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A route to observing ponderomotive entanglement with optically trapped mirrors

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The radiation pressure of two detuned laser beams can create a stable trap for a suspended cavity mirror; here it is shown that such a configuration entangles the output light fields via interaction with the mirror. Intra-cavity, the opto-mechanical system can become entangled also. The degree of entanglement is quantified spectrally using the logarithmic negativity. Entanglement survives in the experimentally accessible regime of gram-scale masses subject to thermal noise at room temperature.

Entanglement both provides a basis for fundamental tests of quantum mechanics, and is an ingredient for applications in quantum information, including cryptography and teleportation. Producing entanglement in a macroscopic mechanical system has become a prominent experimental objective, and progress in the fabrication and cooling of small mechanical resonators is quickly bringing this objective within reach [1–8], as highlighted in a series of recent proposals treating these systems [9–18].

Meanwhile, the improving sensitivity of gravitational-wave interferometers is opening a new regime for macroscopic quantum mechanics, and may reveal quantum features such as squeezing and entanglement of their mirrors' motion [19]. A novel and defining property of this regime is that radiation pressure effects, in particular the optical spring [20–24], can play a dominant role in the dynamics.

A stable optical trap for a macroscopic mirror has been presented in Ref. 25, exploiting the radiation pressure of two laser fields detuned from cavity resonance to create simultaneously an optical spring and an optical damping force. When mechanical forces coupling the mirror with the outside world are negligible in comparison with optical forces, this system becomes nearly immune to the deleterious interaction with its thermal environment. This makes it a promising candidate to exhibit quantum effects including entanglement, a prospect to be evaluated here.

Entanglement criterion.—It is known that entanglement of a bipartite continuous-variable system can be recognized by inspecting its variance matrix for evidence of non-classical correlation. This 4×4 symmetric matrix contains the second order moments between elements of a vector of observables $\mathbf{u} = [Q_1, P_1, Q_2, P_2]^T$ (i.e., the canonical positions Q_j and momenta P_j of subsystems $j \in \{1,2\}$), and is defined as follows:

$$\boldsymbol{V} = \begin{bmatrix} \boldsymbol{V}_{11} & \boldsymbol{V}_{12} \\ \boldsymbol{V}_{12}^T & \boldsymbol{V}_{22} \end{bmatrix}; \quad \boldsymbol{V}_{jk} = \begin{bmatrix} \langle Q_j Q_k \rangle_+ & \langle Q_j P_k \rangle_+ \\ \langle P_j Q_k \rangle_+ & \langle P_j P_k \rangle_+ \end{bmatrix}. \quad (1)$$

Here $\mathbf{u}$ is assumed to have zero mean (the steady-state value $\bar{u}_j$ of each element has been subtracted, leaving only fluctuating terms). The quantity $\langle uv \rangle_+$ denotes the symmetrized average $\langle uv + vu \rangle / 2$.

The Peres-Horodecki entanglement criterion [26], as stated for continuous-variable systems by Simon [27], establishes that the system is entangled whenever the time reversal of one subsystem only (e.g. $P_2 \to -P_2$) would result in a variance matrix that no longer satisfies the uncertainty principle. Stated mathematically, separability constrains the variance matrix by requiring $4 \det \boldsymbol{V} > \Sigma - \frac{1}{4}$, where $\Sigma = \det \boldsymbol{V}_{11} + \det \boldsymbol{V}_{22} - 2 \det \boldsymbol{V}_{12}$. Further, one may define the logarithmic negativity [28] in terms of $\boldsymbol{V}$:

$$E_N = \max \left[0, -\frac{1}{2} \ln \left(2\Sigma - 2\sqrt{\Sigma^2 - 4 \det \boldsymbol{V}} \right) \right]. \quad (2)$$

This entanglement measure quantifies the degree to which the Peres-Horodecki criterion has been violated [29]. Note that the preceding statements presume (dimensionless) canonical commutation relations between the elements of $\mathbf{u}$: $[Q_j, P_k] = i\delta_{jk}$, and $[Q_j, Q_k] = [P_j, P_k] = 0$.

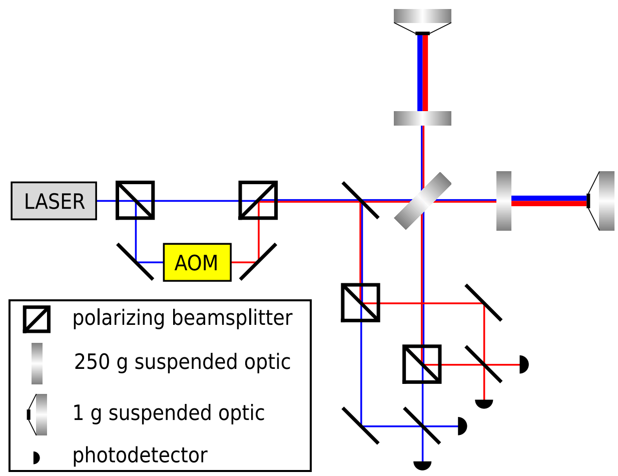

FIG. 1: Schematic of an optical trapping and homodyne readout apparatus for the differential mode of a Fabry-Perot Michelson interferometer. Each arm cavity comprises a highly reflective, low-mass end mirror and a massive input mirror of finite transmissivity. The system is driven by two orthogonally polarized laser beams: a strong "carrier" field, and a weaker frequency-shifted "subcarrier" created by an acousto-optic modulator (AOM). Each optical field is monitored using a balanced homodyne readout. Feedback loops required to hold the interferometer on resonance are not shown.

TABLE I: Parameters and their nominal values.

mirror resonant frequency	$\omega_m/2\pi$	1 Hz
mirror damping rate	$\gamma_m/2\pi$	1 μHz
mirror reduced mass	m	0.5 g
cavity resonant frequency	$\omega_c/2\pi$	$c/(1064$ nm$)$
cavity linewidth (HWHM)	$\gamma_c/2\pi$	9.5 kHz
cavity length	L	1 m
carrier power	I_1	5 W
carrier detuning	Δ_1	$-3\gamma_c$
subcarrier power	I_2	0.3 W
subcarrier detuning	Δ_2	$\gamma_c/2$
ambient temperature	T	300 K

Opto-mechanical dynamics.—A schematic of a trapped mirror system is shown in Fig. 1. To compute its second order moments, we first write down its linearized, Heisenberg-picture equations of motion, which are derived using the quantum Langevin approach (cf. [14, 18, 30]). They can be expressed in the succinct form

$$\dot{\mathbf{u}}_{\rm ic} = \boldsymbol{K}\mathbf{u}_{\rm ic} + \mathbf{u}_{\rm in}. \tag{3}$$

This operator equation relates a vector of intra-cavity coordinates, $\mathbf{u}_{\rm ic} = [q, p, X_1, Y_1, X_2, Y_2]^T$, and a vector of input noises driving the system, $\mathbf{u}_{\rm in} = [0, F_{\rm th}, \sqrt{2\gamma_c}X_{{\rm in},1}, \sqrt{2\gamma_c}Y_{{\rm in},1}, \sqrt{2\gamma_c}X_{{\rm in},2}, \sqrt{2\gamma_c}Y_{{\rm in},2}]^T$, which arise from coupling with the environment. Elements of $\mathbf{u}_{\rm ic}$ include the coordinates q, p of the mirror, and the cavity optical mode quadrature operators defined by $X = (a^\dagger + a)/\sqrt{2}, Y = i(a^\dagger - a)/\sqrt{2}$. Elements of $\mathbf{u}_{\rm in}$ include a Langevin force $F_{\rm th}$ driving Brownian motion of the mirror, and the vacuum noises $X_{{\rm in},j}, Y_{{\rm in},j}$ entering each cavity mode. The coupling matrix is

$$\boldsymbol{K} = \begin{bmatrix} 0 & 1/m & 0 & 0 & 0 & 0 \\ -m\omega_m^2 & -\gamma_m & \hbar G_1 & 0 & \hbar G_2 & 0 \\ 0 & 0 & -\gamma_c & \Delta_1 & 0 & 0 \\ G_1 & 0 & -\Delta_1 & -\gamma_c & 0 & 0 \\ 0 & 0 & 0 & 0 & -\gamma_c & \Delta_2 \\ G_2 & 0 & 0 & 0 & -\Delta_2 & -\gamma_c \end{bmatrix}. \tag{4}$$

Here the cavity mode operators are represented in the frame rotating with their drive fields, so that only their detunings appear in the equations, and $G_j = \alpha_j\omega_c/L$ parametrizes the opto-mechanical coupling. The intra-cavity amplitude near resonance is related to the incident power I_j by $\alpha_j^2 = 4I_j\gamma_c/[\hbar\omega_c(\gamma_c^2 + \Delta_j^2)]$, and the detuning of each field is $\Delta_j = (1-\bar{q}/L)\omega_c - \omega_j$. All other parameters are defined in Table I.

Taking the Fourier transform $\mathcal{F}\{f(t)\} = (2\pi)^{-1/2} \times \int dt\, f(t)e^{-i\Omega t}$, it is straightforward to solve Eq. 3 algebraically for $\mathbf{u}_{\rm ic}$ in terms of $\mathbf{u}_{\rm in}$ [14]. To gain insight into the solution, we begin with the case where $G_1 = G_2 = 0$, decoupling the subsystems. Then the mirror's equation of motion is that of a thermally driven pendulum, $q(\Omega) = \chi_m(\Omega)F_{\rm th}(\Omega)$, where the mechanical susceptibility to force is given by $\chi_m(\Omega) = [m(\omega_m^2 + i\gamma_m\Omega - \Omega^2)]^{-1}$.

Turning on the interaction has two effects on the mirror. First, it introduces new driving terms due to radiation pressure noise. Second, it alters the mirror's response function. When motion is slow on the cavity timescale ($\Omega \ll \gamma_c$), the opto-mechanical susceptibility may still be written in the form $\chi_{\rm eff}(\Omega) = \left[m(\omega_{\rm eff}^2 + i\gamma_{\rm eff}\Omega - \Omega^2)\right]^{-1}$, but the system's new resonance parameters are:

$$\begin{aligned} \omega_{\rm eff}^2 &= \omega_m^2 + \sum_j \omega_{{\rm eff},j}^2; \quad \omega_{{\rm eff},j}^2 = -\frac{\hbar G_j^2\Delta_j}{m(\gamma_c^2 + \Delta_j^2)} \\ \gamma_{\rm eff} &= \gamma_m + \sum_j \gamma_{{\rm eff},j}; \quad \gamma_{{\rm eff},j} = -\frac{2\gamma_c\omega_{{\rm eff},j}^2}{\gamma_c^2 + \Delta_j^2} \end{aligned}. \tag{5}$$

The coupling strengths and detunings of the two optical fields can be chosen so that the effective resonant frequency and damping rate have positive sign, and are dominated by terms of optical origin. These are the conditions needed to realize a stable optical trap [25].

Output variances.—The optical fields exiting the cavity are potentially quantum-correlated, due to the coupling of their intra-cavity amplitude and phase with the motion of a common mirror. To study these correlations, the variance matrix of the output fields is obtained from the solution to Eq. 3 via the cavity input-output relation, $a_{\rm in} + a_{\rm out} = \sqrt{2\gamma_c}a_{\rm ic}$. First, as $i\Omega$ occurs asymmetrically in the frequency-domain equations, the operators must be made Hermitian by combining the positive and negative frequency parts: $O^H(\Omega) = (O(\Omega) + O(-\Omega))/\sqrt{2}$. Subsequently, one finds the variance matrix of the output spatial mode at sideband frequency Ω, in terms of the correlation spectra of the noise inputs, which are [30]:

$$\begin{aligned} \langle F_{\rm th}(\Omega)F_{\rm th}(\Omega')\rangle &= 2\gamma_m m\hbar\Omega N(\Omega)\delta(\Omega + \Omega') \\ \langle a_{{\rm in},j}(\Omega)a_{{\rm in},k}^\dagger(\Omega')\rangle &= \frac{1}{2}\delta_{j,k}\delta(\Omega + \Omega') \\ \langle a_{{\rm in},j}(\Omega)a_{{\rm in},k}(\Omega')\rangle &= \langle a_{{\rm in},j}^\dagger(\Omega)a_{{\rm in},k}^\dagger(\Omega')\rangle = 0 \end{aligned} \tag{6}$$

with $N(\Omega) = (e^{\hbar\Omega/k_BT} - 1)^{-1}$. Moreover, the only non-vanishing commutator among the output fields is $[X_{{\rm out},j}^H(\Omega), Y_{{\rm out},j}^H(\Omega')] = i\delta(\Omega - \Omega')$.

Applying Eq. 2 to these modes, one can show that for $\Omega \ll \omega_{\rm eff}$, the logarithmic negativity of the output fields is approximately constant and can be written simply:

$$E_{N,{\rm out}} = -\frac{1}{2}\ln\left(1 + 2\xi\left[\Theta - \sqrt{\Theta^2 + \xi^{-1}}\right]\right), \tag{7}$$

where ξ and Θ are dimensionless quantities parametrizing the entangler strength, and the degradation due to thermal noise, respectively. They are defined as:

$$\begin{aligned} \xi &= \frac{4\gamma_c^2}{\Delta_1\Delta_2}\frac{\omega_{{\rm eff},1}^2\omega_{{\rm eff},2}^2}{\omega_{\rm eff}^4} \\ \Theta &= 1 - \frac{\gamma_m}{2\gamma_c}\frac{k_BT}{\hbar(\Delta_1/\omega_{{\rm eff},1}^2 + \Delta_2/\omega_{{\rm eff},2}^2)^{-1}} \end{aligned}. \tag{8}$$

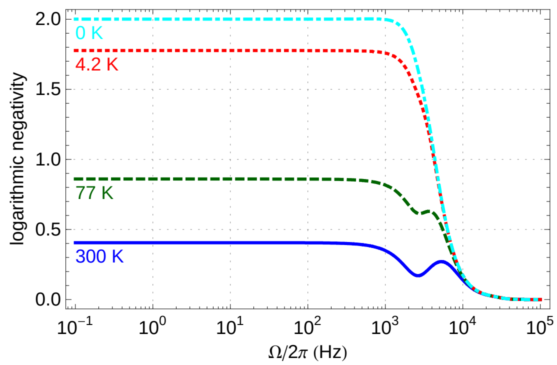

FIG. 2: Predicted logarithmic negativity spectra for entanglement of the output carrier and subcarrier fields, plotted for various ambient temperatures T. Additional parameters are specified in Table I.

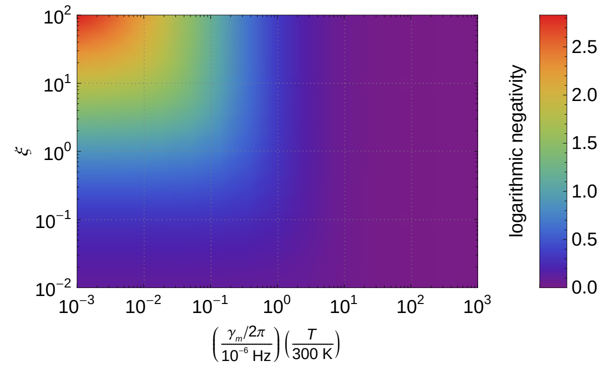

FIG. 3: Logarithmic negativity of output carrier-subcarrier entanglement in the frequency-independent regime. The independent variable on the horizontal axis corresponds to the thermal noise degradation parameter $\Theta - 1$.

Experimental prospects.—An experiment must contend with technical noise sources such as seismic and laser noise, as well as the fundamental noises (vacuum and suspension thermal) that are included in the treatment given here. A detailed noise study exists for the case where light sensing the motion of two gram-scale mirrors is optically recombined in a Fabry-Perot Michelson interferometer [31]. A schematic description of a proposed experiment is shown in Fig. 1, and relevant parameters are summarized in Table I. In such a configuration, the differential motion degree of freedom may be treated as a single cavity wherein common mode laser technical noise largely cancels. In addition, strong restoring and damping forces are supplied to the suspended mirrors by radiation pressure of two detuned optical fields. Consequently the resonant frequency is shifted by 3 orders of magnitude, from $\omega_m/2\pi = 1$ Hz to $\omega_{\text{eff}}/2\pi \approx 2.3$ kHz, with no concomitant increase in the mechanical coupling to the environment. The mirror's response to all external force noises at frequencies well below $\omega_{\text{eff}}/2\pi$ is thereby suppressed by the factor $\omega_m^2/\omega_{\text{eff}}^2$. This combination of noise cancellation and suppression should expose the fundamental noises in a frequency band below $\Omega/2\pi \sim 1$ kHz. Within this spectral window, prospects for entanglement can be evaluated using the analysis described above.

Results of numerical evaluation of $E_{N,\text{out}}(\Omega)$ are presented in Fig. 2, showing that entanglement of the output light should be produced within the frequency band of interest, and that it is remarkably robust against thermal noise — even surviving a room-temperature environment. The spectra are flat until a thermally-induced depression at the effective resonant frequency $\omega_{\text{eff}} \approx 2.3$ kHz, with a cut-off at the cavity linewidth $\gamma_c/2\pi \approx 9.5$ kHz; the magnitude at low frequency is well approximated by Eq. 7.

Given the assumptions of Table I, the entangler strength parameter is $\xi \approx 13.2$, and the thermal degradation parameter is $\Theta \approx 1.8$ at room temperature. In this "strong entangler" limit, one finds

$$E_{N,\text{out}} \xrightarrow{\xi \gg 1} -\frac{1}{2}\ln\left(1 - \frac{1}{\Theta} + \frac{1}{4}\frac{\xi^{-1}}{\Theta^3}\right), \tag{9}$$

from which it is evident that the magnitude of the negativity is being constrained solely by Θ, as depicted in Fig. 3. Although within the limits of our approximations the output entanglement never totally vanishes, a soft, low-loss suspension is necessary to avoid diminution of the logarithmic negativity by thermal noise. To capture an appreciable fraction of the available entanglement, for a suspension with $\omega_m/2\pi \sim 1$ Hz a quality factor $Q_m = \omega_m/\gamma_m \sim 10^6$ is required. This is experimentally challenging but can be achieved, for example, in suspensions constructed of monolithic fused silica [32].

Finally, we remark that homodyne detection of both output optical fields provides a way to measure their covariance in any desired quadrature, permitting the entanglement borne by these fields to be quantified in an experimental setting. Such techniques have been demonstrated on entangled light produced by optical parametric oscillator systems [33].

Opto-mechanical entanglement.—The mirror not only generates the optical entanglement described above, but also can itself become entangled with the intra-cavity fields. The intra-cavity variance matrix can be recovered either from the above analysis via the inverse Fourier transform, or by applying the Lyapunov equation (as in Ref. 17), with correspondence between the two methods providing a valuable check on the numerics. The results, plotted in Fig. 4, are subject to the caveat that the noise model considered here is expected to be valid only in a limited frequency band below 1 kHz, due to the presence of unmodeled technical noise sources at other frequencies. We note, however, that fundamental noise sources do not preclude generating this form of entanglement of the system. It has been proposed to verify the opto-

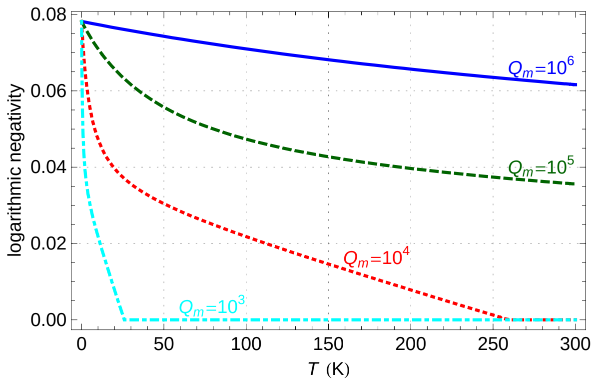

FIG. 4: Logarithmic negativity for bipartite entanglement of the mirror with the intra-cavity carrier field.

mechanical entanglement with the use of an auxiliary cavity and homodyne detection on the output light [17].

Concluding remarks.—We have evaluated the capabilities of a ponderomotive entangler in a novel parameter regime that we believe is experimentally achievable. A singular feature of the system under consideration is the production of entanglement by gram-scale mechanical objects, while immersed in a room-temperature environment. Notable attributes of the apparatus that should allow observation of this entanglement include differential mode noise cancellation in the Fabry-Perot Michelson interferometer configuration, and the isolation from external forces supplied by the stiff optical trap. Construction and operation of this apparatus are underway at our laboratory.

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