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Alignment signal extraction of the optically degenerate RSE interferometer using the wave front sensing technique

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Abstract. The alignment sensing and control scheme of the resonant sideband extraction interferometer is still an unsettled issue for the next-generation gravitational wave antennas. The issue is that it is difficult to extract separate error signals for all 12 angular degrees of freedom, which is mainly arising from the complexity of the optical system and cavity “degeneracy”. We have suggested a new sensing scheme giving reasonably separated signals which is fully compatible with the length sensing scheme. The key of this idea is to resolve the “degeneracy” of the optical cavities. By choosing an appropriate Gouy phase for the degenerate cavities, alignment error signals with much less admixtures can be extracted.

1. Introduction

Modern technology of gravitational wave (GW) antennas is based on a precision measurement using laser interferometry. Only when the relative alignments of the interfering laser beams are optimized, can the maximum sensitivity of the instruments be expected. There are several terrestrial laser interferometer GW antennas operating at present[1, 2], and the alignment sensing and control of the interferometers is, in fact, recognized as an essential technique for maximum performance as well as a stable and robust operation of the instruments.

The next generation GW antennas are going to adopt the resonant sideband extraction (RSE) scheme as an optical configuration to have better sensitivity to GW signals. Meanwhile, this complex design of the interferometer makes it hard to extract appropriate error signals for alignment sensing. In addition, the conventional optical design of ground-based GW antennas has so called optically “degenerate” cavities (or geometrically marginally-stable cavities) for power recycling cavity and a signal extraction cavity for RSE. As is the case for first-generation GW antennas, this degeneracy usually happens when a much shorter power recycling or signal extraction cavity (compared with the arm cavity) is connected to a non-degenerate arm cavity sharing a common laser beam mode. Under this condition, it is in general quite hard to extract separate error signals for mirror misalignments of the degenerate cavity.

We started to study the degenerate optical cavity for better understanding of the relation between signal separation and cavity degeneracy, then developed a new sensing scheme for zero-detuned RSE (LCGT[3]) in an effort to have reasonably diagonalized sensing matrix. In this paper, we propose one of the solutions to have reasonably diagonalized alignment sensing signals by resolving the cavity degeneracy.

2. Alignment signal separation and cavity degeneracy

The investigation of the simple optical cavity (Fabry-Perot cavity) gives a clear understanding of the issue of alignment signal separation and the cavity degeneracy. The schematic of the conventional optical cavity and the alignment sensing scheme is shown in figure 1.

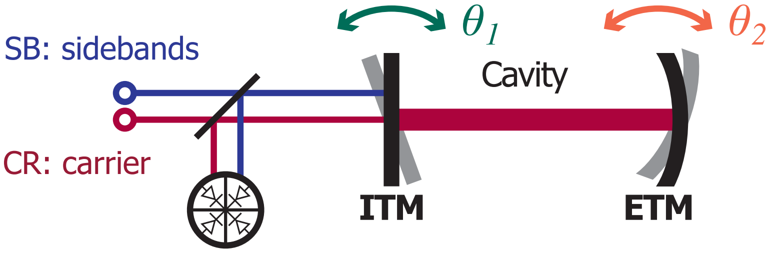

Figure 1. A simple Fabry-Perot cavity illuminated by carrier light of the laser and phase modulation sidebands. The angular motion (misalignment) of the input test mass (ITM) and end test mass (ETM) are referred to as θ_1 and θ_2. The reflected light from the cavity is picked off, detected by a segmented photodetector then demodulated to give a wave front sensing (WFS) signal.

The modal picture is commonly used to deal with interfering laser beams with misaligned optics. Any spatial shape of the laser beam is expanded in a modal space and its expression is given as a superposition of higher order spatial modes. In this picture, the misaligned optics behave as a scattering object, from one mode to the other, and in this sense, they are sometimes called "generators" (of higher-order modes). Thus the expressions of the laser field at any point can be given in a series of higher-order modes; and, in a small angle misalignment approximation, the expansion up to only the first-order mode can give good enough approximation.

The difference of a phase evolution between different spatial modes, which is called the "Gouy phase", is a key parameter for alignment signal sensing. Fundamental and first order mode of Hermit-Gaussian modes are given as

$$\begin{aligned} U_{00}(x,y,z) &= U_0(x,z)U_0(y,z)e^{i(-kz+\eta(z))} \\ U_{10}(x,y,z) &= U_1(x,z)U_0(y,z)e^{i(-kz+2\eta(z))}, \end{aligned}$$

which shows that the essential difference in the terms of the phase evolution is the Gouy phase η. The laser beam has its waist at $z=0$ and is supposed to propagate in the $+z$ direction. Therefore, the x-y plane represents the cross section of the laser beam. The wave number of the laser field is $k=2\pi/\lambda$, $U_l(x(\text{or}\, y),z)$ is a spatial distribution function of the fields in the x-y plane defining the planar mode pattern of the order l, which is given by

$$U_l(x,z) = \left(\frac{2}{\pi\omega(z)^2}\right)^{1/4}\left(\frac{1}{l!\,2^l}\right)^{1/2} H_l\left(\frac{\sqrt{2}x}{\omega(z)}\right)\exp\left[-\left(\frac{x}{\omega(z)}\right)^2 - i\frac{k}{2R(z)}x^2\right].$$

The beam radius $\omega(z)=\omega_0\sqrt{1+(z/z_0)^2}$ at $z=z$ is given as a function of waist size ω_0 and Rayleigh range $z_0=k\omega_0^2/2$. The beam pattern is determined by Hermite polynomials $H_l(x)$ and the radius of curvature of the wave front is given by $R(z)=(z^2+z_0^2)/z$. The Gouy phase $\eta=\arctan(z/z_0)$ is defined as a function of Rayleigh range and the propagation distance z. In a region where $z\simeq z_0$, η has finite values, whereas in the limit of $z\ll z_0$, η goes to zero, which is usually stated as "the cavity is degenerate". In other words, the difference between fundamental and first-order modes becomes negligible in terms of phase evolution with beam propagation for a degenerate cavity.

The Wave Front Sensing (WFS) technique is assumed as the baseline of the alignment sensing method for existing and planned GW antennas so far. The sensitivity of WFS signals to a normalized misalignment angle $\Theta_i = \theta_i/\theta_{\rm D}$ ($\theta_{\rm D} = \lambda/\pi\omega_0$ is the beam divergence angle) after demodulation at segmented photodiode is given as

$$\frac{\partial V}{\partial \Theta_j} = \Re\left[\frac{\partial \boldsymbol{V}}{\partial \Theta_j} e^{i\eta_D} e^{i\delta}\right],$$

where $\boldsymbol{V}$ is the complex demodulated signal, η_D is the Gouy phase shift between detection port and photodiode through the Gouy phase telescope, and δ is a demodulation phase. Thus the separability of the signals is uniquely determined by the difference of the complex vector $\partial\boldsymbol{V}/\partial\Theta_j$ for $j = 1, 2$. Analytical expressions of WFS sensitivities for each mirror are given as

$$\begin{aligned}\frac{\partial \boldsymbol{V}}{\partial \Theta_1} &\simeq (r_{\rm s00} - r_{\rm c00}) + r_{\rm S00}\, g_{\rm c00}\, g_{\rm c10}\, e^{2i\eta} \\ \frac{\partial \boldsymbol{V}}{\partial \Theta_2} &\simeq r_{\rm s00}\, g_{\rm c00}\, g_{\rm c10}\, e^{i\eta}.\end{aligned}$$

Let $r_{\rm xnn} = -r_{\rm ITM} + r_{\rm ETM}/(1 - r_{\rm ITM}\, r_{\rm ETM} e^{i\phi})$ be the cavity reflectivity for the carrier ($x = c$, ϕ=0) and the phase modulation sidebands ($x = s$, $\phi\sim\pi$) of the fundamental ($nn = 00$) mode, while $g_{\rm xnn} = t_{\rm ITM}/(1 - r_{\rm ITM}\, r_{\rm ETM} e^{i\phi})$ is the field enhancement factor inside the cavity for the carrier and the sidebands of the fundamental ($nn = 00$) and first order ($nn = 10$) modes; $r_{\rm ITM}$ and $r_{\rm ETM}$ are amplitude reflectivities for input and end test masses, respectively.

The usual way of the WFS technique to obtain independent signals for cavity mirror misalignments is to make use of their different response to Θ_1 and Θ_2. Therefore, from the above expressions, one may notice that the cavity Gouy phase η is a key and also the only parameter to make a difference between them, for given optical design. Signal amplitude and phase for the Θ_1 and Θ_2 as functions of η are shown in figures 2 and 3. For a degenerate cavity ($\eta \ll 1$), the signal amplitude is getting bigger whereas the relative phase difference is quite small. The demodulated signal behaves almost the same for variations of Θ_1 and Θ_2. On the

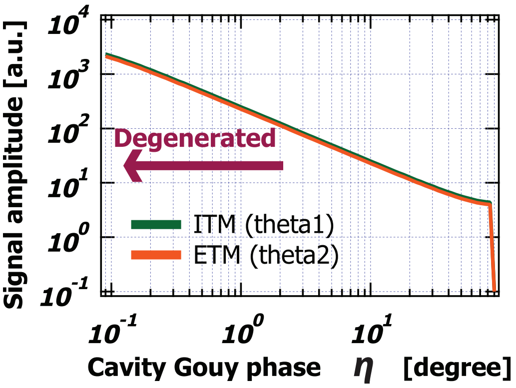

Figure 2. Amplitude of a complex signal vector $\partial\boldsymbol{V}/\partial\Theta_j$ in a function of Gouy phase η. Both misalignment information for Θ_1 and Θ_2 behave almost identically.

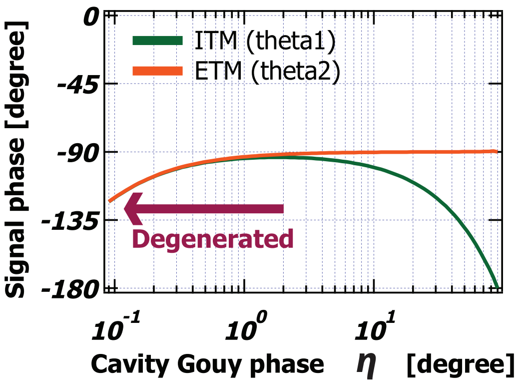

Figure 3. Phase of a complex signal vector $\partial\boldsymbol{V}/\partial\Theta_j$ in a function of Gouy phase η. When η has reasonable value, the response of WFS signal to Θ_1 and Θ_2 can be different.

other hand, for a non-degenerate, conventional-design cavity, the signals are relatively small but the degeneracy of the relative phase dissolves. This means that separate error signals for two optical misalignments are available by taking an appropriate cavity Gouy phase η. This is the idea of this letter.

3. Application to RSE interferometer

The description of the RSE interferometer can be found in references[4, 5] and references therein. As the simplified schematic of the RSE is shown in figure rse, there are five longitudinal degrees-of-freedom and twelve angular degrees-of-freedom for pitch and yaw to be controlled, excepting the beam splitter.

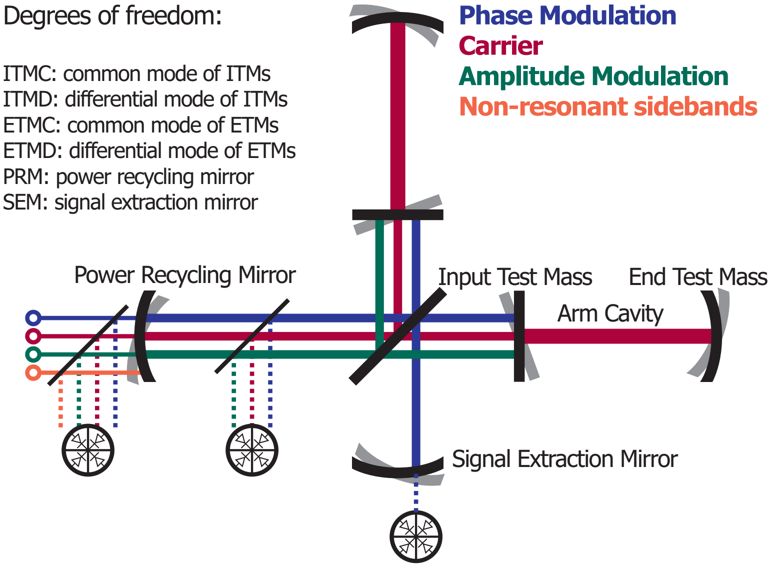

Figure 4. Simplified schematic of the RSE interferometer with laser fields. A total of four different frequencies of light fields (carrier, phase and amplitude modulation and non-resonant sidebands) are illuminating the interferometer. Three signal ports, bright, pick off and dark ports are dedicated for signal extraction. Angular degrees of freedom of each arm cavity mirror are translated to common- and differential-mode components and re-defined as shown in the figure.

The LCGT concept forms the baseline for designing the sensing scheme in this paper. Optical and geometrical parameters, modulation scheme and signal extraction ports are based on a “LCGT design book ”[6]. Key parameters in terms of alignment signal sensing are listed in Table 1.

Table 1. Key parameters of optical design of LCGT.

Reflectivity		Gouy phase	Degenerate	*Non-degenerate*
r^2_{ITM}	0.99995	arm cavity	$0.30\,\pi$	*$0.30\,\pi$*
r^2_{ETM}	0.996	power recycling cavity	$0.0067\,\pi$	*$0.56\,\pi$*
r^2_{PRM}	0.80	signal extraction cavity	$0.0067\,\pi$	*$0.56\,\pi$*
r^2_{SEM}	0.77			

In order to extract reasonably separated error signals, the idea described in the previous section was extended to the case of RSE. The points of scope for the alignment sensing scheme are

- Use optimum cavity Gouy phases η for PRC and SEC [1]
- Additional non-resonant sidebands.

The sensing signals were calculated analytically in order to give an intuitive understanding of the nature, then evaluated numerically using LCGT parameters to give a sensing matrix, as shown in Table 2. The matrix is normalized so that the diagonal elements become unity. The sensing matrix with non-degenerate cavity (optimized Gouy phase for degenerate cavity) looks reasonably diagonal. There are still three significant off-diagonals (in bold fonts), for example the admixture of the ITMD signal into the ETMD signal in the extracted signal at the dark port. However, this admixture can be easily compensated and signals are well-separated by signal manipulations. In addition, sensing matrix elements are also listed for a degenerate-cavity RSE interferometer (conventional design) in *slanted* fonts for comparison. Serious numbers of off-diagonals that have a considerable quantity of admixture are reduced and the matrix is improved to reasonably diagonal shape.

For a design of the instruments, quantitative evaluation and comparison of the quality of the sensing matrix should be done, which, however, it is beyond the scope of this letter. The aim of this paper is to show the newly proposed alignment sensing scheme using the novel idea of resolving the cavity degeneracy. Therefore, the Gouy phases of the degenerate cavities were optimized qualitatively. As can be seen from the sensing matrix for the degenerate case, the admixture of ITMC degree-of-freedom to ETMC, PRM and SEM (and vice versa) are the main contaminant elements. The Gouy phases of the power recycling and signal extraction cavity were carefully chosen so that these off-diagonals disappear. These Gouy phases are common for two cavities, firstly because these two cavities should be identical in length (which is a constraint for the length sensing scheme), and secondly because lense-like ITMs can be supposed as focusing device to produce an appropriate Gouy phase. In this sense, there is still a free parameter to have different Gouy phases for power recycling and signal extraction cavities, and thus there can be some room to further optimize the sensing matrix.

4. Summary

There are two major factors why it is not so easy to extract separate error signals for angular control of RSE interferometer with extension of WFS technique. One stems from the intrinsic complexity of the intricate coupled cavity system of RSE. Another one comes from the fact that the conventional design of RSE employs nearly-degenerate recycling cavities.

We proposed a new alignment sensing scheme based on a WFS technique to give reasonably separated, diagonalized signals. The key is to resolve the degeneracy of the cavities, which might be the only straightforward way to obtain such a signals, because the Gouy phase evolution inside the cavity is an unique and essential parameter. In the application of this nobel idea to the RSE interferometer, the resolved degeneracy of the recycling cavities reduced and improved the admixture of the signals, and accordingly enabled well diagonalizing of the sensing matrix.

Acknowledgments

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[1] There are several ways to resolve the degeneracy of an optical cavity. Having a long recycling cavity or mode conversion telescope inside the cavity are examples of these ways. Another one is to have lens-like ITMs which focus the beam inside the cavity to produce a reasonable Gouy phase.

Table 2. The alignment sensing matrix for non-degenerate (upper matrix) and degenerate (lower matrix in *slanted* fonts) RSE interferometers. Signals are from either bright, pick-off or dark ports, with single or double demodulation[5] with appropriate demodulation phases. In the lower matrix, mainly common mode degrees of freedom are mixing each other for any port signals due to the degeneracy of the recycling cavities. On the other hand, in the upper matrix, the resolved degeneracies enabled reasonable diagonalizing of the sensing matrix.

Port	Demod.	$ITMD$	$ETMD$	$ITMC$	$ETMC$	PRM	SEM
Bright	PM-AM	1	-2×10^{-3}	3×10^{-4}	-5×10^{-7}	2×10^{-3}	-1×10^{-5}
Dark	CR-PM	**0.6**	1	-5×10^{-5}	-1×10^{-2}	6×10^{-2}	0
Bright	AM-NR	-9×10^{-3}	1×10^{-5}	1	-2×10^{-3}	0	0
Pickoff	CR-NR	1×10^{-4}	3×10^{-4}	0	1	**2**	0
Bright	AM-NR	5×10^{-5}	6×10^{-7}	0	1×10^{-5}	1	0
Pickoff	PM-AM	2×10^{-4}	4×10^{-7}	0	6×10^{-6}	**0.9**	1

Bright	*PM-AM*	*1*	-2×10^{-4}	-2×10^{-3}	4×10^{-6}	2×10^{-4}	5×10^{-4}
Dark	*CR-PM*	**0.6**	*1*	5×10^{-5}	3×10^{-5}	-1×10^{-4}	*0*
Bright	*PM-AM*	4×10^{-5}	-7×10^{-8}	*1*	-2×10^{-3}	**−0.6**	**−0.4**
Bright	*CR-NR*	-3×10^{-3}	-8×10^{-3}	**0.6**	*1*	-2×10^{-2}	*0*
Bright	*CR-AM*	-2×10^{-4}	*0*	**−1**	2×10^{-4}	*1*	*0*
Bright	*CR-PM*	-7×10^{-4}	-9×10^{-4}	**−2**	1×10^{-2}	**1**	*1*

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