A Cross-Correlation Technique to Search for Periodic Gravitational Waves

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UWM Center for Gravitation and Cosmology Seminar
2009 June 16
LIGO-G0900946-v2

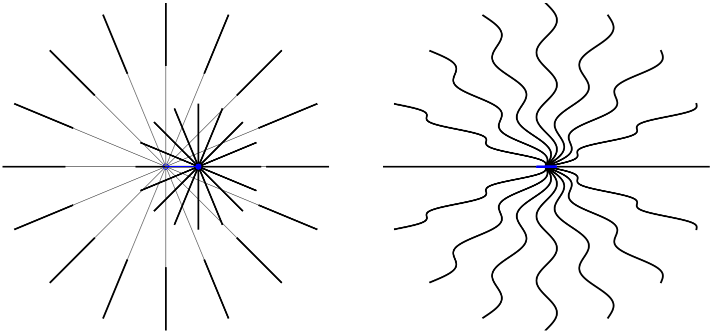
Outline

1. Searches for Gravitational Waves
   - Crash Course in Gravitational Wave Physics
   - Gravitational-Wave Sources & Signals
   - Gravitational-Wave Observations & Detectors
2. Cross-Correlation Method
   - Application to Stochastic Background
   - Application to Quasiperiodic Gravitational-Wave Signals
   - Choice of SFT Pairs for Correlation
3. Applications and Outlook
   - Directed Search for Young Neutron Stars
   - Accreting Neutron Stars in Low-Mass X-Ray Binaries
   - Summary

Adapted from Amaldi talk LIGO-G0900536
by JTW, Chung, Krishnan, Melatos, Peralta

Outline

Motivation



- In Newtonian gravity, force dep on distance btwn objects
- If massive object suddenly moved, grav field at a distance would change instantaneously
- In relativity, no signal can travel faster than light
  ⟶ time-dep grav fields must propagate like light waves

Gravity as Geometry

- Minkowski Spacetime:

$$ds^2 = -(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2$$

$$= \begin{pmatrix} dx^0 \\ dx^1 \\ dx^2 \\ dx^3 \end{pmatrix}^{\mathrm{tr}} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} dx^0 \\ dx^1 \\ dx^2 \\ dx^3 \end{pmatrix} = \eta_{\mu\nu} dx^\mu dx^\nu$$

- General Spacetime:

$$ds^2 = \begin{pmatrix} dx^0 \\ dx^1 \\ dx^2 \\ dx^3 \end{pmatrix}^{\mathrm{tr}} \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{pmatrix} \begin{pmatrix} dx^0 \\ dx^1 \\ dx^2 \\ dx^3 \end{pmatrix} = g_{\mu\nu} dx^\mu dx^\nu$$

Gravitational Wave as Metric Perturbation

- For GW detection, spin-2 “graviton tensor” $h_{\mu\nu}$ is difference btwn actual metric $g_{\mu\nu}$ & flat metric $\eta_{\mu\nu}$:

$$g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$$

($h_{\mu\nu}$ “small” in weak-field regime, e.g. for GW detection)

- E.g. Plane wave propagating in z direction

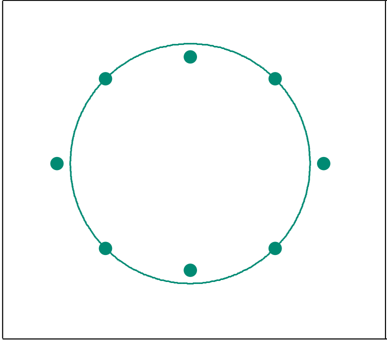
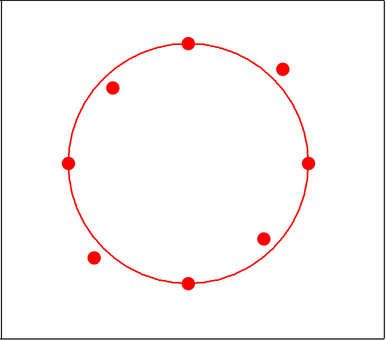
$$\{h_{\mu\nu}\}=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} e^{i2\pi f(z-t)}$$

h_+ and $h_\times$ are amplitudes of “plus” and “cross” pol states.

$$\overleftrightarrow{h} = \left[h_+ \overleftrightarrow{e}_+ + h_\times \overleftrightarrow{e}_\times\right] e^{i2\pi f(\hat{k}\cdot\vec{r}-t)}$$

Effects of Gravitational Wave

Fluctuating geom changes distances btwn particles in free-fall:

Plus (+) Polarization	Cross (×) Polarization
	

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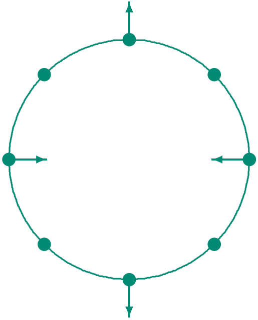
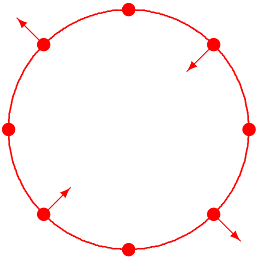
Fluctuating geom changes distances btwn particles in free-fall:

Plus $(+)$ Polarization	Cross $(\times)$ Polarization
	

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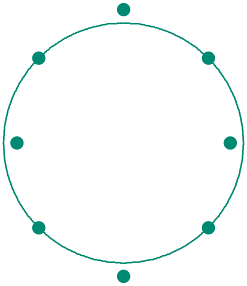	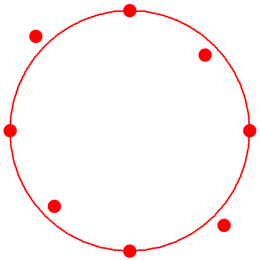

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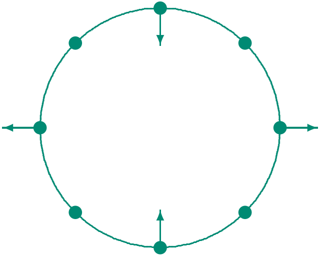	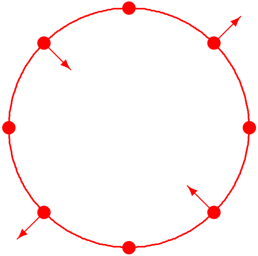

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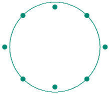
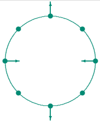
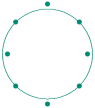
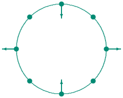
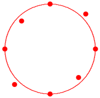
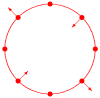
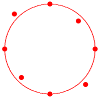
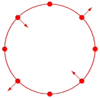
Plus ($+$) Polarization				
Cross ($\times$) Polarization				

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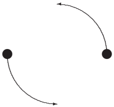
Gravitational Wave Generation

- Generated by moving/oscillating mass distribution
- Lowest multipole is quadrupole
- Classic example: orbiting binary system



(e.g., Binary Pulsar 1913+16
– Observed energy loss agrees w/GW prediction)

Classification of GW Signals

In LIGO band (10s-1000s of Hz), natural division of sources:

	modelled	unmodelled
long	Periodic Sources (e.g., Rotating Neutron Star)	Stochastic Background (Cosmological or Astrophysical)
short	Binary Coälescence (Black Holes, Neutron Stars)	Bursts (Supernova, BH Merger, etc.)

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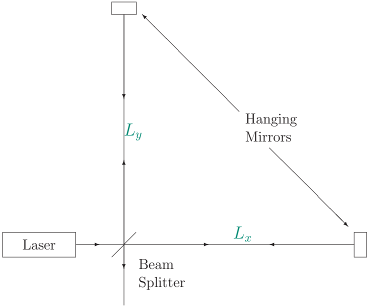
Measuring GWs w/Laser Interferometry

Interferometry: Measure GW-induced distance changes



- Measure small change in
$$L_x - L_y = \sqrt{g_{11} L_0^2} - \sqrt{g_{22} L_0^2}$$
$$= \sqrt{(1+h_{11}) L_0^2} - \sqrt{(1+h_{22}) L_0^2}$$
$$\approx L_0 \frac{h_{11} - h_{22}}{2} \sim L_0 h_+$$
- More gen,
$$(L_1 - L_2)/L_0 = \overleftrightarrow{h} : \overleftrightarrow{d}$$
with “response tensor”
$$\overleftrightarrow{d} = \frac{\hat{n}_1 \otimes \hat{n}_1 - \hat{n}_2 \otimes \hat{n}_2}{2}$$

(also when $\hat{n}_1$ & $\hat{n}_2$ not $\perp$)

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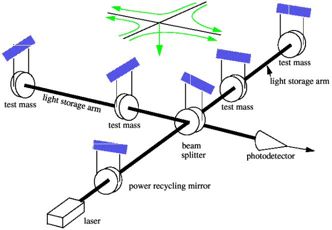

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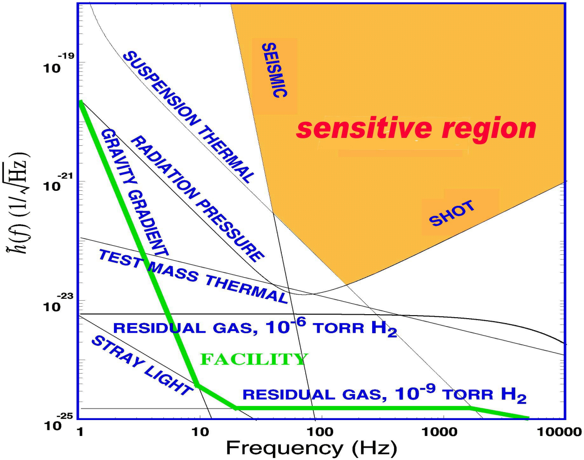
LIGO's Sensitive Frequency Band



Rogues' Gallery of Ground-Based Interferometers

LIGO Hanford (Wash.)

LIGO Livingston (La.)

GEO-600 (Germany)

Virgo (Italy)

Outline

Cross-Correlation Search for Stochastic Background

- Noisy data from GW Detector:
  $x(t) = n(t) + h(t) = n(t) + \overleftrightarrow{h}(t) : \overleftrightarrow{d}$
- Correlate data btwn detectors (Fourier domain)

$$\langle \tilde{x}_1^*(f)\tilde{x}_2(f') \rangle = \langle \tilde{h}_1^*(f)\tilde{h}_2(f') \rangle = \overleftrightarrow{d}_1 : \langle \overleftrightarrow{\tilde{h}_1^*}(f) \otimes \overleftrightarrow{\tilde{h}_2}(f') \rangle : \overleftrightarrow{d}_2$$

- For stochastic backgrounds

$$\langle \tilde{h}_1^*(f)\tilde{h}_2(f') \rangle = \delta(f-f')\gamma_{12}(f)\frac{S_{\mathrm{gw}}(f)}{2}$$

$S_{\mathrm{gw}}(f)$ encodes spectrum; $\gamma_{12}(f)$ encodes geometry

Detection Statistic

- Optimally filtered cross-correlation statistic

$$Y = \int df\, \tilde{x}_1^*(f)\, Q(f)\, \tilde{x}_2(f)$$

- Filter encodes expected spectrum & spatial distribution (isotropic, pointlike, spherical harmonics . . .)

$$Q(f) \propto \frac{\gamma_{12}^*(f) S_{\text{gw}}^{\text{exp}}(f)}{S_{n1}(f) S_{n2}(f)}$$

- “Radiometer” search for ptlike srcs incl targeting Sco X-1: known sky location, unknown frequency
Ballmer, *CQG* **23**, S179 (2006); LSC, *PRD* **76**, 082003 (2007)

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Gravitational Waves from Quasiperiodic Sources

- Sco X-1 is Low-Mass X-Ray Binary:
  accreting neutron star in orbit w/companion
- Rotating NS w/deformation emits nearly sinusoidal signal

$$\overleftrightarrow{h}(t) = h_0 \left[\frac{1+\cos^2\iota}{2} \cos\Phi(\tau(t))\overleftrightarrow{e}_{+} + \cos\iota \sin\Phi(\tau(t))\overleftrightarrow{e}_{\times} \right]$$

  - $\Phi(\tau)$: phase evolution in rest frame;
  - $\tau(t)$: Doppler mod from detector motion (& binary orbit)
- Features of signal model missing from stoch search:
  - Doppler shift @ each detector:
    correlations peaked @ different freqs
  - Long-term coherence:
    can correlate data @ different times

Cross-Correlation of Continuous GW Signals

- Cross-correlation of signal w/intrinsic frequency f_0:

$$\begin{aligned}\langle \tilde{x}_I^*(f_I)\,\tilde{x}_J(f_J)\rangle &= \tilde{h}_I^*(f_I)\,\tilde{h}_J(f_J)\\ &= h_0^2\,\tilde{\mathcal{G}}_{IJ}\,\delta_{\Delta T}(f_0-f_I-\delta f_I)\,\delta_{\Delta T}(f_0-f_J-\delta f_J)\end{aligned}$$

  - $\tilde{h}_I(f)$ is Short Fourier Transform, duration ΔT
  - $\delta_{\Delta T}(f-f') = \int_{-\Delta T/2}^{\Delta T/2} dt\, e^{i2\pi(f-f')t}$
  - $\tilde{h}_I$ & $\tilde{h}_J$ can be same or different times or detectors
  - δf_I is relevant Doppler shift
- For given set of params, can add products of all SFT pairs

$$Y = \sum_{IJ} Q_{IJ}\tilde{x}_I^*(f_0-\delta f_I)\tilde{x}_J(f_0-\delta f_J) \qquad Q_{IJ} \propto \frac{\tilde{\mathcal{G}}_{IJ}^*}{S_{n,I}(f_0)S_{n,J}(f_0)}$$

Dhurandhar, Krishnan, Mukhopadhyay & JTW *PRD* **77**, 082001 (2008)

Computational Costs and Frequency Resolution

- If freq, sky pos etc known, can do most sensitive fully coherent search (correlate all data)
- If some params unknown, have to search over them
- Long coherent observation → fine resolution in freq etc → need too many templates → computationally impossible
- Most CW searches semi-coherent: deliberately limit coherent integration time & param space resolution to keep number of templates manageable

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Synchronous Cross-Correlation Search

	$x_1(t_0)$	$x_2(t_0)$	$x_1(t_1)$	$x_2(t_1)$	$x_1(t_2)$	$x_2(t_2)$	$x_1(t_3)$	$x_2(t_3)$
$x_1(t_0)$	N	Y	N	N	N	N	N	N
$x_2(t_0)$	Y	N	N	N	N	N	N	N
$x_1(t_1)$	N	N	N	Y	N	N	N	N
$x_2(t_1)$	N	N	Y	N	N	N	N	N
$x_1(t_2)$	N	N	N	N	N	Y	N	N
$x_2(t_2)$	N	N	N	N	Y	N	N	N
$x_1(t_3)$	N	N	N	N	N	N	N	Y
$x_2(t_3)$	N	N	N	N	N	N	Y	N

"Stochastic-style": correlate data @ same time, diff detectors

Fully Coherent Search

	$x_1(t_0)$	$x_2(t_0)$	$x_1(t_1)$	$x_2(t_1)$	$x_1(t_2)$	$x_2(t_2)$	$x_1(t_3)$	$x_2(t_3)$
$x_1(t_0)$	Y	Y	Y	Y	Y	Y	Y	Y
$x_2(t_0)$	Y	Y	Y	Y	Y	Y	Y	Y
$x_1(t_1)$	Y	Y	Y	Y	Y	Y	Y	Y
$x_2(t_1)$	Y	Y	Y	Y	Y	Y	Y	Y
$x_1(t_2)$	Y	Y	Y	Y	Y	Y	Y	Y
$x_2(t_2)$	Y	Y	Y	Y	Y	Y	Y	Y
$x_1(t_3)$	Y	Y	Y	Y	Y	Y	Y	Y
$x_2(t_3)$	Y	Y	Y	Y	Y	Y	Y	Y

Combine all SFT pairs; as with standard $\mathcal{F}$-statistic, quadratic combination of all SFTs

Excess Power Search

	$x_1(t_0)$	$x_2(t_0)$	$x_1(t_1)$	$x_2(t_1)$	$x_1(t_2)$	$x_2(t_2)$	$x_1(t_3)$	$x_2(t_3)$
$x_1(t_0)$	Y	N	N	N	N	N	N	N
$x_2(t_0)$	N	Y	N	N	N	N	N	N
$x_1(t_1)$	N	N	Y	N	N	N	N	N
$x_2(t_1)$	N	N	N	Y	N	N	N	N
$x_1(t_2)$	N	N	N	N	Y	N	N	N
$x_2(t_2)$	N	N	N	N	N	Y	N	N
$x_1(t_3)$	N	N	N	N	N	N	Y	N
$x_2(t_3)$	N	N	N	N	N	N	N	Y

Only consider “diagonal” auto-correlations

Semi Coherent Search

	$x_1(t_0)$	$x_2(t_0)$	$x_1(t_1)$	$x_2(t_1)$	$x_1(t_2)$	$x_2(t_2)$	$x_1(t_3)$	$x_2(t_3)$
$x_1(t_0)$	Y	Y	Y	Y	N	N	N	N
$x_2(t_0)$	Y	Y	Y	Y	N	N	N	N
$x_1(t_1)$	Y	Y	Y	Y	N	N	N	N
$x_2(t_1)$	Y	Y	Y	Y	N	N	N	N
$x_1(t_2)$	N	N	N	N	Y	Y	Y	Y
$x_2(t_2)$	N	N	N	N	Y	Y	Y	Y
$x_1(t_3)$	N	N	N	N	Y	Y	Y	Y
$x_2(t_3)$	N	N	N	N	Y	Y	Y	Y

Coherently combine within epochs

Lag-Limited Cross-Correlation Search

	$x_1(t_0)$	$x_2(t_0)$	$x_1(t_1)$	$x_2(t_1)$	$x_1(t_2)$	$x_2(t_2)$	$x_1(t_3)$	$x_2(t_3)$
$x_1(t_0)$	Y	Y	Y	Y	N	N	N	N
$x_2(t_0)$	Y	Y	Y	Y	N	N	N	N
$x_1(t_1)$	Y	Y	Y	Y	Y	Y	N	N
$x_2(t_1)$	Y	Y	Y	Y	Y	Y	N	N
$x_1(t_2)$	N	N	Y	Y	Y	Y	Y	Y
$x_2(t_2)$	N	N	Y	Y	Y	Y	Y	Y
$x_1(t_3)$	N	N	N	N	Y	Y	Y	Y
$x_2(t_3)$	N	N	N	N	Y	Y	Y	Y

“Sliding” semi-coherent search

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Supernova 1987A Remnant

Credit: NASA/ESA, P. Challis, R. Kirshner (Harvard-Smithsonian Center for Astrophysics) and B. Sugerman (STScI)

Searching for Young Neutron Stars

- Young ($\lesssim 100\,\mathrm{yr}$) NSs should be spinning rapidly
  LIGO/Virgo band $50\,\mathrm{Hz} \lesssim f_{\mathrm{GW}} \lesssim 1500\,\mathrm{Hz}$
- Look in likely sky locations for NSs not seen as pulsars:
  SN1987A should have one; galactic ctr could have $\mathcal{O}(1)$
- Spinning down rapidly; inefficient to search over f, $\dot{f}$, $\ddot{f}$, . . .
  Phase model: GW spindown $\propto f^5$; EM spindown $\propto f^{\approx 3}$

$$\frac{df}{d\tau} = Q_{\mathrm{GW}} \left(\frac{f}{f_{\mathrm{ref}}}\right)^5 + Q_{\mathrm{EM}} \left(\frac{f}{f_{\mathrm{ref}}}\right)^n$$

Search over f_0, Q_{GW}, Q_{EM}, n

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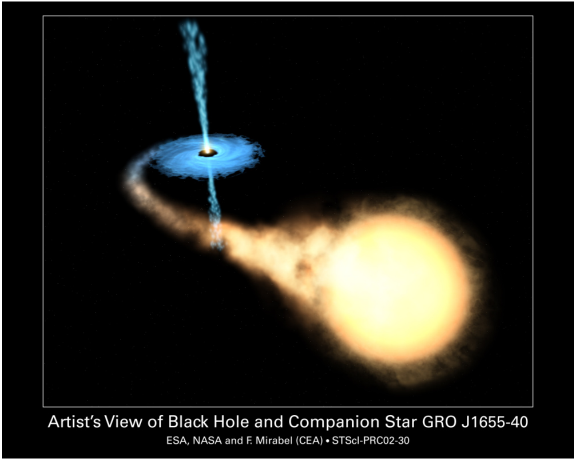
Low-Mass X-Ray Binary



Artist's View of Black Hole and Companion Star GRO J1655-40

ESA, NASA and F. Mirabel (CEA) • STScI-PRC02-30

Compact object accreting mass from companion star

Searching for Neutron Stars in LMXBs

- LMXB: BH/NS/WD accreting mass from companion star
- Accretion spinup may be balanced by GW spindown [Bildsten *ApJL* **501**, L89 (1998)] → no $\dot{f}$
- Scorpius X-1: $1.4M_\odot$ NS w/$0.4M_\odot$ companion unknown params are f_0, $a \sin i$, orbital phase
- LSC searches for Sco X-1:
    - Coherent search w/6 hr of S2 data *PRD* **76**, 082001 (2007)
    - Directed stochastic cross-corr ("radiometer") search w/simultaneous S4 H1 & L1 data *PRD* **76**, 082003 (2007)
- Can use improved cross-corr method to search including wider range of correlated segments

Outline

Summary

- Cross-correlation method adapted to periodic GWs
- Tuning max time-lag between cross-correlated data allows tradeoff of sensitivity for computing time
- Can search for young NSs (e.g., SN1987A) (search over f_0 & braking model params)
- Can search for LMXBs (e.g., Sco X-1) (search over f_0 & binary orbit params)