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Gas Damping of the Final Stage in the Advanced LIGO Suspensions

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Abstract The finding by the Trento group (Cavalleri 1 (2009), Cavalleri 2 (2009)) of increased gas damping in a torsion oscillator when moving in a constrained volume and the subsequent application of this finding to the Advanced LIGO suspension (Robertson and Hough (2009)) are analysed in this note. The damping from the residual gas has been reestimated from first principles for the Advanced LIGO suspension geometry and outgassing by the surfaces has been included. The damping is not as large as estimated by Robertson and Hough but is not a negligible component of the Advanced LIGO noise budget.

Unconstrained flow The residual gas has long been known to assert a damping force on the suspended mirrors through the exchange of momentum with the residual gas. The damping calculation is particularly simple in the gas density regime where the mean free path is much larger than any of the dimensions of the test mass and if one assumes unconstrained mean molecular flow in the chamber holding the test mass. The results are a gas damping force ,F, proportional to the test mass velocity, v, given by

$$F = -2\pi\kappa\rho_{\#}a^2\sqrt{m_{mol}k_B T}v$$

where $\rho_{\#}$ is the number density of the residual gas molecules which have a mass m_{mol}. a is the radius of the test mass and κ is a factor approximately 1 which is only important if one wants to really get a result to a few percent as in a kinetic vacuum gauge. The damping mechanism is also a source of the fluctuations of the suspension around equilibrium. The fluctuation-dissipation theorem expressed as the force noise power spectrum relates the damping to the thermal noise as

$$F^2(f) = 4k_B T\,\mathrm{Re}\{Z_{mech}(f)\}$$

where $\mathrm{Re}\{Z_{mech}f)\} = \mathrm{Re}\{\frac{F(f)}{v(f)}\}$ is the real part of the mechanical impedence at the place where the gas damping is impressed on the test mass. The test mass displacement noise power spectrum due to the fluctuating force is $x^2(f) = \frac{F^2(f)}{(m_{tm}\omega^2)^2}$ for frequencies higher than the suspension pendulum frequency. The strain noise from gas damping including the contributions from the four test masses becomes

$$h(f) = \frac{\{32\pi\rho_{\#}a^2(k_B T)^{\frac{3}{2}}\sqrt{m_{mol}}\}^{\frac{1}{2}}}{(2\pi f)^2 L\, m_{tm}}$$

L is the arm length of the interferometer and m_{tm} is the mirror mass. **Figure 1** shows an estimate for the gas damping noise with parameters appropriate for Advanced LIGO : test mass 40kg, test mass radius 17 cm, a residual molecular Hydrogen pressure of 1 x 10^{-8} torr ($\rho_{\#}$ 3 x 10^8 molecules/cc) and an arm length of 4km.

Constrained flow: The measurements by the Trento group show that the gas damping increases when the gas is constrained to move in channels around the test mass even though the pressure is low enough to have free molecular flow (no gas/gas collisions). I found this mysterious and possibly a wrong explanation of their observation. It turns out the Trento group is correct, here is a simple way to see it for the Advanced LIGO test mass.

The Advanced LIGO test masses are suspended with one side next to a compensating and control plate of equal diameter leaving a small space for gas to move between them. The gap between the test mass and the compensating plate is h = 0.5cm. When the test mass and compensating plate move relative to each other the molecular density changes between them since the gas cannot flow in or out of the gap immediately. There is a diffusive flow radially opposite to the direction of the molecular density gradient which satisfies the one dimensional diffusion equation in cylindrical coordinates.

$$D\left(\frac{d^2\rho_{\#}}{dr^2} + \frac{d\rho_{\#}}{r\,dr}\right) = \frac{d\rho_{\#}}{dt}$$

The diffusion constant for free molecular flow between the plates is $D = h\,v_{th}$ where $v_{th} = \sqrt{\frac{k_B T}{m_{mol}}}$ is the thermal velocity of the molecule. The solution of the diffusion equation is separable into a product of a radial function (simple polynomials in r) and a function only of time. The time solutions are exponentials with time constant $\tau = \frac{a^2}{h\,v_{th}} = \frac{a^2}{D}$. The most useful variables to estimate the gas flow and the drive impedance on the test mass are the average molecular density in the gap at a specific time and the flow time constant. These are most easily represented in a mechanical impedance approach by the dynamics of a spring in series with a dashpot. The spring models the gas compression between the test mass and the compensating plate while the dashpot models the gas flow in and out of the gap. The impedance of the dashpot is real $Z_{dp}(f) = \frac{F(f)}{v(f)} = \beta$ while the impedence of the spring is pure imaginary $Z_{sp}(f) = -j\frac{k_{sp}}{\omega}$. The spring constant, the change in the force on the test mass surface due to the gas pressure as the gap is changed, is

$k_{sp} = \frac{\rho_{\#} k_B T \pi a^2}{h}$ The time constant of the model is $\tau = \frac{\beta}{k_{sp}}$ which is the same as the time constant associated with the solution of the one dimensional diffusion equation. The total impedance at the test mass is calculated by combining the individual impedences as parallel circuit elements

$Z_{total} = \frac{Z_{sp} Z_{dp}}{Z_{sp} + Z_{dp}} = \frac{\beta(1\text{-}j\omega\tau)}{(1 + (\omega\tau)^2)}$. At low frequencies, the gas flow dominates the impedance while the gas spring is not much compressed. At high frequencies, the dynamics is dominated by the gas compression and the gas flow plays little role. The gas flow provides the dissipation while the gas compression is conservative.

The calculation of the noise due to the constrained gas damping follows the same procedure as for the unconstrained flow. The real part of the drive impedence is

$$\mathrm{Re}\{Z(f)\} = \frac{\beta}{(1+(\omega\tau)^2)} = \frac{k_{sp}\tau}{(1+(\omega\tau)^2)} = \frac{\rho_{\#}(m_{mol}k_BT)^{\frac{1}{2}}\pi a^4}{h^2(1+(\omega\tau)^2)} .$$

Applying to the fluctuation-dissipation theorem and including the four test masses in the interferometer, as well as that only one face of the test mass experiences constrained gas flow, the estimated strain noise from the constrained gas damping becomes

$$h(f) = \frac{\{8\pi\rho_{\#}a^4(k_BT)^{\frac{3}{2}}\sqrt{m_{mol}}\}^{\frac{1}{2}}}{(1+(\omega\tau)^2)^{\frac{1}{2}}h(2\pi f)^2 L\, m_{tm}}$$

At equal ambient gas density the ratio of constrained to unconstrained gas damping noise becomes $\frac{a}{2h\sqrt{(1+(\omega\tau)^2)}}$ a factor of 17 at frequencies less than $\frac{1}{2\pi\tau}$ (31Hz for H_2 and 11 Hz for H_2O) in the current Advanced LIGO suspension design. The ratio and also the calculations no longer apply when a and h become comparable. **Figure 1** shows a variety of cases for the gas damping noise as a function of frequency.

Outgassing by the surfaces A situation that needs to be considered is the additional pressure increase in the constrained space of the gap if the surfaces outgas. Initially, I thought this might have been part of the reason for the observations in Trento. The estimate for the effect of outgassing follows easily from the cylindrically symmetric diffusion equation. If one assumes a uniform outgassing rate, J, with only a slow change with time as compared to the diffusion time between the walls of the gap, the solution for the almost time independent molecular density in the gap is

$$\rho_{\#} = \frac{\pi J}{2h^2 v_{th}}(a^2 - r^2) + \rho_{\#\ ambient} .$$

The expression includes a factor of 2 for outgassing by both surfaces facing the gap. The noise calculations use the average molecular number density in the gap which is

$\langle\rho_{\#}\rangle = \frac{\pi J a^2}{3h^2 v_{th}} + \rho_{\#\ ambient}$. A good estimate for the outgassing rate, J ,by fused quartz after the processing we do is not available. There is data in the literature (O'Hanlon (1989), Schram(1963), Dushman (1962)) which indicates that water outgassing dominates from cleaned but not baked glasses. The outgassing has the typical

$J(t) = \frac{\alpha(\text{torr liters/sec cm}^2)}{t(\text{hours})}$ dependence from surfaces with a distribution of adsorption site energies. α ranges from 10^{-7} to 10^{-8} torr liters/sec cm^2 after 1 hour under vacuum. **Figure 1** shows the noise bounds for the best and worst case water outgassing after 100 hours of pump-down.

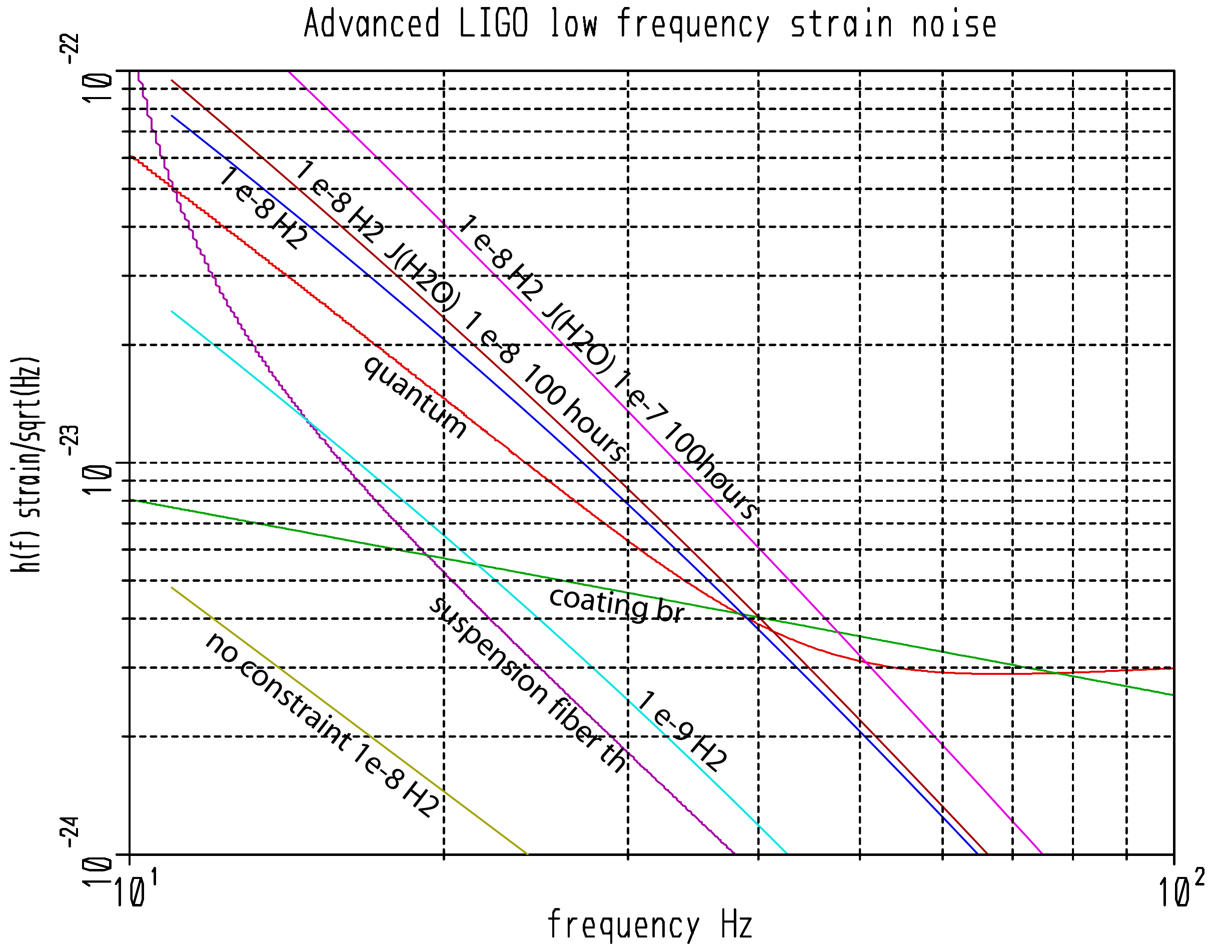

Figure 1 The Advanced LIGO low frequency strain noise estimates from "Advanced LIGO Systems Design" LIGO-T010075-v2 September 18, 2009 and the various estimates for gas damping noise. The curve labeled "no constraint" is most likely the gas damping noise contribution used in making the suspension noise estimate labeled "suspension fiber th". The remaining curves are a variety of estimates of the gas damping noise with the gap constraint included. The curves labeled 1 e-9 H2 and 1 e-8 H2 show the gas damping of molecular Hydrogen at 10^{-9} and 10^{-8} torr. The design pressure of Hydrogen, the major gas species after 100 hours of pumping of the LIGO chambers, is 10^{-8} torr. The increased noise due to the outgassing of water by the test mass and compensating plate surfaces facing the gap is shown for an ambient Hydrogen pressure of 10^{-8} torr. The two curves represent the noise after 100 hours of pumping for water outgassing rates that were 10^{-7} and 10^{-8} torr liters/sec cm^2 after 1 hour of pumping .

Strategies to deal with the increased gas damping noise

Increasing the gap width , h , between the test mass and the compensating plate will reduce the strain noise from damping by the ambient gas as 1/h and the noise from outgassing by $1/h^2$.If there are no serious other constraints to hold the gap at 0.5 cm, increasing the gap may well be the most economical and robust strategy. I could guess that the forces required to acquire lock are one of the drivers for the small gap although the control forces during operation can be easily accomodated. Increasing the hydrogen pumping speed in the chambers with titanium sublimation pumps or other pumps that are effective in pumping hydrogen will reduce the strain noise from gas damping as $1/\sqrt{F_{pump}}$ but will not reduce the noise from the outgassing. One could reduce the noise from outgassing by local heating of the testmass and compensating plate. Improvement expected in the outgassing rate needs further modeling but I could well imagine a reduction in the outgassing rate at room temperature by a factor of 2 for every 8 degrees increment in temperature held for several days.

References

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O’Hanlon (1989) “A User’s Guide to Vacuum Technology” John Wiley & Sons

Roberts (1971) The diffusive flow of gases between parallel plates J. Phys A :Gen Phys **4** 401
a not very informative paper with a seductive title, it does show how one performs the kinetic theory integrals

Robinson and Hough (2009) Gas Damping in Advanced LIGO Suspensions LIGO-T0900416-v1

Schram (1963) Le Vide **103** 55

I did run a finite difference one dimensional diffusion program and got the same results for the molecular density distributions and time dependences as the analytic solutions being presented here. So if the assumptions are correct the mathematics at least is consistent.