Structure and suspension DC compliance. J. Giaime, 4/2004

Coordinates: Let's assume that for low frequencies, we are using feedback in terms of horizontal (x, y) and vertical (z), platform displacement, measured in meters, and platform rotations about these axes (rx, ry, rz), measured in radians.

To implement this, we produce signals in terms of these variables from the non-orthogonal sensor outputs, using a constant matrix. These signals are then filtered (for servo compensation) and re-projected into the actuator basis for feedback. At the low frequencies that are the concern of this document, simple geometric projection will be used to carry out these basis transforms. For each stage, the origin is the center of plane defined by the lower zero-moment points of the flexures that support it. Using these coordinates, the horizontal actuators located at their nominal positions cause no torque about x and y, and pure torque applied via combinations of actuators causes no displacement.

Each stage's DC stiffness matrix is to be evaluated when the stage that supports it is fixed in 6 DOF's at DC, either mechanically or via servo controls.

Stiffness and compliance matrices: The stiffness matrix is generally written with the symbol K. It is conceptually easier to specify requirements for the inverse of the stiffness matrix, for no good reason here called $A = K^{-1}$. For static deflections, Hooke's law can be written:

$$\begin{pmatrix} x \\ y \\ z \\ rx \\ ry \\ rz \end{pmatrix} = \begin{pmatrix} A_{x\,x} & A_{x\,y} & A_{x\,z} & A_{x\,rx} & A_{x\,ry} & A_{x\,rz} \\ A_{y\,x} & A_{y\,y} & A_{y\,z} & A_{y\,rx} & A_{y\,ry} & A_{y\,rz} \\ A_{z\,x} & A_{z\,y} & A_{z\,z} & A_{z\,rx} & A_{z\,ry} & A_{z\,rz} \\ A_{rx\,x} & A_{rx\,y} & A_{rx\,z} & A_{rx\,rx} & A_{rx\,ry} & A_{rx\,rz} \\ A_{ry\,x} & A_{ry\,y} & A_{ry\,z} & A_{ry\,rx} & A_{ry\,ry} & A_{ry\,rz} \\ A_{rz\,x} & A_{rz\,y} & A_{rz\,z} & A_{rz\,rx} & A_{rz\,ry} & A_{rz\,rz} \end{pmatrix} \begin{pmatrix} F_x \\ F_y \\ F_z \\ \tau_x \\ \tau_y \\ \tau_z \end{pmatrix}, \tag{1}$$

where F_i and τ_i are the force along the i axis and the torque about the i axis, respectively.

In setting constraints on the off-diagonal elements of A, our goal is to reduce cross-coupling among the servo loops at low frequencies. Cross-coupling can lead to servo instabilities or overly constrain our controller design choices. It is easiest (for me) to set limits on the ratio of a compliance matrix element to its associated diagonal element. For example, $A_{rx\,y}/A_{y\,y} = \Delta rx/\Delta y$, the amount of rotation accompanying horizontal motion due to a nominal horizontal actuation, with units m^{-1}.

The requirements listed below will be upper limits to fractions of the form $A_{i\,j}/A_{j\,j}$ for the *elements in the lower left of* A. Since A should be symmetric about its diagonal, the upper right elements are implicitly constrained as well, and shouldn't need to be calculated.

Horizontal-tilt coupling: A pure horizontal force in general causes motion in all DOF's, including tilt. Horizontal seismometers measure tilt as well as horizontal motion, creating the potential to couple the servo loops unstably. Consider a horizontal seismometer as a mass-spring oscillator with a natural frequency ω_0. The instrument's readout is sensitive to the relative displacement,

Δx, between its inner test mass and the stage on which it rests. At frequencies well below ω_0, the instrument magnitude response to horizontal stage motion, x_0, at frequency ω, is $|\Delta x/x_0| \simeq \omega^2/\omega_0^2$. If horizontal motion also causes the stage to *tilt* slightly as though it were pivoting from a distance, R, away, then gravity pulls the seismometer mass against its restoring force, and the instrument responds with an additional term: $|\Delta x/x_0| \simeq g/R\omega_0^2$. These two terms are equal and can cause a zero in the overall response when $\omega \simeq (g/R)^{1/2}$. We require $R > 500$ m.

Based on this, we constrain the lower-left **red** matrix elements of A by $A_{i\,j}/A_{j\,j} < 0.002\ \mathrm{m}^{-1}$.

Horizontal-vertical coupling: Horizontal actuation that causes vertical motion can lead to servo instability, and without a point design to explore, I will fall back on prior experience with SISO control of 6-DOF platforms. Generally, the loop gain is limited to the inverse of the cross coupling, so for now, let's require that the lower-left **blue** elements of A by $A_{i\,j}/A_{j\,j} < 0.02$.

Vertical-tilt coupling: Vertical actuation that causes tilt can cross-couple the vertical with the horizontal loops via the tilt-pickup mechanism in the seismometer at low frequencies, as described above. However, it appears that we can tune out this term via gain adjustments in the vertical actuation, so let's use a relatively relaxed requirement, constraining lower-left **green** matrix elements of A by $A_{i\,j}/A_{j\,j} < 0.02\ \mathrm{m}^{-1}$.

Vertical-yaw coupling: Vertical actuation that causes yaw can cross-couple the vertical with the horizontal sensors that are used to measure yaw. This should not be a problem if the lower-left **purple** element of A by $A_{i\,j}/A_{j\,j} < 0.02\ \mathrm{m}^{-1}$.

Others: The other elements are either not expected to cause problems, or are likely constrained indirectly by the conditions above.