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Alignment signal extraction of the optically degenerated RSE interferometer using wave front sensing technique

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Abstract. The alignment sensing and control scheme of the resonant sideband extraction interferometer is still unsettled issue for next generation gravitational wave antennas. The difficulty is that it is not easy to extract separate error signals for 12 angular degrees of freedom, which is mainly arising from the complexity of the optical system and cavity "degeneracy". We have suggested new sensing scheme giving reasonably separated signals which is fully compatible with length sensing scheme. The key of this ides is to resolve "degeneracy" of the optical cavities. By choosing appropriate Gouy phase for the degenerated cavities, the alignment error signals with much less admixtures can be extracted.

1. Introduction

Modern technology of the gravitational wave (GW) antennas is based on a precision measurement using laser interferometory. Only when the relative alignments of the interfering laser beams are optimized, the maximum sensitivity of the instruments can be expected. There are several terrestrial laser interferometer GW antennas operating at present[1], and the alignment sensing and control of the interferometers is, in fact, recognized as an essential technique for maximum performance, stable and robust operation of the instruments.

The next generation of GW antennas are going to adopt resonant sideband extraction (RSE) scheme as an optical configuration to have better sensitivity to GW signals. Meanwhile, this complex design of the interferometer makes it hard to extract appropriate error signals for alignment sensing. In addition, the conventional optical design of ground-based GW antennas have so called optically "degenerated" cavities (or geometrically unstable cavity) for power recycling cavity and a signal extraction cavity for RSE. This certainly happens when the power recycling (signal extraction) cavity is much shorter than the arm cavity. Under this condition, it is quite hard to extract separated error signals for mirror misalignments of degenerated cavity, in general.

We started to study the degenerated optical cavity for better understanding of the relation between signal separation and cavity degeneracy, then developed new sensing scheme for zero-detuning RSE (LCGT[2]) in an effort to have reasonably diagonalized sensing matrix. In this paper, we propose one of the solutions to have reasonably diagonalized alignment sensing signals by resolving the cavity degeneracy.

2. Alignment signal separation and cavity degeneracy

The investigation of the simple optical cavity (Fabry-Perot cavity) gives clear understanding on the issue of alignment signal separation and the cavity degeneracy. The schematic of the conventional optical cavity and the alignment sensing scheme is shown in figure 1.

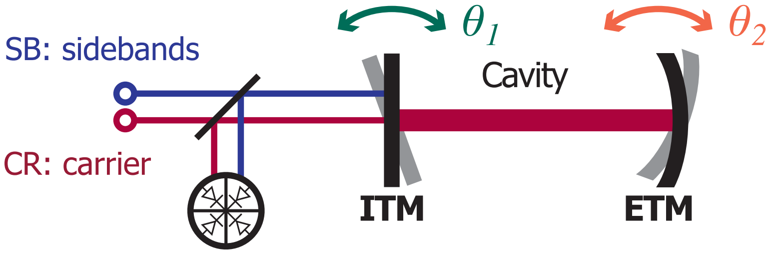

Figure 1. A simple Fabry-Perot cavity illuminated by carrier light of the laser and phase modulation sidebands. The angular motion (misalignment) of the input test mass (ITM) and end test mass (ETM) are referred as θ_1 and θ_2. The reflected light from the cavity is picked off, detected by segmented photodetector then demodulated to give a wave front sensing (WFS) signals.

The modal picture is commonly used to deal with an interfering laser beams with misaligned optics. Any spatial shape of the laser beam is expanded in a modal space and its expressioi is given as a superposition of the higher order spatial modes. In this picture, the misaligned optics behave as a scattering object, from one mode to the other, and in this sense, they are sometimes called as "generator" (of higher order modes). Thus the expressions of laser field at any points can be given in a series of higher order modes, however, in a small angle misalignment approximation, the expansion up to first order mode can give good enough approximation.

The difference of a phase evolution between different spatial modes, which is called as "Gouy phase", is a key parameter for alignment signal sensing. Fundamental and first order mode of Hermit-Gaussian modes are given as

$$\begin{aligned} U_{00}(x,y,z) &= U_0(x,z)U_0(y,z)e^{i(-kz+\eta(z))} \\ U_{10}(x,y,z) &= U_1(x,z)U_0(y,z)e^{i(-kz+2\eta(z))}, \end{aligned}$$

which is showing that characteristic difference is Gouy phase η in terms of phase evolution. The laser beam is supposed to propagate to $+z$ direction and x-y plane gives cross section of the laser beam. $k = 2\pi/\lambda$ is wave number of the laser field. $U_l(x(\text{or}\,y), z)$ is a spacial distribution function of the fields in a x-y plane, which defines the mode pattern on the plane, is given as

$$U_0(x,z) = \left(\frac{2}{\pi\omega(z)^2}\right)^{1/4}\left(\frac{1}{l!\,2^l}\right)^{1/2} H_l\left(\frac{\sqrt{2}x}{\omega(z)}\right)\exp\left[-\left(\frac{x}{\omega(z)}\right)^2 - i\frac{k}{2R(z)}x^2\right].$$

$\omega(z) = \omega_0\sqrt{1+(z/z_0)^2}$ is a beam radii at $z = z$ with waist size of ω_0 and Rayleigh range of $z_0 = k\omega_0^2/2$. $H_l(x)$ is a Hermit polynomials and $R(z) = (z^2+z_0^2)/z$ is a radii of curvature of the wave front.

Gouy phase $\eta = \arctan(z/z_0)$ is defined as a function of Rayligh range and the propagation distance. In a region where $z \simeq z_0$, η has some finite quantities. On the other hand, in a limit of $z \ll z_0$, η goes zero, which is usually stated as "the cavity is degenerated". In other words, the deference between fundamental and first order modes becomes negligible in terms of phase evolution with beam propagation for degenerated cavity. Wave Front Sensing (WFS) technique is assumed as a baseline of alignment sensing method for existing and planned GW antennas so far. WFS signals after demodulation at segmented photodiode are given as

$$\frac{\partial V}{\partial \theta_i} = \Re\left[\frac{\partial \boldsymbol{V}}{\partial \theta_i} e^{i\eta_D} e^{i\delta}\right],$$

therefore, the separability of the signals is uniquely determined by the difference of complex vector $\partial \boldsymbol{V}/\partial\theta_j$ for $j = 1, 2$. Analytical expressions of WFS sensitivities for each mirrors are given as

$$\begin{aligned} \frac{\partial \boldsymbol{V}}{\partial \theta_1} &\simeq (r_{\mathrm{S00}} - r_{\mathrm{C00}}) + r_{\mathrm{S00}}\, g_{\mathrm{C00}}\, g_{\mathrm{C10}}\, e^{2i\eta} \\ \frac{\partial \boldsymbol{V}}{\partial \theta_2} &\simeq r_{\mathrm{S00}}\, g_{\mathrm{C00}}\, g_{\mathrm{C10}}\, e^{i\eta}. \end{aligned}$$

$r_{\mathrm{Xnn}} = -r_{\mathrm{ITM}} + r_{\mathrm{ETM}}/(1 - r_{\mathrm{ITM}}\, r_{\mathrm{ETM}} e^{i\phi})$ is a cavity reflectivity for the carrier (X=C) and phase modulation sidebands (X=S) of the fundamental (nn=00) mode, while $g_{\mathrm{Xnn}} = t_{\mathrm{ITM}}/(1 - r_{\mathrm{ITM}}\, r_{\mathrm{ETM}} e^{i\phi})$ is a field enhancement factor inside the cavity for the carrier and the sidebands of the fundamental (nn=00) and first order (nn=10) modes.

The usual way of WFS technique to obtain independent signals for cavity mirror misalignments is to make use of different response to θ_1 and θ_2. Therefore, from above expressions, one may notice that cavity Gouy phase η is a key and also an only parameter to make difference between them, with given optical design. Signal amplitude and phase for the θ_1 and θ_2 in a function of η are shown in figures 2, 3. As for degenerated cavity ($\eta \ll 1$), the signal amplitude is getting bigger whereas relative phase difference is quite small. The demodulated signal behave almost same for the variance of θ_1 and θ_2. On the other hands, for non-degenerated, conventional design cavity, the signals are relatively small but the degeneracy of the relative phase dissolves. This means that separate error signals for two optics misalignments are available by taking appropriate cavity Gouy phase η. This is an idea of this letter.

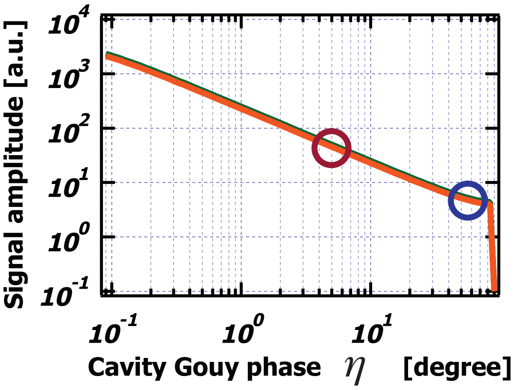

Figure 2. Amplitude of a complex signal vector $\partial \boldsymbol{V}/\partial\theta_j$ in a function of Gouy phase η. Both misalignment information for θ_1 and θ_2 behave almost identically.

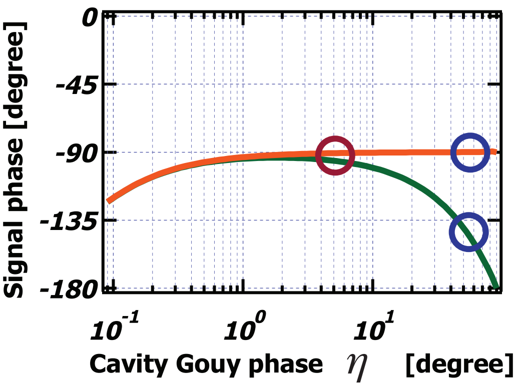

Figure 3. Phase of a complex signal vector $\partial \boldsymbol{V}/\partial\theta_j$ in a function of Gouy phase η. When η has reasonable value, the response of WFS signal to θ_1 and θ_2 can be different.

3. Application to RSE interferometer

The description of the RSE interferometer can be found in a reference[3, 4] and the references therein. As the simplified schematic of the RSE is shown in figure 4 , there are five longitudinal degrees-of-freedom and twelve angular degrees-of-freedom for pitching and yawing to be controlled, except the beam splitter.

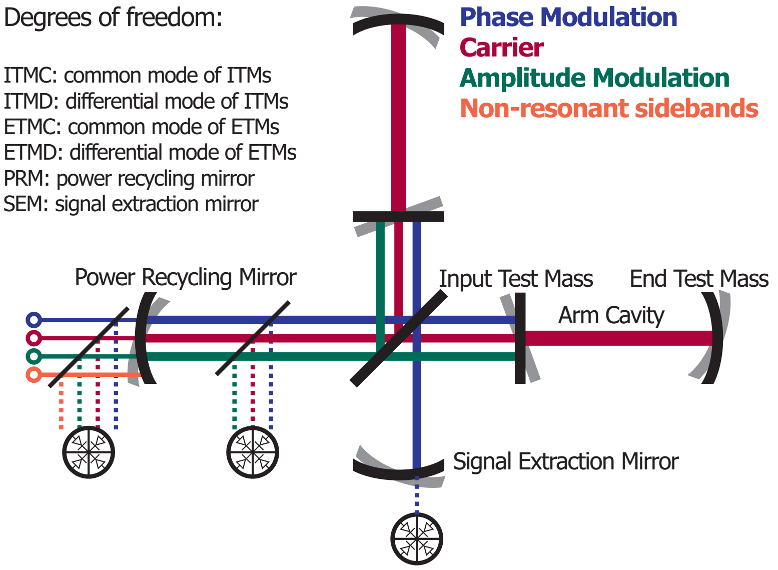

Figure 4. Simplified schematic of RSE interferometer with laser fields. Total four different frequencies of light fields (carrier, phase and amplitude modulation and non-resonant sidebands) are illuminating the interferometer. Three signal ports, bright, pick off and dark ports are dedicated for signal extraction. Angular degrees of freedom of each arm cavity mirrors are translated to common- and differential-components and re-defined as shown in a figure.

The LCGT conceptual design is a baseline for designing of sensing scheme in this paper. Optical and geometrical parameters, modulation scheme and signal extraction ports are based on a “LCGT design book ”[5]. In order to extract reasonably separated error signals, the idea described in previous section was extended to the case of RSE. The points of scope for the alignment sensing scheme are

- Use optimum cavity Gouy phases η for PRC and SEC [1]
- Additional non-resonant sidebands.

The sensing signals were calculated analytically in order to give an intuitive understanding of the nature, then evaluated numerically using LCGT parameters to give a sensing matrix. The matrix is normalized so that the diagonal elements become unity. The sensing matrix with resolved degenerated-cavity (optimized Guoy phase for degenerated cavity) looks reasonably diagonal. There are still some significant off-diagonals, for example admixture of ITMD signal into ETMD signal in extracted signal at dark port, however, this admixture can be easily compensated and signals are well-separated by signal manipulations. In addition, sensing matrix elements are also listed for degenerated cavity RSE interferometer (conventional design) in *slanted* fonts. Serious numbers of off diagonals which have considerable quantity of admixture are reduced and the matrix is improved to reasonably diagonal shape.

4. Summary

There are two major factors why it is not so easy to extract separated error signals for angular control of RSE interferometer with extension of WFS technique. One is from the intrinsic

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[1] There are several ways to resolve the degeneracy of the optical cavity. Having long recycling cavity or mode conversion telescope inside the cavity are example of the ways. Another one is to have lens-like ITMs which focus the beam inside the cavity to have reasonable Gouy phase.

Table 1. The alignment sensing matrix for non-degenerated (upper) and degenerated (lower in *slanted* fonts) RSE interferometers. Signals are from either bright, pick-off or dark ports, with single or double demodulation[4].

Port	Demod.	*ITMD*	*ETMD*	*ITMC*	*ETMC*	*PRM*	*SEM*
Bright	PM-AM	1	-2×10^{-3}	3×10^{-4}	-5×10^{-7}	2×10^{-3}	-1×10^{-5}
Dark	CR-PM	**0.6**	1	-5×10^{-5}	-1×10^{-2}	6×10^{-2}	0
Bright	AM-NR	-9×10^{-3}	1×10^{-5}	1	-2×10^{-3}	0	0
Pickoff	CR-NR	1×10^{-4}	3×10^{-4}	0	1	**2**	0
Bright	AM-NR	5×10^{-5}	6×10^{-7}	0	1×10^{-5}	1	0
Pickoff	PM-AM	2×10^{-4}	4×10^{-7}	0	6×10^{-6}	**0.9**	1

Bright	*PM-AM*	*1*	-2×10^{-4}	-2×10^{-3}	4×10^{-6}	2×10^{-4}	5×10^{-4}
Dark	*CR-PM*	**0.6**	*1*	5×10^{-5}	3×10^{-5}	-1×10^{-4}	*0*
Bright	*PM-AM*	4×10^{-5}	-7×10^{-8}	*1*	-2×10^{-3}	**−0.6**	**−0.4**
Bright	*CR-NR*	-3×10^{-3}	-8×10^{-3}	**0.6**	*1*	-2×10^{-2}	*0*
Bright	*CR-AM*	-2×10^{-4}	*0*	**−1**	2×10^{-4}	*1*	*0*
Bright	*CR-PM*	-7×10^{-4}	-9×10^{-4}	**−2**	1×10^{-2}	**1**	*1*

complexity of the intricate coupled cavity system of RSE. Another one is from the fact that conventional design of RSE are employing nearly degenerated recycling cavity. We proposed new alignment sensing scheme based on a WFS technique to give a reasonably separated, diagonalized signals. The key is to resolve the degeneracy of the cavity, which might be an only straightforward way to give such a signals.

Acknowledgments

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