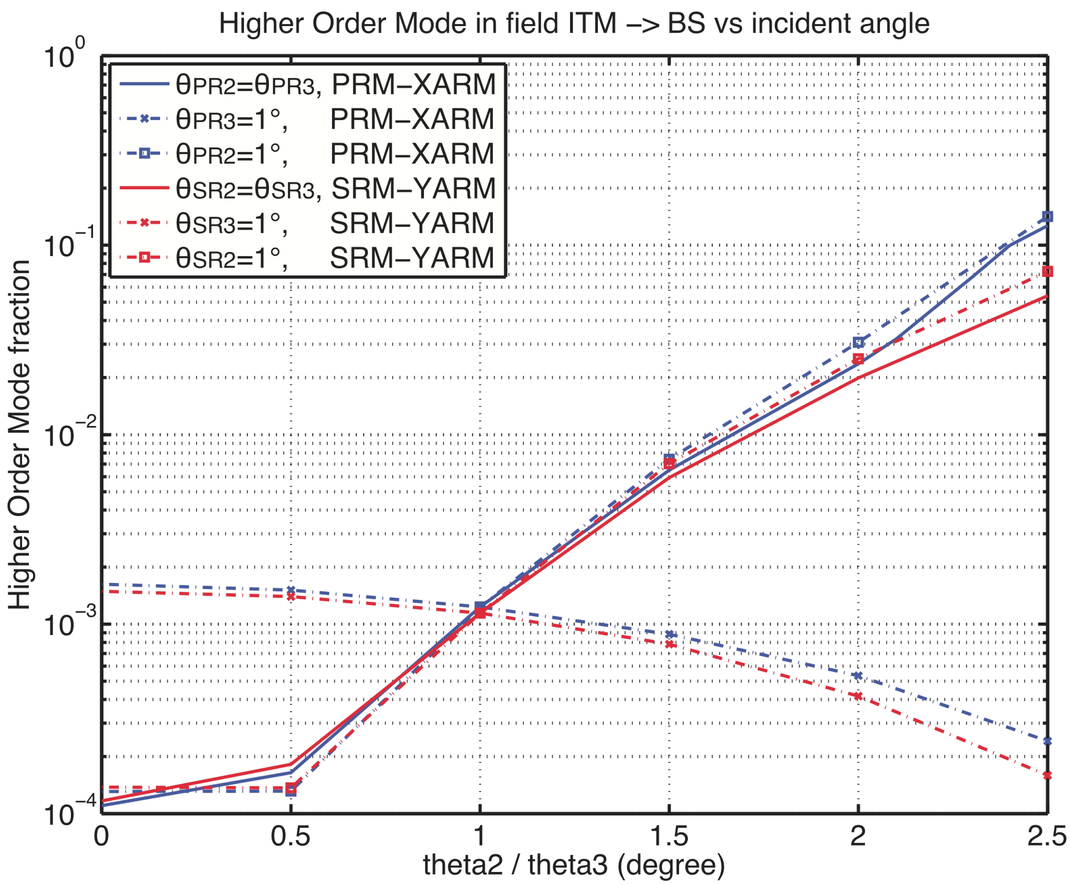

(a) phase : 0 deg – 0 deg : locked vs detuned
P(x) locked
P(x) detuned
P(y) detuned
0.2
0.15
0.1
0.05
0
−0.05
−0.1
−0.15
−0.2
−0.1
−0.05
0
0.05
0.1
(b) phase : 2 deg – 2 deg : locked vs detuned
P(x) locked
P(y) locked
P(x) detuned
P(y) detuned
0.2
0.15
0.1
0.05
0
−0.05
−0.1
−0.15
−0.2
−0.1
−0.05
0
0.05
0.1


(a) Power : 0 deg – 0 deg : locked vs detuned
P(x) locked
P(x) detuned
P(y) detuned
1.3
1.2
1.1
1
0.9
0.8
−0.1
−0.05
0
0.05
0.1
(b) phase : 2 deg – 2 deg : locked vs detuned
P(x) locked
P(y) locked
P(x) detuned
P(y) detuned
1.3
1.2
1.1
1
0.9
0.8
−0.1
−0.05
0
0.05
0.1


Astigmatism in the Signal Recylcing caviy with Y arm
(a) (Wx−Wy)/(Wx+Wy)
x 10^−3
(b) (Rx−Ry)/(Rx+Ry)
(c) Higher Order Mode
(d) Power
θSR2
θSR3


Astigmatism in the Power Recylcing caviy with X arm
(a) (Wx−Wy)/(Wx+Wy)
(b) (Rx−Ry)/(Rx+Ry)
(c) Higher Order Mode
(d) Power
θPR2
θPR3

5



6 Parameters

Cavity parameters from ref. 3.

Power recycling cavity gouy phase = 25 degree

Signal recycling cavity gouy phase = 20 degree

```
Arm length = 3994.75 m

ETM ROC = 2245 m
ITM ROC = 1934 m

ITM thickness = 0.20 m
Compensation plate thickness = 0.13 m
bevel = 0.002 m

space between CP and ITM = 0.005 m

extra arm loss = 70 ppm
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ETM transmittance = 5ppm
ITM transmittance = 0.014

ETM and ITM reflectance = 1 – transmittance – loss / 2

SRM transmittance = 0.20
PRM transmittance = 0.03

No Wedge

(D_A_B means distance between mirror A and B)

Power recycling cavity
D_PRM_PR2 = 16.6129
D_PR2_PR3 = 16.1709
D_PR3_BS  = 19.511
D_BS_PCP  = 4.8526

PRM ROC = -11.268
PR2 ROC = -4.5160
PR3 ROC = 36

Signal recycling cavity
D_SRM_SR2 = 15.727
D_SR2_SR3 = 15.4606
D_SR3_BS  = 19.3645
D_BS_SCP  = 4.8073

SRM ROC = -5.3507
SR2 ROC = -6.445
SR3 ROC = 36
```

7 References

1. T070039, SIS (Stationary Interferometer Simulation) Manual
2. T0900331, SIS Physics : Reflection by curved mirror with arbitrary incident angle
3. T0900043-0x, Optical Layout and Parameters for the Advanced LIGO Cavities