The SRC shows a similar behavior, any minor mismatch in the ROCs can be recovered by adjusting the telescope inside the SRC. However, these adjustments will change the Gouy phases inside both recycling cavities by up to 10°. This range has to be included in any follow-on analysis to calculate potential resonances of higher order modes inside the recycling cavities. Based on our results, the tolerances in the manufactured rOCs of these mirrors could be set by the available space to position the mirrors inside the vacuum chamber (typically on the order of 20 cm). This translates into a tolerance of 0.5% in ROC of $PR_3$ and $SR_3$. Much tighter tolerances have to be put on our knowledge of the ROCs before the mirrors can be installed or installation procedures have to be developed which allow to place the mirrors in the appropriate position for the as-build ROCs.

### *5.2. Test masses*

The expected tolerances in the ROCs for the Advanced LIGO test masses are $\pm 10$ m or about $\pm 0.5\%$ of the $\sim 2000$ m ROCs. These deviations from the nominal ROCs will change the eigenmode inside the arm cavities and will reduce the mode matching between the recycling cavities and the arm cavities. The left graph in Fig. 6 shows the mode matching between the eigenmode of the PRC and the eigenmode of the arm cavity as a function of the ROCs of the two test masses assuming that the recycling cavity mirrors are at their nominal position. Note that the ranges for the ROCs are already a factor of two larger than the above mentioned tolerances. Even without any corrections in the mode matching telescope, the mode matching will stay above 99.5%. This figure can also be used to estimate the mode mismatch between the two arm cavities. In the worst case, the mismatch will be twice what is shown in the graph when one set of mirrors is off by $+10$ m and the other by $-10$ m, respectively. The SRC shows a similar behavior.

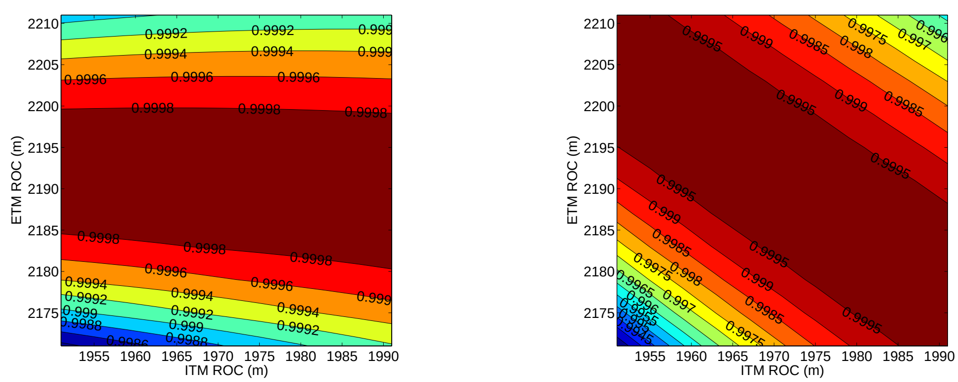


Fig. 6. The mode matching (in power) between the power recycling cavity and the (average) arm cavity as a function of ITM and ETM ROC. The left graph shows the mode matching w/o length adjustments. The mode matching between recycling cavity and arm cavity after adjusting the distances inside the beam expanding telescope becomes essentially 100.00%. The right graph shows the mode matching between the input mode cleaner and the power recycling cavity/arm cavity after adjusting the distances inside the power recycling cavity. These adjustments were made without changing the overall length of the recycling cavity and without changing the mode matching from the input mode. The mode matching between the output mode cleaner, the signal recycling cavity, and the arm cavities shows a similar behavior.

#96414 - $15.00 USD Received 19 May 2008; revised 17 Jun 2008; accepted 17 Jun 2008; published 23 Jun 2008

Adjusting the distance between $PR_2$ and $PR_3$ makes the mode matching between recycling cavity and arm cavity to virtually 100%. However, the mode matching from MC to the recycling cavity remains greater than 99.1% as shown in the right graph of Fig. 6. This assumes that the input mode is fixed. However, we can adjust the input mode by adjusting the mirrors present before $PR_1$ or by increasing the power from the laser. A 1% decrease can easily by adjusted by increasing the laser power without worrying about any additinal thermal effects. Similarly, for SRC, the output MC can be adjusted to the new SRC mode.

## 6. Summary

The PRC in the current LIGO detector consists essentially of flat mirrors and has a transversal mode spacing well below the linewidth of the cavity. Consequently, the spatial eigenmodes of the RF sidebands which are used to control all longitudinal and angular degrees of freedom are not well confined. Only the installation of a thermal correction system allowed LIGO to reach its current design sensitivity. The next major upgrade of LIGO, Advanced LIGO, will use power and signal recycling to enhance the carrier and the signal sidebands. In this paper we describe the new design for both recycling cavities which have well defined spatial eigenmodes and transversal mode spacings well above the linewidth of the cavities. We also discussed the allowed mode matching losses between the recycling cavities and the arm cavities. The main part of the paper shows that this new design is flexible enough and can be adjusted to easily accommodate ROC mismatches as long as the mismatches stay within some tolerances.

## 7. Appendix

A different concept for the stable recycling cavities with less optical components is often mentioned as an alternative to the three-mirror design presented in section 3. This concept uses only one focusing element in addition to the nominal recycling mirror. One version of this concept is shown in Fig. 7. A focusing lens ($PR_2$) which could be polished either into the substrate of the ITM or in the substrate of a compensation plate which will be located directly in front of the ITM. This lens would focus the beam over the length of the recycling cavity. The second element $PR_1$ would then be placed inside the Rayleigh range near the waist of the mode to accumulate a reasonable Gouy phase. An alternative design simply replaces the lens with a curved mirror similar to the curved mirror used in the three-mirror design.

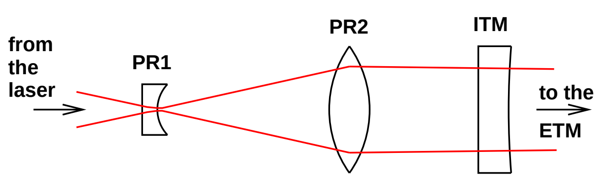


Fig. 7. A two element design for the stable recycling cavities. This design uses a focusing lens (PR2) and one curved mirror (PR1). The focusing lens could be formed inside the thermal compensation plate or inside the ITM substrate. Instead of a focusing lens, it is also possible to use a large curved mirror to focus the beam.

The main disadvantage of this design is that the divergence angle of the Gaussian mode would have to be:

$$\alpha = \frac{w_{ITM}}{L+\Delta} = \frac{w_0}{z_R} = \frac{\lambda}{\pi w_0}$$

where $w_{ITM} \approx 5.5\,\mathrm{cm}$ would be the beam size on $PR_2$, $L$ is the distance to $PR_1$, $L+\Delta$ is the distance to the waist, $w_0$ is the waist of this mode, and $z_R$ is the Rayleigh range. A distance $L$

#96414 - $15.00 USD Received 19 May 2008; revised 17 Jun 2008; accepted 17 Jun 2008; published 23 Jun 2008

that could fit into the LIGO vacuum envelope without folding the recycling cavity furthermore (which makes this a 3 mirror design) is $L \approx 25\,\mathrm{m}$. The distance between $PR_1$ and the waist of this mode $\Delta$ has to be in the order of the Rayleigh range to have any appreciable Gouy phase or transversal mode spacing inside the recycling cavity:

$$\Delta \approx z_R = \frac{\pi w_0^2}{\lambda} \qquad \Rightarrow \qquad \frac{w_{ITM}}{L + \frac{\pi w_0^2}{\lambda}} = \frac{\lambda}{\pi w_0}$$

Solving this for $w_0$ gives:

$$w_0 = \frac{w_{ITM}}{2} \pm \sqrt{\frac{w_{ITM}^2}{4} - \frac{L\lambda}{\pi}}$$

Using

$$\frac{L\lambda}{4} \approx 8.5 \times 10^{-6}\mathrm{m}^2 \ll 7.6 \times 10^{-4}\mathrm{m}^2 \approx \frac{w_{ITM}^2}{4}$$

we can expand the square root and use only the minus sign as the only reasonable physical solution:

$$w_0 = \frac{w_{ITM}}{2} - \frac{w_{ITM}}{2}\left(1 - \frac{\lambda L}{\pi}\frac{2}{w_{ITM}^2}\right) = \frac{\lambda L}{\pi w_{ITM}} \approx 154\,\mu\mathrm{m}$$

to calculate the waist of this mode. The beamsize on $PR_1$ would then be:

$$w_{PR1} = \sqrt{2}w_0 = 218\,\mu\mathrm{m}$$

The Rayleigh range of such a mode is:

$$z_R(\Delta = \pm z_R) = 7\,\mathrm{cm} \ll L$$

As this is much smaller than the distance between $PR_2$ and $PR_1$, the waist of this mode will also not change when we move it closer to $PR_1$ to change the Gouy phase. In general, any solution which generates a reasonable transversal mode spacing starting with a 5.5 cm and having only 25 m to work with will have to have a Rayleigh range of about 7 cm and beamsizes on $PR_1$ below 250 $\mu$m.

Small beamsizes such as this are usually associated with several potential problems. First of all, the intensity on $PR_1$ inside the power recycling cavity will reach a few $\mathrm{MW/cm}^2$. This might cause life time problems with the coatings. Similar to the three mirror cavity, the short Rayleigh range makes this design very sensitive to ROC mismatches. This can also be compensated by changing the distance between $PR_1$ and $PR_2$. This length change would have to be compensated by also changing the distance between $PR_2$ and the ITM to maintain the overall length to keep the RF-sidebands resonant. Such a change is impossible when the focusing lens is polished into the ITM substrate and virtually impossible when it is polished into the compensation plate (CP) as the CP is suspended from the same suspension system than the ITM. The second design which uses the large curved mirror could accomodate this. However, this design requires to relay the laser beam to the other vacuum chamber and inject the beam from the other side which does not reduce the number of optical components in the entire setup, it tends to increase it.

Another problem of the small beamsizes is associated with alignment sensing and control. An angular motion of a cavity mirror will change the apparent length of the cavity if the beam is not centered on the rotation axis. This piston effect scales with the offset from the axis and the angle by which the mirror rotates. This is independent from the beamsize. But interferometric gravitational wave detectors use wavefront sensing to measure and suppress the angular motion.

#96414 - $15.00 USD Received 19 May 2008; revised 17 Jun 2008; accepted 17 Jun 2008; published 23 Jun 2008

These wavefront sensors measure the amplitude of the generated (1,0) and (0,1) Hermite Gauss mode. The amplitude scales with the beamsize on the rotated mirror:

$$a_{10} = \frac{\delta\alpha}{\lambda}\pi w.$$

Consequently, our sensing signals will be reduced proportional to the beamsize while the piston effect is independent from the beamsize. Larger beams make it easier to measure and control the rotation.

None of the above arguments completely rules out the use of a 2-mirror design for the recycling cavities, however, it does not appear to have any advantages over the three mirror design given the constrains of the current vacuum system and the current general layout.

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