

LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY  
- LIGO -  
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Technical Note	LIGO-T0900385-v06	2010/03/31
<h1>The Advanced LIGO ETM transmission monitor</h1>		
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# 1 Introduction

Here we describe the Transmon optical layout. The Transmon assembly is an in vacuum optics table suspended immediately behind the End Test Mass (ETM) of each interferometer arm. The Transmon incorporates a telescope for reducing the large diameter arm cavity beam to a manageable size, beam steering optics, and in-vacuum QPDs. During science mode, all infrared beams transmitted through the ETM are dumped on the Transmon table. During lock acquisition the Transmon relays the interferometer beam to an in-air optics table. Finally, the Transmon table provides the input optics and alignment reference for the green beams of the Lock Acquisition Interferometer (LAI) and the Hartmann wavefront sensor.

In the following sections, we describe the overall layout of the Transmon table, calculate the beam reducing telescope parameters, calculate the sensitivity of the QPDs with an ideal telescope, evaluate the effects of astigmatism, determine the layout tolerances, and describe alignment procedures for each of the subsystems. We conclude with a parts list for the optical table.

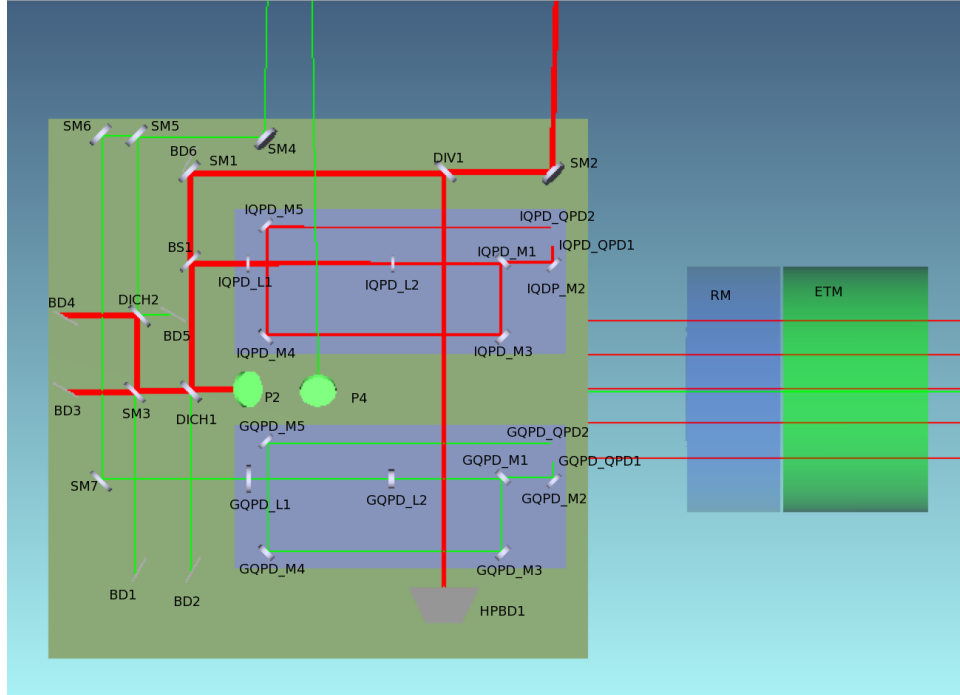


Figure 1: Annotated Transmon top view.

## 2 Overall layout

The Transmon table is designed as a single plate with the folded beam reducing telescope mounted below the table and the ISC optics mounted above. The layout is shown in Figs. 1 and 2. The red beam traces the infrared beam from the arm cavity, through the ETM and reaction mass (here labeled “RM”), and into the telescope. The primary role of the Transmon

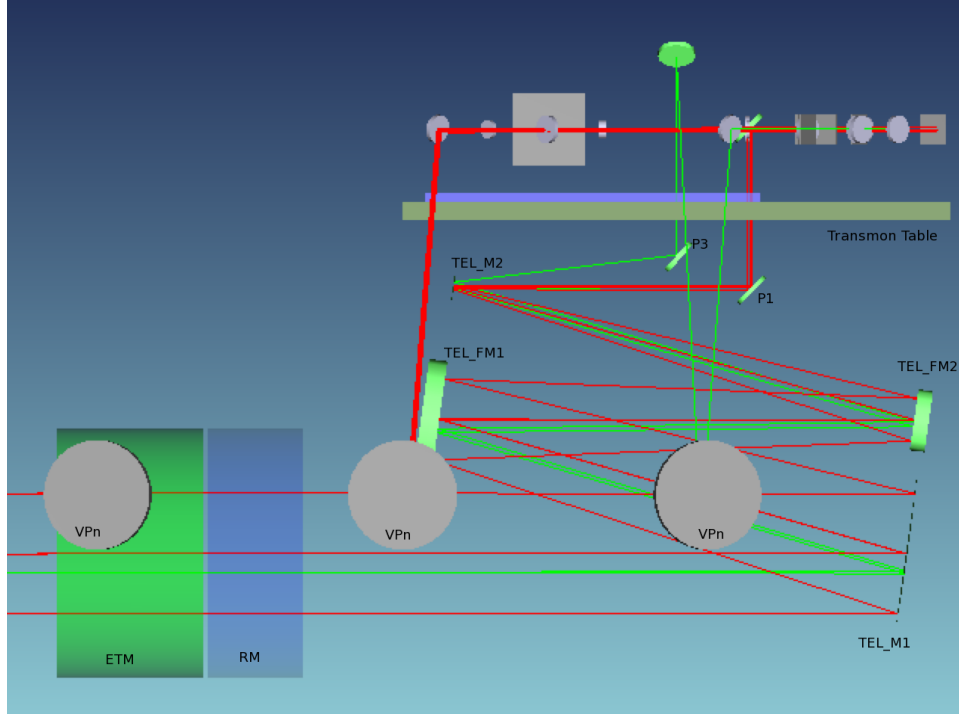


Figure 2: Transmon side view.

table is to sense the position and angle of this IR beam using two quadrant photodiodes. In Fig. 1, the diodes are mounted on the purple QPD breadboard with a Gouy telescope that sets the angle and position sensitivity of the two QPDs and labeled “IQPD\_QPD1” and “IQPD\_QPD2”. The QPDs are arranged symmetrically around a focus to set the Gouy phase difference at  $90^\circ$  and minimize sensitivity to length errors. Together, the two QPDs are sensitive to the arm cavity waist position and angle. During lock acquisition, the Transmon table delivers the beam through a viewport to an in-air table with high gain sensors. The viewport beam selection and beams are described in the viewport interface document Ref. [4]. During science mode, the IR beam is diverted to a beam dump using an actuated beam diverter (labeled “DIV1” in Fig. 1 and drawn as a beam splitter) .

The green beams shown in the layouts are the auxiliary 532 nm beam used for the Lock Acquisition Interferometer (LAI) and the Hartmann sensors. The beam is injected from a viewport (the middle viewport off the bottom of Fig. 1) and is used to lock the arm cavity using a PDH lock. The beam is aligned to the Transmon table (and hence the arm cavity) using a second QPD breadboard, shown by the lower purple rectangle in Fig. 1. The beam is then overlapped with the IR beam on a dichroic beam splitter (labeled “DICH1”) and sent through the beam reducing telescope. The beam retro-reflected off the ETM is returned to the in-air table through the entrance viewport.

The green beam incident on the ETM is partially reflected off the ETM AR surface which is oriented with a vertical wedge of  $0.07^\circ$ . Note that the vertical wedge is specified to have a maximum wedge less than  $0.1^\circ$ . This “Hartmann reference beam” emerges from the beam reducing telescope at a large vertical angle as shown in Fig. 2 and is picked up by a second periscope and delivered to the top of the table. This reference beam is then delivered to a

Optic	Radius of curvature [mm]
Telescope parabolic T1	4000
Telescope parabolic T2	-200
Gouy tel. plano-convex L1, f 343.6	154.5, (4 mm thick)
Gouy tel. plano-concave L2, f -114	-51.5, (2 mm thick)

Table 1: Transmon optics and their radii of curvature.

viewport here shown as the rightmost viewport in Fig. 2 but which could also be the central viewport.

The Transmon table size is 85 cm long and 70 cm wide. The thickness is not here specified, but should be sufficient to preserve the table alignment during assembly, testing and installation. The beam reducing telescope has a 20x magnification and consists of an positive (converging) off-axis parabolic primary and a negative (diverging) off-axis parabolic secondary. The telescope is folded to fit the Transmon table and has a total length of approximately 2 m. Each of the QPD breadboards is 50 cm long by 25 cm wide. The Transmon optics have been modeled using the Zemax physical optics propagation, using Mathematica models, and using Matlab ABCD matrices. For consistency and clarity, the physical lengths stated in the following are based on the Matlab model in *TransmonTable.m*, listed in §A. The optics used in the model are listed in Table 1. The matlab model treats the off-axis parabolic mirror as standard on-axis mirrors and the plano-convex lenses as an ideal thin lens. The length between the BRT output and Gouy telescope input is arbitrarily chosen. The lengths are quoted accordingly in Table 2.

Description	length [mm]
Telescope M1 to M2	1902.6
Telescope M2 to Lens L1	1097.4
Lens L1 to Lens L2	246.7
Lens L2 to QPD1	437
QPD1 to QPD2	369

Table 2: The optic positions as modeled in TransmonTable.m and used in the following calculations.

### 3 Telescope parameters

The nominal Advanced LIGO arm cavity consists of an ITM with a radius of curvature (ROC) of 1934 m and an ETM with a radius of curvature of 2245 m.[1]. The resulting arm cavity waist radius is  $\omega_0 = 1.2$  cm located 2160 m in front of the ETM. On its way to the BRT, the beam passes through the ETM which acts like a diverging lens with -5.0 km focal length. As a result, the beam at the telescope is described by a waist  $\omega_{in} = 8.4$  mm located 1520 m in front of the telescope. At the telescope, the beam waist is 6.2 cm. The complex beam parameter at the telescope input is

$$q_{in} = 1522 + 207.3i. \quad (1)$$

The beam at the output of the BRT should have a Rayleigh range of  $\geq 10$  m to minimize the effects of beam divergence between the Transmon table and the in-air table. We choose a BRT magnification of 20x, such that the BRT output beam has a waist of  $\simeq 3$  mm and a Rayleigh range of 28 m. The input beam at the BRT has a divergence angle of  $40 \mu\text{rad}$ . After an infinity adjusted telescope, the divergence angle would be 1 mrad and the beam would grow significantly during transport. Consequently, the telescope must be focusing so that the beam has a waist at the BRT output. In effect, the telescope must have a focal length of 1522 m. The ray transfer matrix for the BRT is:

$$\begin{aligned} M_{BRT} &= \begin{pmatrix} 1 & 0 \\ -2/R_2 & 1 \end{pmatrix} \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -2/R_1 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 - 2L/R_1 & L \\ -2/R_1(1 - 2L/R_2) - 2/R_2 & 1 - 2L/R_2 \end{pmatrix}, \end{aligned} \quad (2)$$

where  $R_1$  and  $R_2$  are the ROC of the primary and secondary, respectively.  $L$  is the distance between the telescope mirrors. To set a waist at the telescope output, the cavity length must satisfy

$$\left[ -\frac{2}{R_1} \left( 1 - 2\frac{L}{R_2} \right) - \frac{2}{R_2} \right] \Re(q_{in}) + 1 - 2\frac{L}{R_2} = 0, \quad (3)$$

$$L = \frac{R_1}{2} \frac{1}{1 - \frac{R_1}{2} \frac{1}{\Re(q_{in})}} + \frac{R_2}{2}, \quad \text{and}$$

$$L \approx \frac{R_1 + R_2}{2} + \frac{R_1^2}{4} \frac{1}{\Re(q_{in})}. \quad (4)$$

The approximation is valid in the limit that  $\Re(q_{in}) \gg R_1$ . For the telescope optics listed in Table 1, the telescope length is  $L = 1902.6 \pm 0.3$  mm. Note that the telescope length changes linearly with changes in the mirrors' ROC. With the telescope set to its nominal focus, the output beam has a waist of 3.1 mm located at the telescope output.

Figure 3 shows the output waist position and size as a function of the telescope length. The waist position is normalized to the Rayleigh range. To create an output waist within one Rayleigh range of the telescope output, the telescope length must be set to  $1902.6 \pm 0.35$  mm, over which range the output waist varies from 2.2 to 3.1 mm. This represents a telescope detuning from infinite conjugate of almost 3 mm. Note that a similar constraint,  $\pm 0.3$  mm is placed on the relative focus for the infrared and green beams.

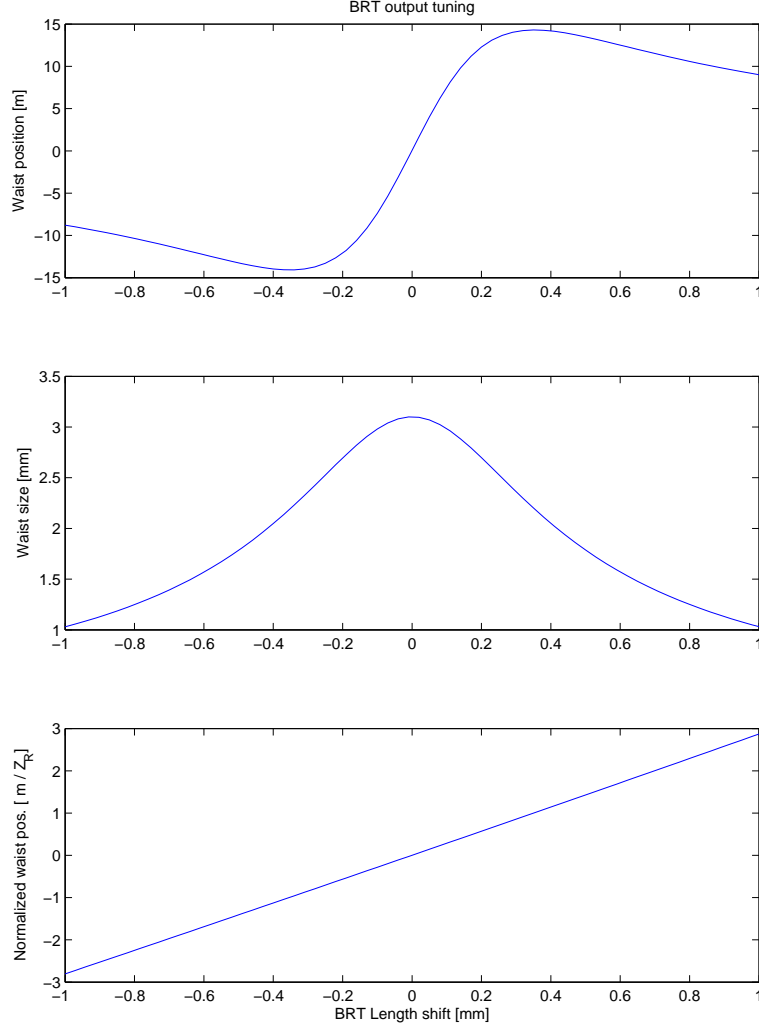


Figure 3: Variation of the output waist position and radius as a function of the telescope length. The calculation assumes perfect optics.

## 4 Gouy telescope

The Transmon table has two QPDs to sense the displacement and angle of the arm cavity. Naively the detectors would be set in the near field and the far field. Unfortunately, this configuration maximizes the sensitivity to the BRT length and ROC errors. More generally, the two QPDs should be  $90^\circ$  of Gouy phase apart. This condition is most robustly met by setting the QPDs at  $\pm z_R$  from a beam focus. The focused waist sets the QPD optical gain and should be matched to the 3 mm QPD diameter. Obviously, the waist must be located more than a Rayleigh range after the telescope. We choose an output waist size of  $\omega_{gouy} \approx 250 \mu\text{m}$  which has a Rayleigh range of 0.18 m, well matched to the size of the Transmon optical table.

To design the Gouy telescope, we assume a telescope formed with standard lenses from CVI having focal lengths of 343 mm and -114 mm. Such a telescope produces an appropriate output beam at an appropriate distance from the output. We find the telescope spacing using

the Matlab package ALM. Finally, we adjust the telescope focus and the QPD positions to maximize the range of BRT lengths that provide a good error signal for the horizontal and vertical DOFs.

The optimum Gouy telescope configuration has a lens separation of 247 mm, a lens to QPD1 spacing of 437 mm, and a spacing between the QPDs of 369 mm. Fig. 4 shows the position of the optics and the waist size along the beam path. Also shown is the spacing from the telescope output to the first QPD, assuming that QPD is set one Rayleigh length from the waist. The spot size at the QPDs will be  $350 \mu\text{m}$ .

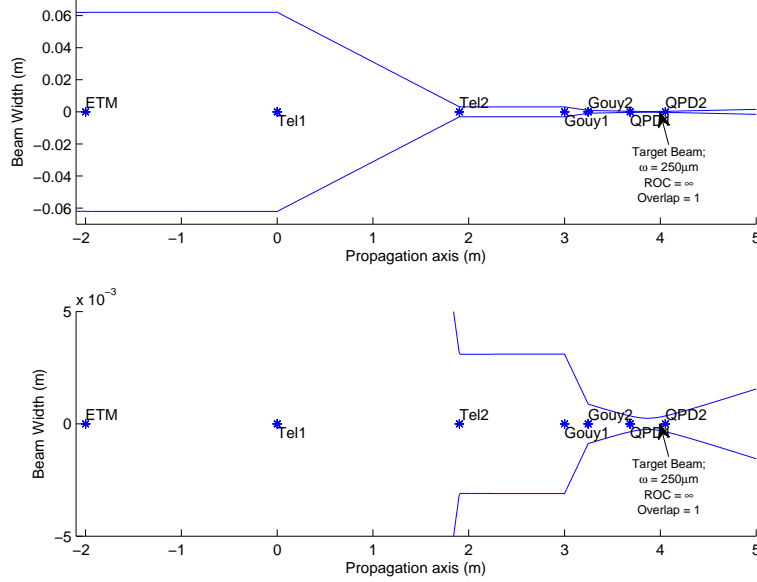


Figure 4: The beam waist as it propagates through the telescope and Gouy optics to the QPDs.

## 5 QPD sensitivity

Assuming an ideal beam reducing telescope (ie. one without astigmatism), the signals measured on each QPD for the cavity displacement and angle are:

$$\begin{pmatrix} QPD_1 \\ QPD_2 \end{pmatrix} = \begin{pmatrix} -43 & 47 \times 10^3 \\ -110 & -18 \times 10^3 \end{pmatrix} \begin{pmatrix} x \\ \theta \end{pmatrix}_{cavity}, \quad \text{and} \quad (5)$$

$$\begin{pmatrix} x \\ \theta \end{pmatrix}_{cavity} = \begin{pmatrix} -3.1 \times 10^{-3} & -7.9 \times 10^{-3} \\ 1.8 \times 10^{-5} & -7.3 \times 10^{-6} \end{pmatrix} \begin{pmatrix} QPD_1 \\ QPD_2 \end{pmatrix}. \quad (6)$$

The motion at each QPD,  $QPD_1$  and  $QPD_2$ , is a function of the arm cavity degrees of freedom,  $x_{cavity}$  and  $\theta_{cavity}$ , defined at the cavity waist. The units for the cavity motion are meters and radians, the dimensionless QPD signals have been normalized by the waist size such that  $QPD = x_{QPD}/\omega_{QPD}$ . This matrix has been calculated using the Matlab script TransmonTable.m included in §A.

Similarly, the QPD signals as a function of the table motion are:

$$\begin{pmatrix} QPD_1 \\ QPD_2 \end{pmatrix} = \begin{pmatrix} -71 & 140 \times 10^3 \\ -150 & 220 \times 10^3 \end{pmatrix} \begin{pmatrix} x \\ \theta \end{pmatrix}_{table}. \quad (7)$$

Note that QPD1 is roughly twice as sensitive to table motion, both position and angle, as it is to cavity motion. QPD2 is slightly more sensitive to table position and much more sensitive to table angle than it is to the cavity motion. These signals are not orthogonal; in fact, the two QPDs respond almost identically to motion of the table, up to a relative gain. Together,

$$\begin{pmatrix} x \\ \theta \end{pmatrix}_{cavity} = \begin{pmatrix} 1.4 & -2200 \\ -2.0 \times 10^{-4} & 1.0 \end{pmatrix} \begin{pmatrix} x \\ \theta \end{pmatrix}_{table}. \quad (8)$$

The very large and very small off-diagonal terms are a function of the arm cavity length; the 2 km between the arm cavity waist and the telescope sets the displacement to angle coupling.

## 6 Suspension requirements

The QPD sensitivities from §5, the cavity motion predicted from Ref. [2], and the ISI table displacement predicted from Ref. [3] can be combined to place a requirement on the Transmon telescope suspension. The predicted ETM pitch motion, together with the corresponding cavity waist displacement and angle are shown in Fig. 5. The displacement and angle are the signals we must detect with the QPDs.

Several noise sources contribute to QPD signals independent of the cavity motion. Motion of the ISI seismic isolation table dominates the QPD signals at low frequency with shot noise contributing above 10 Hz. Fig. 6 shows all of the predicted contributions to the QPD signals from the cavity and the ISI table. Above 10 Hz, the QPD signal is totally dominated by the ISI table's angular motion. Below 10 Hz, the table motion is closer to the cavity noise than desired.

These curves can be used to place a requirement on the Transmon suspension. Namely, we required the apparent telescope motion to be 5 times less than the smallest of the cavity signals, QPD1 displacement, and 10 times less than shot noise. This required motion is compared to the predicted ISI motion in Fig. 7. Not shown in the figure is the shot noise limited QPD signal at  $1 \times 10^{-15}$  above 20 Hz. This curve places a requirement on the suspension of roughly a factor of 1000 above 10 Hz, and a factor of a few between 1 and 10 Hz.



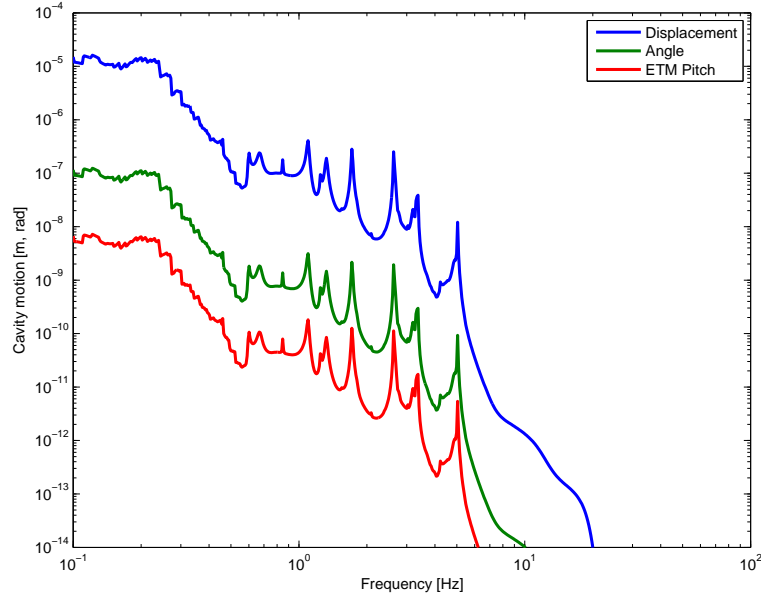


Figure 5: The predicted ETM pitch motion, together with the corresponding cavity waist displacement and angle.

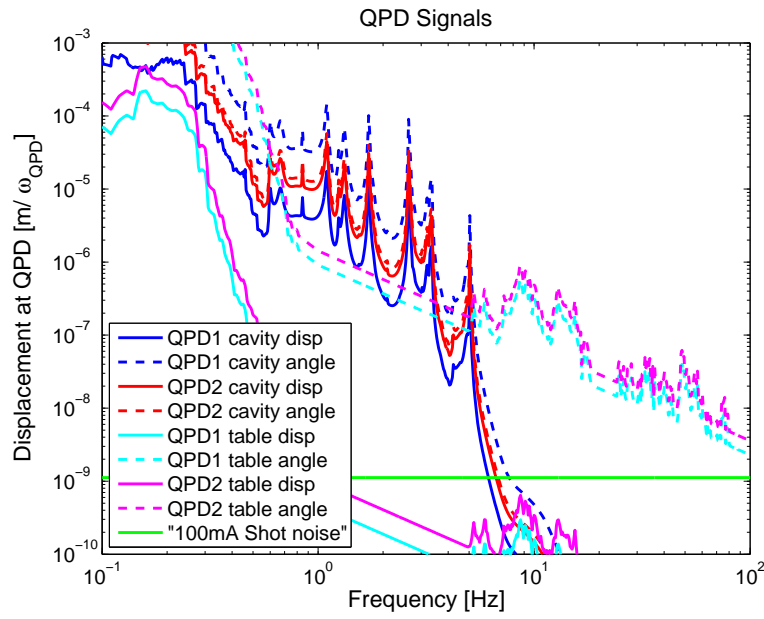


Figure 6: Contributions to the QPD signals.

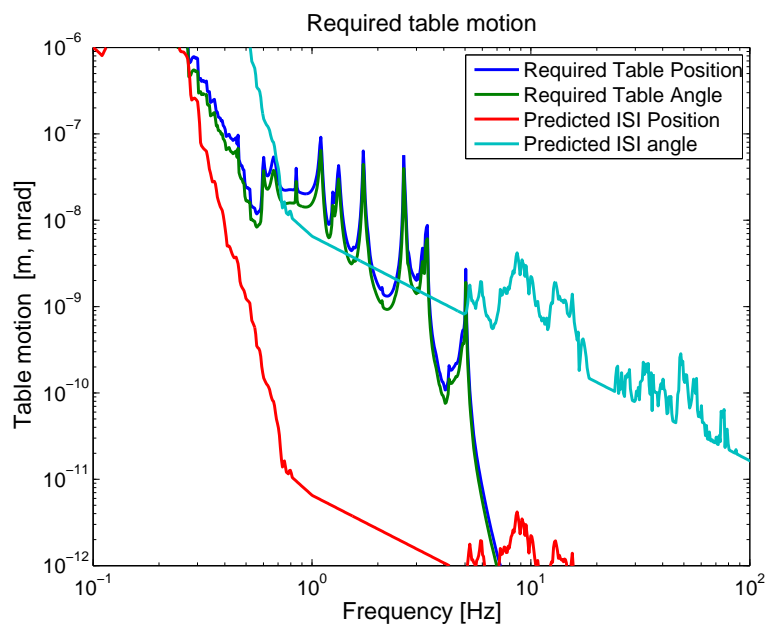


Figure 7: The required telescope motion compared with the predicted ISI table motion

## 7 Setup tolerances

The Transmon table signals are sensitive to both the Gouy phase difference at the two QPDs and the spot size at the QPDs, which varies the optical gain. This section discusses the alignment tolerance required to get a Gouy phase difference of  $\delta\phi_G = 90 \pm 20^\circ$  at the QPDs. The exact matrix is not relevant to the setup since it will be measured *in situ*. The variations of the matrix once installed are dealt with in §8.

There are only two sensitive lengths within the transmon assembly, the BRT length and the Gouy telescope. The Gouy phase is shown in Fig. 8, assuming a shift from the nominal, ideal tuning described above. The Rayleigh range of the beam in the BRT is smaller than the Gouy telescope ( $350 \mu m$  vs.  $4.2 mm$ ), with a corresponding increase in sensitivity. The setup tolerance for the two telescopes is:

$$\begin{aligned}\delta z_{BRT} &= -0.5, +0.2 \text{ mm} \\ \delta z_{gouy} &= -6.0, +2.1 \text{ mm}\end{aligned}\tag{9}$$

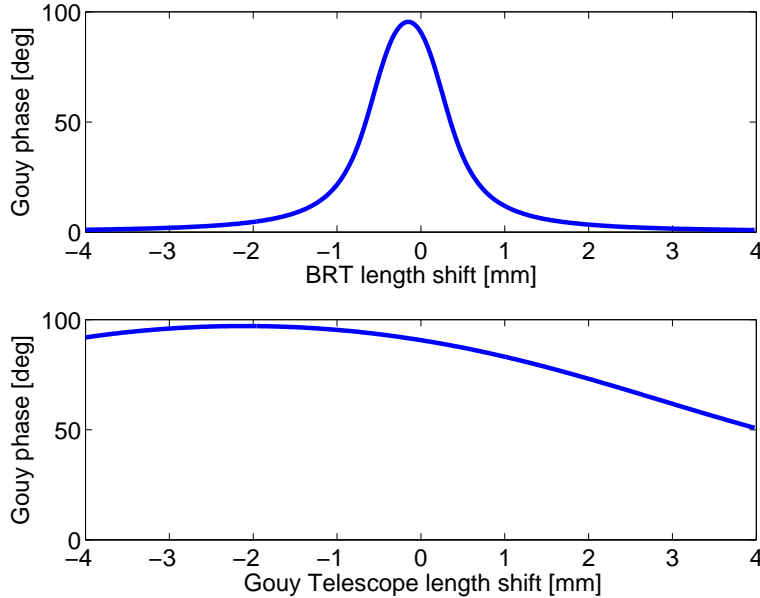


Figure 8: The setup length tolerance required for the Beam Reducing Telescope and the Gouy phase telescope to maintain a gouy phase difference of  $90 \pm 20^\circ$  between the QPDs.

If anything, the output beam focusing places a tighter requirement on the BRT focus than the Gouy phase. As shown in Fig. 3, the tolerances of Eq. 9 correspond to a change in output waist size from 3.8 mm to 2.4 mm and a shift in focus position of as much as 20 m. These output beam changes are mainly relevant for the in-air optics and mode matching and don't affect the performance of the Transmon table itself. Consequently, they are not used to set a tolerance *requirement*, but we can suggest that a best effort should be made for  $|\delta z_{BRT}| \leq 100 \mu m$ .

## 8 Length variation

Once the telescope lengths have been set, we can measure the cavity to QPD transfer matrix *in situ* and diagonalize the displacement and angle signals accordingly. Because the cavity angular motion is expected to be larger than the displacement by a factor of 30, we have to take care that the diagonalization remains stable.<sup>1</sup> In order to avoid polluting the displacement signal with angle, the off-diagonal angle to displacement matrix element must be a small fraction, 1% or less, of the displacement to displacement element. The off-diagonal element has units of  $m/rad$ , so we scale the term by the factor  $\theta_{div}/\omega_0$ .

As with the setup alignment, the only sensitive lengths are the BRT length and the Gouy telescope length. To evaluate the acceptable variation, we apply the QPD to arm matrix from Eq. 6 to the arm to QPD matrix from a Transmon model with variations in the telescope lengths. The results are shown in Fig. 9. At the nominal position, the sensitivity is 1% per  $3\mu m$  BRT motion and 1% per  $44\mu m$  of Gouy telescope motion.

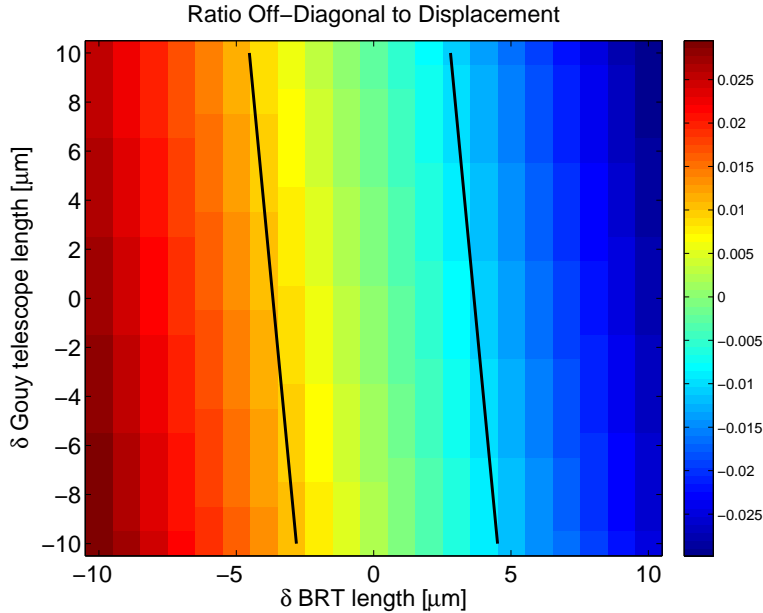


Figure 9: The variation in the transmon sensitivity of angle to length as a function of the telescope lengths. The 1% contour is marked with the solid black contour. To keep the angle to length coupling to less than 1%, the BRT length must be held within  $\pm 3\mu m$ .

If we assume the length variation arises from the thermal expansion of the telescope, the sensitivity specification can be turned into an environmental temperature requirement. Assuming Invar has a temperature coefficient of  $1.2 \times 10^{-6}/K$ , the telescope temperature change must be  $\delta T \leq 1.2C$ . Another source of variation is the primary mirror itself which has a lossy mirror reflecting a 5 W beam. Even a modest absorption of 1% will deposit 50 mW into the mirror substrate and could lead to a lensing and length change. Finally, the arm cavity to QPD beam path will be affected, perhaps even dominated, by the thermal lens

<sup>1</sup>Based on modeling by L. Barsotti

generated in the ETM itself, independent of the transmon. Consequently, the arm to QPD matrix element should be measured after the thermal state has equilibrated.

## 9 Required apertures

The Matlab code *TransmonTable.m* calculates the ray tracing matrices from the table motion to each of the optics listed in Table 1. Recall that the OAP primary mirror has a clear aperture of 190 mm for the input 62 mm beam width, a clear aperture 3 times larger in diameter than the beam width. In order to make this the limiting aperture, the following optics must have a clear diameter at least 3 times larger than the beam width at the optic. For the purpose of this estimate, we consider the required aperture to be 6 times larger. The apertures must also accommodate arm cavity motion equivalent to  $\leq 1$  cm on the ETM. Fig. 10 shows the required aperture as it travels through the Transmon assembly. Prior to the Gouy telescope, the required clear aperture is  $< 1$  inch, implying 2 inch diameter optics required for a 45 degree angle of incidence beamsplitter.

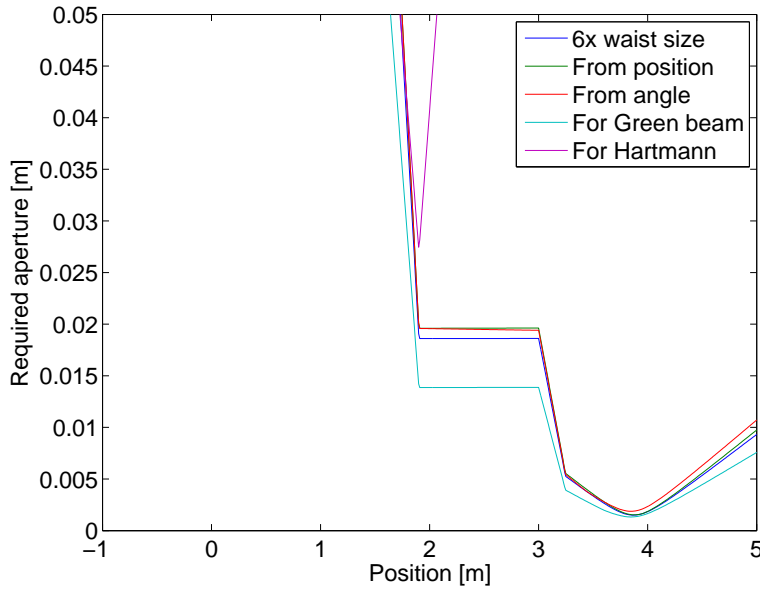


Figure 10: The required apertures through the Transmon optics.

The required aperture for green is also shown, calculated by scaling the maximum IR aperture by  $\sqrt{2}$  to account for the difference in wavelength. With this calculation, the largest required aperture is 1.7 cm, close to the clear aperture through a 1 inch optic at 45°. Given the less sensitive nature of the green beam, a clear aperture six times larger than the beam width is probably not necessary and 1 inch optics will be sufficient.

Finally, the Hartmann reference beam is a green beam reflected off of the ETM Anti-Reflection surface. The AR surface is wedged by 0.07° in the vertical plane. The input beam is angled by 0.1° with respect to this surface. Thus, the Hartmann reference beam is retro-reflected at an angle of 0.2° from the main beam. Fig. 10 includes the required aperture for this reference beam assuming a  $3 \times \omega$  clearance at all locations.

## 10 Air to vacuum shifts

The index of refraction of air,  $n_{air} - 1 = 2.926 \times 10^{-4}$ , is sufficiently large to change the telescope focus between air and vacuum. The path length change for the 2.1 m telescope is  $610\mu m$ , more than the allowed range set in §7. The in air alignment procedure must account for this factor. In contrast, the Gouy lenses' index shift is well within the tolerance of the Gouy telescope.

## 11 Telescope focusing procedure

It is impractical to generate a test beam matched to the arm cavity to set the telescope focus. Instead, we launch a smaller beam in from the telescope output and retro-reflect it with a flat mirror at the telescope input, as shown schematically in Fig. 11. The  $\approx 1\text{ mm}$  beam from a green laser is beam expanded, injected into the telescope, retro-reflected, and measured after a second beam expander. The ALM model for this configuration is in §B, and the beam path propagation is shown in Fig. 12.

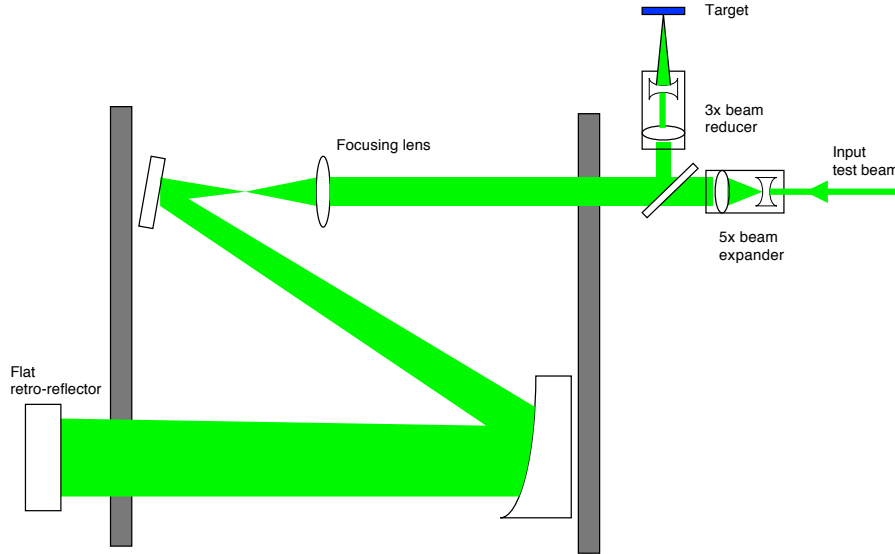


Figure 11: Simplified schematic of the telescope alignment test setup. A small diameter test laser at 532 nm is sent through the telescope from the output and retroreflected at the input.

With this configuration, the output waist position – labeled “target” in Fig. 12 – shifts as a function of the telescope focus. As shown in Fig. 13, the waist position shifts from -5 cm to 17 cm across the range of acceptable telescope focusing. For the  $\approx 100\mu m$  waist of the test setup output, this corresponds to a shift of  $-6$  to  $2$  Rayleigh lengths. To limit the measurement systematics to an error of less than  $100\mu m$  in the BRT length, the output

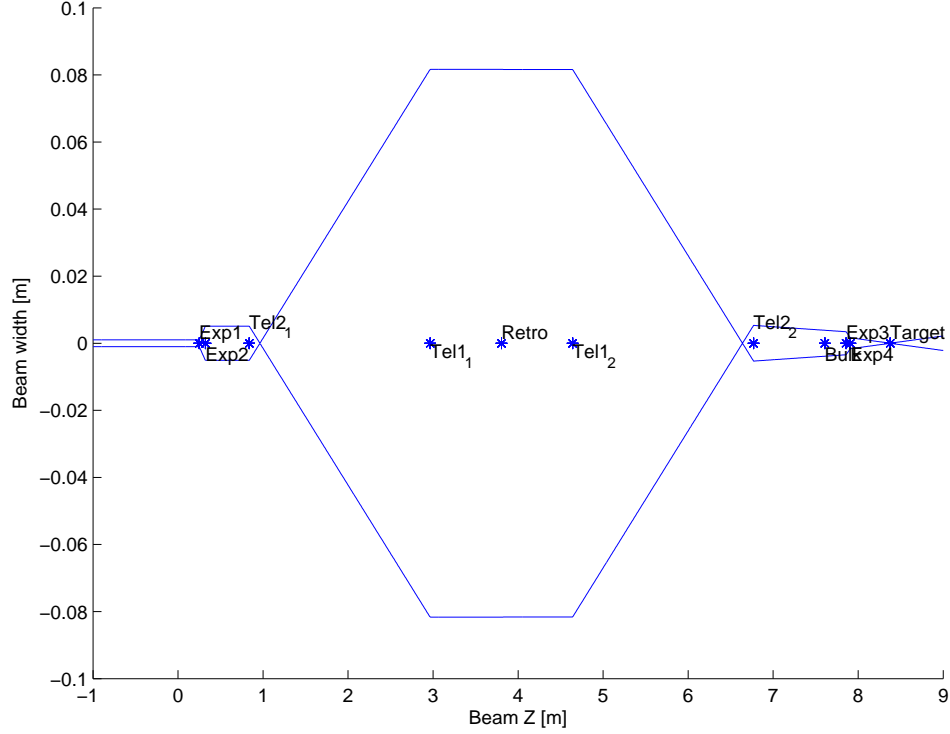


Figure 12: The beam width through the telescope focusing apparatus. A 1 mm radius, 532 nm beam is propagated through a 5:1 beam expander before entering the telescope. A flat retroreflector returns the beam to the secondary to a 2:1 beam reducer for measurement.

waist measurement must be accurate to better than 25 mm. Obviously, the accuracy of the focusing measurement will depend on the optics used, particularly the input laser waist and the beam expanders. Fig. 14 shows the systematic shift in output waist as a function of the test optics. By far the most sensitive optic is the input 5:1 beam expander, followed by the output 2:1 expander. These optics will have to be characterized to ensure measurement accuracy. The measurement is relatively insensitive to errors in the input beam. All of the characterizations and measurements should be accomplished with standard beam scanning hardware, either CCD or scanning slit.

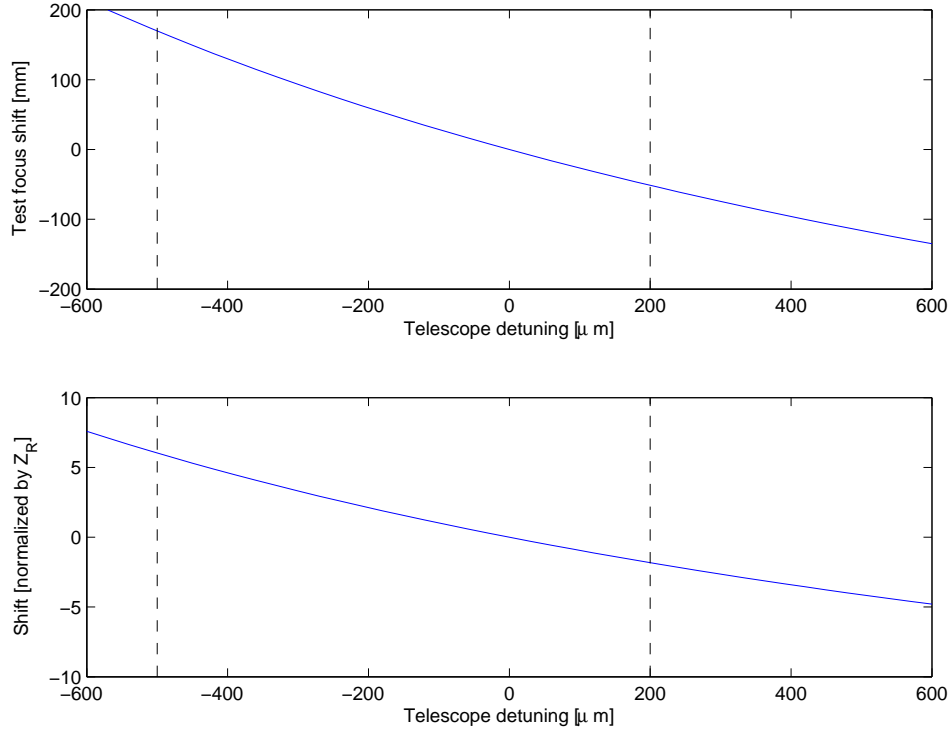


Figure 13: The shift of the test setup output beam as a function of the BRT length. The black vertical lines denote the required sensitivity given by Eq. 9.

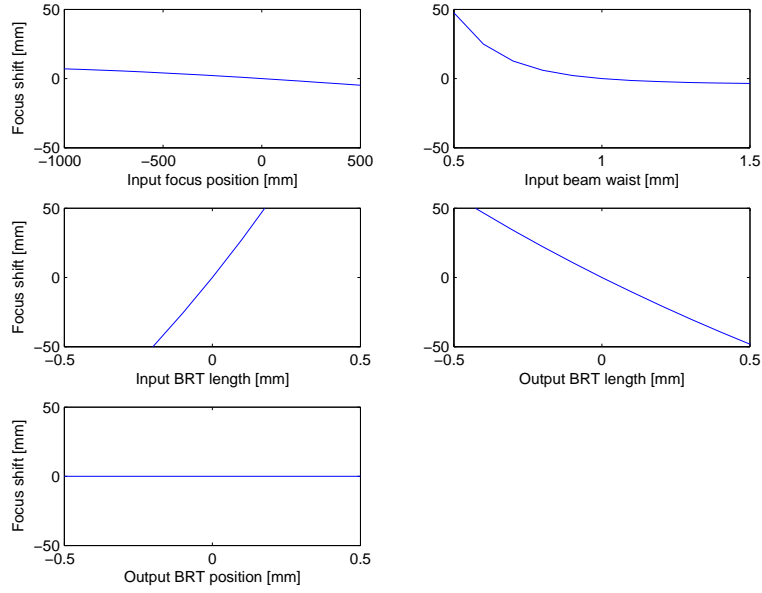


Figure 14: The systematic shifts in the measurement output waist as a result of optic imperfections. The shifts should be less than 25 mm.



## 12 Gouy optics alignment procedure

A similar measurement technique to that described in §11 will be used to align each Gouy telescope and QPD sled. Since the Gouy telescopes are significantly less sensitive than the BRT, we leave this as an exercise to the viewer until the next version of this document.

## 13 Optics and components lists

The optics are listed in Table 3. Note the pick off for the IR QPDs, *IRTRANS B1*, was chosen assuming 800 kW of arm power, 5 ppm ETM transmission, and 100 mW per QPD. The corresponding green pick off, *GNTRANS B1*, was arbitrarily chosen to be 5%. Where the reflectivity is not explicitly specified, it should be OK to leave it uncontrolled. The beam diverter is in the optics list and Zemax model as *IRTRANS M3* and is chosen to be 25.4 mm in diameter. If it should be changed to 50.8 mm diameter, both the optic for the diverter, the diverter itself, and the steering mirror before the beam dump should be change.

Label	Description	Qty	ROC	Reflectivity IR / GRN	Diameter
ETMXPO M1	OAP primary	1	4.00 m	HR / HR	200 mm
ETMXPO FM1-2	BRT folding mirror	2		HR / HR	
ETMXPO M2	OAP secondary	1	0.200 m	AR / AR	25 mm
ETMXPO P1-Pn	Periscope mirrors	n		HR / HR	50.8 mm
IRTRANS D1, D2	Dichroic mirrors	2		HR / AR	50.8 mm
IRTRANS B1	Beamsplitter	1		95% /	50.8 mm
IRTRANS M1-M4	beam steering	4		HR /	50.8 mm
IRQPD L1	Gouy telescope lens	1	154.4 mm	AR /	50.8 mm
IRQPD L2	Gouy telescope lens	1	-51.5 mm	AR /	25.4 mm
IRQPD M1-M4	beam steering	4		HR /	25.4 mm
IRQPD B1	beam splitter	1		50% /	25.4 mm
GNTRANS B1	Beamsplitter	1		/ 5%	25.4 mm
GNTRANS M1-M3	beam steering	3		HR /	25.4 mm
GNQPD L1	Gouy telescope lens	1	154.4 mm	/ AR	25.4 mm
GNQPD L2	Gouy telescope lens	1	-64.4 mm	/ AR	25.4 mm
GNQPD M1-M4	beam steering	4		/ HR	25.4 mm
GNQPD B1	beam splitter	1		/ 50%	25.4 mm

Table 3: Optics required for Transmon table.

Description	Quantity
Telescope mounting	1
45° kinematic periscope mirror mounts	4
50.8 mm kinematic mirror mounts	7
50.8 mm lens mount	1
25.4 mm lens mount	3
25.4 mm kinematic mirror mount	9
25.4 mm actuated mirror mount	4
25.4 mm beam diverter	1
3 mm QPD and mount	4
High power beam dump	1

Table 4: Components required for Transmon table.

## 14 References

### References

- [1] M. Arain and G. Mueller; Optical Layout and Parameters for the Advanced LIGO Cavities; T0900043-v10
- [2] L. Barsotti; Modeling of Alignment Sensing and Control for Advanced LIGO; T0900511
- [3] F. Matichard and *et al.*; PROTOTYPING, TESTING AND PERFORMANCE OF THE TWO-STAGE P1000029
- [4] Viewport Parameters **Please cite me!**

## A TransmonABCD.m

```

% Plot up the Transmon table
close all
clear classes

disp(' Transmon table. ')
disp(' ')

%% Parameters
% Our seedwaist comes from the arm cavity
armlength = 3995;
R_itm = 1934;
R_etm = 2245;
lambda = 1.064e-6;
zETM = 2; % rough distance from the ETM to Telescope

M_tel = 20; % Telescope magnification.
f_m1 = 2.00; % Primary mirror focal length.
f_m2 = - f_m1 / M_tel; % Secondary mirror focal length (diverging)

f_qpd1 = 0.344; % lenses for the qpds.
f_qpd2 = -0.114;

w_target = 250e-6; % target waist size.

[z0,z1,z2] = cavHG(armlength, R_itm, R_etm); % The waist from by the arm

%% Functions
% Equation to calculate effective focal length of ETM
lensmakers = @(n, R1, R2, d) ( (n-1)*(1/R1 - 1/R2 + (n-1)*d/(n*R1*R2)) ) ^ -1;

%% Telescope design
myBeam = beamPath;
myBeam.seedWaist(sqrt(z0*lambda/pi), -z2-zETM);

% Put down fixed components. OAPs and mirror are treated as lenses
f_ETM = lensmakers(1.45, -R_etm, inf, 0.2);
myBeam.addComponent(component.lens(f_ETM, -zETM, 'ETM'));
myBeam.addComponent(component.lens(f_m1, 0, 'Tel1'));

% tune the telescope length by matching the beam radius to the f2 ROC.
qB = myBeam.qPropagate(0.1);
z_telR = imag(qB.q);
z_tel0 = -real(qB.q);
% magic math here, to match the beam radii of curvature to the optic
dz_m2 = f_m2 + sqrt(f_m2^2 - z_telR^2);
z_m2 = z_tel0 - dz_m2;
fprintf(1, 'Placing m2 at %.4f\n', z_m2);
myBeam.addComponent(component.lens(f_m2, z_m2, 'Tel2'));

% Now set up the QPD telescope.
myBeam.addComponent(component.lens(f_qpd1, 3, 'Gouy1'));

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myBeam.addComponent(component.lens(f_qpd2, 3.243, 'Gouy2'));

% Tune the QPD telescope length so there is a correct output waist.
myBeam.targetWaist(w_target, 4);
myBeam = myBeam.optimizePath('Gouy2', [3 3.5], 'target', [3 4]);
fprintf(1, 'Placing Gouy2 at %.2f\n', myBeam.component('Gouy2').z);

% To put down the QPDs, we find the output waist location and go +/- Zr
zout = 3.5;
qout = myBeam.qPropagate(zout);
waist_z = zout - qout.waistZ;

myBeam.addComponent(component.lens(inf, waist_z-qout.rayleighRange, 'QPD1'));
myBeam.addComponent(component.lens(inf, waist_z+qout.rayleighRange, 'QPD2'));

%% Plot figure
figure(101)
zdomain = -2.1:.01:5;
clf;
subplot(2,1,1)
hold on;
orighandle=myBeam.plotBeamWidth(zdomain);
myBeam.plotComponents(zdomain)
myBeam.plotBeams(zdomain)
hold off;
axis([zdomain(1) zdomain(end) -7e-2 7e-2]);
ylabel('Beam Width (m)')
xlabel('Propagation axis (m)')

subplot(2,1,2)
hold on;
orighandle=myBeam.plotBeamWidth(zdomain);
myBeam.plotComponents(zdomain)
myBeam.plotBeams(zdomain)
hold off;
axis([zdomain(1) zdomain(end) -5e-3 5e-3]);

ylabel('Beam Width (m)')
xlabel('Propagation axis (m)')
saveas(101, 'Waist.pdf', 'pdf');

%% print the component list to the command window
disp(' ')
disp('===== Optimized Path Component List =====')
display(myBeam.components)

%% And calculate the QPD sensitivities
disp('===== QPD sensitivity =====');

QPD_from_Arm = QPD_TransferMatrix(myBeam, -z2-zETM)
Arm_from_QPD = inv(QPD_from_Arm)

ABCD_qpd1 = myBeam.getTransferMatrix(-0.5, myBeam.component('QPD1').z);
ABCD_qpd2 = myBeam.getTransferMatrix(-0.5, myBeam.component('QPD2').z);

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```

QPD_from_Table = QPD_TransferMatrix(myBeam, -0.5)
Arm_from_Table = Arm_from_QPD * QPD_from_Table

%% Calculate the QPD tolerances
dZ = -4e-3:3e-5:4e-3;
[gTelescope, zTelOut, fTelOut] = gouyLengthScan(myBeam, dZ, 'Tel1');
gTelescope(gTelescope<0) = gTelescope(gTelescope<0)+180;

gGouyTel = gouyLengthScan(myBeam, dZ, 'Gouy1');
gGouyTel(gGouyTel<0) = gGouyTel(gGouyTel<0) + 180;

figure(201)
subplot(2,1,1)
set(gca, 'FontSize', 16);
plot(dZ*1000, gTelescope, 'LineWidth', 3);
xlabel('BRT length shift [mm]');
ylabel('Gouy phase [deg]');
axis([-4 4 0 100])

subplot(2,1,2)
set(gca, 'FontSize', 16);
plot(dZ*1000, gGouyTel, 'LineWidth', 3);
xlabel('Gouy Telescope length shift [mm]');
ylabel('Gouy phase [deg]');
axis([-4 4 0 100])
saveas(201, 'SetupGouy.pdf', 'pdf');

figure(2001)
subplot(3,1,1)
plot(dZ*1000, zTelOut)
xlim([-1 1])
ylabel('Waist position [m]');
title('BRT output tuning');

subplot(3,1,2)
plot(dZ*1000, fTelOut*1000)
xlim([-1 1])
ylabel('Waist size [mm]');

subplot(3,1,3)
myZr = pi*fTelOut.^2/1.064e-6;
plot(dZ*1000, zTelOut./myZr);
xlim([-1 1])
ylabel('Normalized waist pos. [ m / Z-R]');
xlabel('BRT Length shift [mm]');
orient tall;
saveas(2001, 'TransmonTelescopeFocusing.pdf', 'pdf');

%% Calculate the aperture required
figure(301)

dz = -0.1:1e-2:5;
zstart = -z2 -zETM;
ETM_spot_shift = 0.01;

```

```

cavity_angle_shift = ETM_spot_shift/z2;
num_waist_clear = 6;
Hart_angle = 0.2/180*pi;

for ii = 1:length(dz)
    ABCD = myBeam.getTransferMatrix(zstart, dz(ii));
    this_waist = myBeam.qPropagate(dz(ii)).beamWidth;

    WaistAperture(ii) = num_waist_clear*this_waist;

    PosAperture(ii) = 2*abs(ETM_spot_shift*ABCD(1,1)) + WaistAperture(ii);
    AngAperture(ii) = 2*abs(cavity_angle_shift*ABCD(1,2)) + WaistAperture(ii);

    ABCD = myBeam.getTransferMatrix(-zETM+1e-6, dz(ii));
    HartAperture(ii) = 2*abs(Hart_angle*ABCD(1,2)) + WaistAperture(ii)/sqrt(2);
end

set(gca, 'FontSize', 16);
plot(dz, WaistAperture, dz, AngAperture, dz, PosAperture, ...
     dz, max(PosAperture, AngAperture)/sqrt(2), ...
     dz, HartAperture);
legend('6x waist size', 'From position', 'From angle', 'For Green beam', 'For Hartmann');
xlabel('Position [m]');
ylabel('Required aperture [m]');

axis([-1 5 0 0.05]);
saveas(301, 'RequiredAperture.pdf', 'pdf');

%% Calculate the allowable variations.
Ngrid = 21;
Xgrid = 1e-5;
dX = linspace(-Xgrid, Xgrid, Ngrid);
nX = length(dX);
[z_BRT, z_Gouy] = meshgrid(dX);
zTelStart = myBeam.component('Tel1').z;
zGouyStart = myBeam.component('Gouy1').z;
z_BRT = z_BRT + zTelStart;
z_Gouy = z_Gouy + zGouyStart;

q_cavity = myBeam.qPropagate(-z2-zETM);
meritscale = q_cavity.divergenceAngle / q_cavity.waistSize ;
clear figmerit;
for ii = 1:nX
    for jj = 1:nX
        myBeam.component('Tel1').z = z_BRT(ii, jj);
        myBeam.component('Gouy1').z = z_Gouy(ii, jj);

        ABCD = QPD_TransferMatrix(myBeam, -z2-zETM);
        Mact = Arm_from_QPD*ABCD;
        figmerit(ii, jj) = Mact(1,2)/Mact(1,1) * meritscale;
    end
end

myBeam.component('Tel1').z = zTelStart;
myBeam.component('Gouy1').z = zGouyStart;
%%

```

```

figure(401)
set(gca, 'FontSize', 16);
imagesc(dX*1e6, dX*1e6, figmerit);
xlabel('\delta BRT length [\mm]');
ylabel('\delta Gouy telescope length [\mm]');
title('Ratio Off-Diagonal to Displacement');
colorbar;
set(gca, 'YDir', 'normal')
p = polyfit(dX, figmerit(16,:), 1);
fprintf(1, 'Off diagonal term = %.1f%% per BRT um\n', 100*p(1)/1e6);
p = polyfit(dX, figmerit(:,16).', 1);
fprintf(1, 'Off diagonal term = %.1e%% per Gouy telescope um\n', 100*p(1)/1e6);

val = [-0.01 0.01]; % percentage shift between angle and position
for ii = 1:2
    xline(1) = interp1(figmerit(1,:), dX, val(ii));
    xline(2) = interp1(figmerit(end,:), dX, val(ii));
    yline(1) = dX(1);
    yline(2) = dX(end);
    line(xline*1e6, yline*1e6, 'Color', [0 0 0], 'LineWidth', 2)
end
saveas(401, 'MatrixVariance.pdf', 'pdf');

```



## B TelescopeFocusing.m

```

%% Calculate the telescope focusing
%
% The telescope focusing consists of an input beam, beam expander,
% a beam splitter, the telescope, a retroreflector, the telescope,
% and a beam expander.
function TelescopeFocusing

    figure(501);
    clf;
    telfoc = beampath;
    telfoc.seedWaist(1e-3, -1); % seed the test with a 1 mm beam 1 m in front
                                % of the secondary

    f2 = 0.125 - 72e-6;
    zTel = 2.1276;

    zBulkhead = (29 + 4) * 25.4e-3; % Invar is 29", plus a few for slop
    zExpand = -0.25; % position of the input/output beam expanders

    telfoc = makeTelescope(f2, zTel, zBulkhead);

    % Start with a reasonable beam
    telfoc.seedWaist(1e-3, zExpand-0.05);

    % Input 5x beam expander
    telfoc.addComponent(component.lens(-18e-3, -zExpand, 'Exp1'));
    telfoc.addComponent(component.lens(90e-3, -zExpand+72e-3, 'Exp2'));
    zLast = telfoc.component('Tel2_2').z;

    % output 2x beam compressor
    telfoc.addComponent(component.lens(inf, zBulkhead + zLast, 'Bulk'));
    telfoc.addComponent(component.lens(0.10, zBulkhead + zLast -zExpand, 'Exp3'));
    telfoc.addComponent(component.lens(-0.050, 0.050 + zBulkhead + zLast -zExpand, 'Exp4'));

    % Here's the nominal parameters
    f0 = findOutputFocus(telfoc);
    fprintf(1, 'Nominal output waist = %.0f um\n', ...
        telfoc.qPropagate(10).waistSize*1e6);
    nomRay = telfoc.qPropagate(10).rayleighRange;
    fprintf(1, 'Nominal Rayleigh range = %.0f mm\n', ...
        nomRay*1e3);
    telfoc.addComponent(component.lens(inf, f0, 'Target'));

    z = -1:0.01:9;
    telfoc.plotBeamWidth(z);
    hold on;
    telfoc.plotComponents(z);
    hold off;
    focal_length = 10-telfoc.qPropagate(10).waistZ - telfoc.component('Tel2_2').z;
    xlabel('Beam Z [m]');
    ylabel('Beam width [m]');

    % The sensitivity to the telescope length

```

```

figure(601)
clf;
dFocus = -6e-4:3e-5:6e-4;
clear focii dtel;
zStart = [telfoc.component('Tel2_1').z telfoc.component('Tel2_2').z];
for ii = 1:length(dFocus)
    detuneTelescope(telfoc, dFocus(ii), zStart);
    focii(ii) = findOutputFocus(telfoc);
    dtel(ii) = telfoc.component('Tel2_1').z;
end
% Put the telescope back where it started
detuneTelescope(telfoc, 0, zStart);

subplot(2,1,1);
plot(dFocus*1e6, (focii-f0)*1000);
line([-500 -500], [-200 200], 'LineStyle', '--', 'Color', [0 0 0]);
line([200 200], [-200 200], 'LineStyle', '--', 'Color', [0 0 0]);
xlabel('Telescope detuning [ $\mu$ m]');
ylabel('Test focus shift [nm]');
axis([-600 600 -200 200]);

subplot(2,1,2);
plot(dFocus*1e6, (focii-f0)/nomRay);
line([-500 -500], [-200 200], 'LineStyle', '--', 'Color', [0 0 0]);
line([200 200], [-200 200], 'LineStyle', '--', 'Color', [0 0 0]);
xlabel('Telescope detuning [ $\mu$ m]');
ylabel('Shift [normalized by Z_R]');
axis([-600 600 -10 10]);

% The sensitivity to the input beam
dZ_input = -1:0.1:0.5;
for ii = 1:length(dZ_input)
    telfoc.seedWaist(1e-3, zExpand-0.05 + dZ_input(ii));
    input_focii(ii) = findOutputFocus(telfoc);
end

dF_input = 5e-4:1e-4:1.5e-3;
for ii = 1:length(dF_input)
    telfoc.seedWaist(dF_input(ii), zExpand-0.05);
    input_focusSize(ii) = findOutputFocus(telfoc);
end
telfoc.seedWaist(1e-3, zExpand-0.05);

dZ_inBRT = [-1e-3:1e-4:1e-3];
zStart = telfoc.component('Exp1').z;
for ii = 1:length(dZ_inBRT)
    telfoc.component('Exp1').z = zStart + dZ_inBRT(ii);
    input_BRT(ii) = findOutputFocus(telfoc);
end
telfoc.component('Exp1').z = zStart;

dZ_outBRT = [-1e-3:1e-4:1e-3];
zStart = telfoc.component('Exp4').z;
for ii = 1:length(dZ_inBRT)
    telfoc.component('Exp4').z = zStart + dZ_outBRT(ii);

```

```

        output_BRT(ii) = findOutputFocus(telfoc);
    end
    telfoc.component('Exp4').z = zStart;

    dZ_posBRT = [-1e-2:1e-3:1e-2];
    zStart = [telfoc.component('Exp3').z telfoc.component('Exp4').z];
    for ii = 1:length(dZ_posBRT)
        telfoc.component('Exp3').z = zStart(1) + dZ_outBRT(ii);
        telfoc.component('Exp4').z = zStart(2) + dZ_outBRT(ii);
        pos_BRT(ii) = findOutputFocus(telfoc);
    end
    telfoc.component('Exp3').z = zStart(1);
    telfoc.component('Exp4').z = zStart(2);

    figure(701)
    subplot(3,2,1)
    plot(dZ_input*1000, (input_focii-f0)*1000);
    xlabel('Input focus position [mm]');
    ylabel('Focus shift [mm]');
    ylim([-50 50]);

    subplot(3,2,2)
    plot(dF_input*1000, (input_focusSize-f0)*1000);
    xlabel('Input beam waist [mm]');
    ylim([-50 50]);

    subplot(3,2,3);
    plot(dZ_inBRT*1000, (input_BRT-f0)*1000);
    xlabel('Input BRT length [mm]');
    ylabel('Focus shift [mm]');
    axis([-0.5 0.5 -50 50]);

    subplot(3,2,4);
    plot(dZ_outBRT*1000, (output_BRT-f0)*1000);
    xlabel('Output BRT length [mm]');
    axis([-0.5 0.5 -50 50]);

    subplot(3,2,5)
    plot(dZ_posBRT*1000, (pos_BRT-f0)*1000);
    xlabel('Output BRT position [mm]');
    ylabel('Focus shift [mm]');
    axis([-0.5 0.5 -50 50]);

end

function telOut = makeTelescope(f2, zTel, zBulkhead)
% a telescope positioned with the input bulkhead as 0

f1 = 2.0;
telOut = beampath;
telOut.addComponent(component.lens(f2, zBulkhead, 'Tel2_1'));
telOut.addComponent(component.lens(f1, zTel+zBulkhead, 'Tel1_1'));
telOut.addComponent(component.lens(inf, zTel + 2*zBulkhead, 'Retro'));

```

```

        telOut.addComponent(component.lens(f1, zTel+3*zBulkhead, 'Tel1_2'));
        telOut.addComponent(component.lens(f2, 2*zTel+3*zBulkhead, 'Tel2_2'));
    end

function detuneTelescope(in, dZ, zStart)
    in.component('Tel2_1').z = zStart(1) - dZ;
    in.component('Tel2_2').z = zStart(2) + dZ;
end

function focus = findOutputFocus(in)
    zMax = 10;
    q = in.qPropagate(zMax);
    focus = 10 - q.waistZ;
end

```

## C TableRequirements.m

```

% I start by assuming the waist is 2km down the arm, and that the ITM and
% EIM table motions are independent (both reasonable assumptions)

%% Calculate the cavity noise
load('PITCHmotion.mat'); % The EIM pitch motion from LisaB

ETMmot = PitchMot(1,:);
R_etm = 2245;
R_itm = 1934;
armlength = 3995;

waist_x = R_etm*ETMmot; % waist displacement from etm lever arm

d_center = R_etm+R_itm - armlength;
waist_theta = sqrt(2)*waist_x/d_center; % waist angle given by ROC center
% shift relative to spacing.
% sqrt(2) from having two optics

waist = [waist_x; waist_theta];

%% Calculate the QPD shot noise
qpd_waist = 350e-6; % from TransmonTable.m
qpd_slope = sqrt(8/pi)/qpd_waist;
qpd_I = 100e-3; % QPD total photocurrent
ec = 1.6e-19; % electron charge
shotnoise_RIN = sqrt(2)*(qpd_I/ec)^(-1/2); % sqrt(2) from two halves
x_sn = shotnoise_RIN/qpd_slope; % this is about right for shot noise

%% Calculate the table motion
load('seismicX');
x_table = interp1(freq, noiseX, f);
x_table(isnan(x_table)) = min(x_table);

rad_per_meter = 1;
table = [x_table; rad_per_meter*x_table];

%%
% Now to turn this waist motion into motion at the table, we need a ray
% tracing matrix. We take this one from TransmonTable.m.
to_qpd = QPD_from_Arm;
tab_qpd = QPD_from_Table;

figure(1)
clf;
a = loglog(f, abs(to_qpd(1,1)*waist(1,:)), 'b-', ...
    f, abs(to_qpd(1,2)*waist(2,:)), 'b--', ...
    f, abs(to_qpd(2,1)*waist(1,:)), 'r-', ...
    f, abs(to_qpd(2,2)*waist(2,:)), 'r--', ...
    f, abs(tab_qpd(1,1)*table(1,:)), 'c-', ...
    f, abs(tab_qpd(1,2)*table(2,:)), 'c--', ...

```

```

        f, abs(tab_qpd(2,1)*table(1,:)), 'm-', ...
        f, abs(tab_qpd(2,2)*table(2,:)), 'm--', ...
        f, ones(size(f))*x_sn/qpd_waist, 'g-', ...
        'LineWidth', 2);
a = legend('QPD1 cavity disp', 'QPD1 cavity angle', ...
        'QPD2 cavity disp', 'QPD2 cavity angle', ...
        'QPD1 table disp', 'QPD1 table angle', ...
        'QPD2 table disp', 'QPD2 table angle', ...
        '"100mA Shot noise"');
set(a, 'FontSize', 14);
xlabel('Frequency [Hz]', 'FontSize', 16);
ylabel('Displacement at QPD [m/ \omega-{QPD}]', 'FontSize', 16);
set(gca, 'FontSize', 14);
axis([0.1 100 1e-10 1e-3])
title('QPD Signals', 'FontSize', 16);

%%
figure(2)

from_tab = inv(to_qpd)*tab_qpd;
from_shot = inv(to_qpd)*x_sn/qpd_waist;

for ii = 1:2
    noise(ii,:) = sqrt((from_tab(ii,1)*table(1,:)).^2 + ...
        (from_tab(ii,2)*table(2,:)).^2 + ...
        (from_shot(ii,1)).^2 + ...
        (from_shot(ii,2)).^2);
end

loglog(f, noise(1,:), 'k-', ...
        f, 1e3*noise(2,:), 'k--', ...
        f, waist(1,:), 'r-', ...
        f, .9e3*waist(2,:), 'r--', ...
        'LineWidth', 2);

axis([0.1 100 1e-12 1e-4])

a = legend('Disp. noise', 'Angle noise', ...
        'Disp. cavity', 'Angle cavity');
set(a, 'FontSize', 14);

xlabel('Frequency [Hz]', 'FontSize', 16);
ylabel('Waist motion [m, mrad]', 'FontSize', 16);
set(gca, 'FontSize', 14);
title('Inferred cavity motion', 'FontSize', 16);

%% Required isolation

figure(3)
% the allowed table motion in order to be 5 times below the lowest cavity
% signal, and 10 times lower than the shot noise.
qpdsusnoise = sqrt((waist(1,:)*to_qpd(1,1)).^2/25 + x_sn.^2/100);

invtab = inv(tab_qpd);
allowedtab = [abs(qpdsusnoise*invtab(1,1));
        abs(qpdsusnoise*invtab(2,1))];

```

```

loglog(f, allowedtab(1,:)/sqrt(2), ...
      f, 1e3*allowedtab(2,:)/sqrt(2), ...
      f, table(1,:), ...
      f, table(2,:)*1e3,...
      'LineWidth', 2);
legend('Required Table Position', 'Required Table Angle', ...
      'Predicted ISI Position', 'Predicted ISI angle');
xlabel('Frequency [Hz]', 'FontSize', 16);
ylabel('Table motion [m, mrad]', 'FontSize', 16);
set(gca, 'FontSize', 14);
title('Required table motion', 'FontSize', 16);
axis([0.1 100 1e-16 1e-6])

```

```

%% Starting signals
figure(4)
loglog(f, waist, f, ETMmot, 'LineWidth', 2);
legend('Displacement', 'Angle', 'ETM Pitch');
ylabel('Cavity motion [m, rad]');
xlabel('Frequency [Hz]');
axis([0.1 100 1e-14 1e-4]);

```