LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY

-LIGO-

CALIFORNIA INSTITUTE OF TECHNOLOGY

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Excess noise in anti-seismic filters for gravitational wave antennae

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of the LIGO Project.

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Dears Gary and Riccardo,

I took the liberty to present few notes about the possibility (and the doability also) of a small R and D program which seems to me reasonable and useful. I think that you both share my point of view: the more we know in advance about all sources of mechanical noises which act on the mirror's center of mass the better we are prepared to decipher the gravitational wave signal from other noises in the time domain. I also do think that you agree that till now nobody knows what are the random displacements (the spectral density, the features of rare "chirps") of the last platform of the antiseismic filter which may be produced by processes associated with the creep, the slippage etc.).

The key "idea" I am proposing here is to consider the possibility to install on the last platform of the antiseismic filter a sensitive accelerometer which has to record (and even to monitor continuously along with the process of the measurements) all AC displacements of the platform within the projected bandwidth of LIGO II. I do think that some kind of "fee" has to be paid for the substantial gain of the new antiseismic filter in which the better filtering is due to lower eigenfrequencies in the filter and due to higher values of the stresses in the blades and springs.

The following simple estimates may be regarded as arguments in favor of such a program.

I. Initial conditions for the measuring device.

In LIGO-II the expected sensitivity corresponds to the displacements of the mirror itself $\approx 10^{-17} cm$ near $\omega_{grav} \approx 10^{+3} rad/\mathrm{sec}$, and with averaging time $\tau \approx 10^{-2}$ sec (optimistic prognosis). Taking into account that the transfer factor from the platform through the fiber to the mirror itself has to be between $10^{-3} - 10^{-4}$ and assuming that $S/N \approx 10$ we have to have the sensitivity of the meter on the platform near $\Delta x_0 \approx 10^{-14} - 10^{-15} cm$ for $\omega \approx \omega_{grav} \approx 10^{+3}\, rad/\sec$ and averaging time $\tau \approx 10^{-2}\sec$.

II. The properties of the mechanical oscillator used in the meter (accelerometer).

The eigen frequency of the mechanical oscillator ω_0 has to be smaller than ω_{grav} and its quality factor Q_0 has to be enough high to obtain the spectral density of the oscillator's Brownian fluctuations sufficiently smaller to detect Δx_0. This condition may presented in the following form:

$$Q_0 \geq \frac{4\kappa T \omega_0}{\Delta x_0^2 \tau m_0 \omega_{grav}^4} \approx 1.6\times 10^{+7}\left(\frac{\omega_0}{10^2\, \frac{rad}{\sec}}\right)^1 \left(\frac{x_0}{10^{-15} cm}\right)^{-2} \left(\frac{\tau}{10^{-2}\sec}\right)^{-1} \left(\frac{m_0}{10^2 gram}\right)^{-1} \left(\frac{\omega_{grav}}{10^{+3}\, \frac{rad}{\sec}}\right)^{-4} \qquad (*)$$

where κ is Boltzmann constant, T is the temperature, m_0 is the mass of the oscillator. In the numerical estimate (*) $T = 300K$. The above estimate shows that the oscillator may be done from very pure fused silica in the form (e.g.) of tune fork (with not identical "legs").

There is another condition for ω_0 and m_0 which is due to finite value of the dynamical range of a meter which will record the displacement of the oscillator's mass relatively the platform. If an optical Fabri-Perot resonator will be chosen for such a coordinate meter then the finesse $\mathcal{F}$ of the resonator's mirror has not to exceed the limit:

$$\mathcal{F} \le 0.1\lambda_{opt}\sqrt{\frac{m_0\omega_0^2}{\kappa T}}$$
$$= 5\times10^4\left(\frac{\lambda_{opt}}{10^{-4}cm}\right)\left(\frac{m}{10^2 gram}\right)^{\frac{1}{2}}\left(\frac{\omega_0}{10^{+2}\frac{rad}{\sec}}\right)\left(\frac{T}{300K}\right)^{-\frac{1}{2}}, \qquad (**)$$

If the condition (**) will be satisfied then the RMS value of the Brownian oscillation of the oscillator will be approximately equal to 20% of the dynamical range of the coordinate meter.

III. <u>The features of the coordinate meter.</u>

High value of $\mathcal{F}$ (as in numerical example (**) permits to use relatively low pumping power W to beat the shot noise limit:

$$W = \frac{\lambda_{opt}\hbar c\pi}{\mathcal{F}^2\Delta x_0^2\tau}, \qquad (***)$$
$$\approx 4\times10^{+2}\,\frac{erg}{\sec}, = 40 microwatt$$

where c is the speed of light and $\lambda_{opt} = 10^{-4}cm$, $\mathcal{F} = 5\times10^{+4}$, $\Delta x_0 = 1\times10^{-15}cm$, $\tau \approx 10^{-2}\sec$.

Thus the value of W is not a problem: the finesse may be even smaller than 5×10^4.

But it does exist another more serious obstacle: the thermodynamical fluctuations of mirror's surface (the mirrors which forms the Fabri-Perot resonator, used to monitor the Δx_0).

The spectral density of the fluctuations of a spot on such a mirror with radius τ_0 may be presented in the form:

$$S_{TD} = \frac{8}{\sqrt{2\pi}}\alpha_T^2(1+\sigma)^2\frac{\kappa T^2\lambda^*}{(\rho C^*)^2\tau_0^3\omega^2} \qquad (****)$$

where α_T is the heat expansion factor, σ is the Poisson factor, λ^* is heat conductivity, ρ is the density, C^* is specific heat capacity, ω is the frequency of observation (see PHYS. LETT. A 264 (1999)1). It is evident that the value τ_0 must be high enough to satisfy the condition

$$\Delta x_0 \geq \sqrt{S_{TD}\,\tfrac{1}{\tau}}\,. \qquad (*****)$$

If the pure fused silica is used for such a Fabry-Perot resonator, then using the last two equations it is easy to obtain the condition for τ_0. It is necessary (at room temperature) to have in this case $\tau_0 \geq 1\times 10^{-2}\,cm$. This value by the way demonstrates that the idea to use tunnel microscope technique to register the displacements of the test mass of the oscillator is absolutely hopeless!

It is evident that the distance between the two mirrors has to be not too large. If one chooses this distance $\approx 2cm$ then the radius of the mirror's curvature has to larger than $20cm$.

From my point of view the most difficult task in such a program will be the design of elements of this accelerometer = mechanical oscillator + optical readout system. I may recommend the Mach-Zehnder scheme interferometer with two Fabri-Perot resonators in each arms. I also recommend to design all elements of such a scheme that the lower eigenfrequencies of these elements have to be not lower than $\approx 20kHz$ (except the key oscillator). These elements have to be glued to the platform (MSU group prefers Indium as a good glue). The last (?) remaining problem for such a scheme of meter is how to tune the Fabry-Perot resonator. Being an archenemy of any PZT (because of the excess noise) I think that small local heater may help.

yours

V. B.

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