Increasing measurement precision and accuracy, and applying several disparate calibration methods has improved our understanding of systematic errors and increased our confidence in test mass actuator calibration results [8]. However, optimizing the scientific reach of future gravitational wave searches will require further improvements in detector calibration accuracy and precision [13]. We expect that the frequency modulation method will continue to play a role in these efforts.

Acknowledgments

We gratefully acknowledge enlightening discussions with R Adhikari, K Kawabe, M Rakhmanov, P Schwinberg, D Sigg and the LIGO Calibration team. We also thank the National Science Foundation for support under grant PHY-0555406. LIGO was constructed by the California Institute of Technology and Massachusetts Institute of Technology with funding from the National Science Foundation and operates under cooperative agreement PHY-0107417. This paper has LIGO document number LIGO-P0900259.

Appendix. Calibration of the frequency actuator

The AOM at the heart of the laser frequency shifter (see figure 2) uses the first-order beam that is Bragg-diffracted from the acoustic wave driven by the radio frequency (RF) signal from the AOM driver. Energy and momentum conservation dictate that the frequency of the laser light in this beam is up-shifted (or down-shifted) by the frequency of the acoustic wave [15], which is dictated by the frequency of the signal from the AOM driver. The frequency shifter bandwidth is greater than 1 MHz, so for the range of modulation frequencies used (up to 2 kHz) the frequency modulation of the diffracted light is given by the frequency modulation of the RF signal driving the AOM.

To characterize the AOM driver, a phase-locked loop (PLL) is used to lock its output frequency to a frequency standard. This minimizes frequency drifts enabling precise measurement of the amplitudes of the RF carrier (~80 MHz) and modulation sidebands using an RF spectrum analyzer. The unity gain frequency of this PLL is approximately 400 Hz. The AOM driver input monitor signal is calibrated by injecting a sinusoidal frequency excitation, measuring the amplitude of the sinusoidal signal at the S_f monitor point, and using an RF spectrum analyzer (Agilent 4395A) to measure the ratio of the power in one of the induced first-order frequency modulation sidebands with respect to the carrier in the AOM driver output signal.

The time-varying electric field of the frequency-modulated AOM driver output signal can be expressed as

$$E(t) = E_0 \mathrm{e}^{\mathrm{i}(\omega t + \phi(t))}, \tag{A.1}$$

where E_0 is the amplitude of the sinusoidally varying electric field, ω is angular frequency of the RF carrier and

$$\phi(t) = \int_0^t \Delta\omega \cos(2\pi f \tau)\,\mathrm{d}\tau = \Gamma \sin(2\pi f t), \tag{A.2}$$

with the modulation index, Γ, given by $\Gamma = \Delta\omega/(2\pi f)$. The frequency-modulated field can be decomposed into a carrier and a series of frequency-shifted sideband fields by writing it as an infinite series of Bessel functions of the first kind, J_n, as

$$E(t) = E_0 \mathrm{e}^{\mathrm{i}\omega t} \sum_{n=-\infty}^{\infty} J_n(\Gamma) \mathrm{e}^{2\mathrm{i}\pi n f t}. \tag{A.3}$$