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CALIFORNIA INSTITUTE OF TECHNOLOGY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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DAC output signal conditioning: dewhitening and anti-imaging filter design

P Fritschel, J Heefner

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California Institute of Technology
LIGO Project - MS 51-33
Pasadena CA 91125
Phone (818) 395-2129
Fax (818) 304-9834
E-mail: info@ligo.caltech.edu

Massachusetts Institute of Technology
LIGO Project - NW17-161
Cambridge, MA 01239
Phone (617) 253-4824
Fax (617) 253-7014
E-mail: info@ligo.mit.edu

WWW: http://www.ligo.caltech.edu/

1 DEWHITENING FILTER ASSUMPTIONS

The LSC dewhitening filter is an analog filter placed (for each control channel) between the DAC output and the suspension coil driver input. The function of the dewhitening filter is to reduce the voltage noise associated with the DAC sufficiently so that it does not contribute significantly to the interferometer noise budget. From a practical standpoint, it is desirable to design a 'universal' dewhitening filter, such that there is a single design for all instances of the filter. Considering this goal, the following assumptions and constraints are taken:

- the baseline design does not assume any effective suppression of (dewhitening filter) electronics noise due to larger-than-unity loop gain
- the DC displacement range of an LOS controller is assumed to be 20 micron-pk, for an LSC input voltage of 12.5 V-pk (for all core optics)
- for the test masses, the transfer function of LSC input to coil current is assumed to have a 1 Hz pole and a 40 Hz zero; for the beamsplitter and recycling mirror, the transfer function is frequency independent (no filtering)
- the DAC voltage range is 10 V pk-pk, and the noise density is $7\mu V/\sqrt{\mathrm{Hz}}$ (Pentek 6102); this implies there must be a DC gain of 2.5 between the DAC and coil driver LSC input.

  Revision: in this version the DAC voltage noise density is taken to be $2\mu V/\sqrt{\mathrm{Hz}}$, the value measured for the new, 'low-noise' version of the Pentek 6102.

Note: Since the original design work on these filters, is has been decided to imlement digital suspension controllers, where the LSC and ASC control signals will be added digitally to the suspension controller digital signals. There will be 4 DAC outputs for each suspension, one for each coil (5 including the side channel). The filter designs given below apply to each of the DAC output channels; in principle the dewhitening filtering and noise requirements can be a factor of 2 less stringent due to the fact that there are 4 independent channels.

2 REQUIREMENTS

The interferometer displacement sensitivity, given in the SRD, is denoted by x_{SRD}. The position fluctuations x_{TM} of the test masses and x_{BS} of the beamsplitter, produced by the noise under consideration, must satisfy:

$$\begin{aligned} x_{\mathrm{TM}} &\le x_{\mathrm{SRD}}/20 \\ x_{\mathrm{BS}} &\le x_{\mathrm{SRD}} \cdot 5 \end{aligned} \tag{1}$$

The filtering required is computed as:

$$\begin{aligned} T_{\mathrm{dw}}(f) &= \frac{x_{\mathrm{TM}}}{1.6\ \mu\mathrm{m/V}} \cdot \left(\frac{f}{0.74}\right)^2 \cdot \sqrt{\frac{1+f^2}{1+(f/40)^2}} \cdot \frac{1}{2.5 \cdot 2\ \mu\mathrm{V}/\sqrt{\mathrm{Hz}}} \quad \text{(test masses)} \\ &= \frac{x_{\mathrm{BS}}}{1.6\ \mu\mathrm{m/V}} \cdot \left(\frac{f}{0.74}\right)^2 \cdot \frac{1}{2.5 \cdot 2\ \mu\mathrm{V}/\sqrt{\mathrm{Hz}}} \quad \text{(beamsplitter)} \end{aligned} \tag{2}$$

Since x_{BS} is 100x larger than x_{TM}, and the filtering in the test mass coil driver provides a maximum attenuation factor of 40, the filtering requirement for the test masses is more stringent than that for the beamsplitter, and so the former is taken as the universal requirement.

The output voltage noise requirement for the dewhitening filter is found by multiplying equation (2) by $2.5 \cdot 2\ \mu\mathrm{V}/\sqrt{\mathrm{Hz}}$; the result is shown in Figure 1.

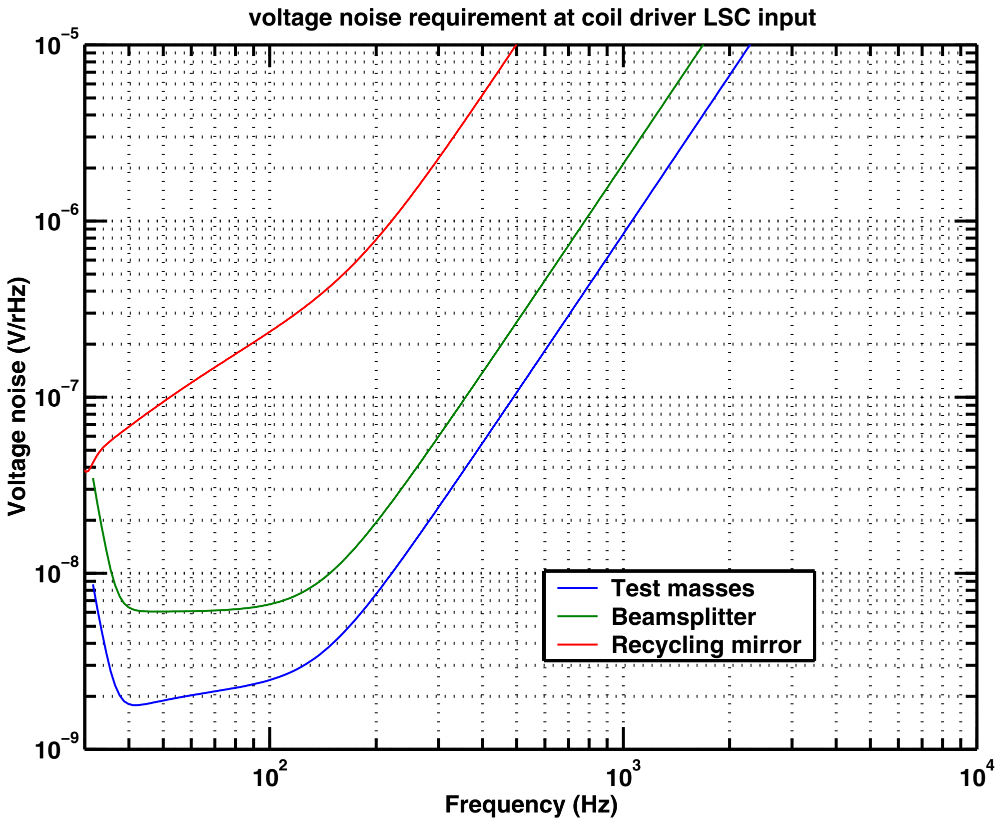

Figure 1: Maximum noise allowed at the output of the dewhitening filter, assuming it is fed directly into the coil driver. The recycling mirror requirement is set so that the equivalent frequency noise is 10x below the frequency noise requirement.

Competing with the requirement on filtering of electronics noise is the requirement of preserving sufficient control range to compensate for ground-induced motion of the optics. This constraint is most severe at the vertical (bounce) mode of the suspension, which is f_v = 13 Hz for a large optic. The coil driver range at the vertical mode frequency is:

$$\begin{aligned} x_{cd}(f_v) &= 20\mu\mathrm{m} \cdot \left(\frac{0.74}{f_v}\right)^2 \cdot \left(\frac{1}{f_v}\right) = 5\ \text{nm-pk} \qquad \text{(test masses)} \\ &= 20\mu\mathrm{m} \cdot \left(\frac{0.74}{f_v}\right)^2 = 65\ \text{nm-pk} \qquad \text{(BS \&RM)} \end{aligned} \tag{3}$$

Estimates for the optic-axis motion at the vertical mode are given in the LSC FDD; they range from 6×10^{-11} m-rms for the long arm cavities, to 1-2 nm-rms for the Michelson and recycling cavities (corrected for by the beamsplitter and recycling mirror, respectively). These estimates were made assuming a vertical ground motion of 10^{-9} m/$\sqrt{\text{Hz}}$ at 13 Hz; this may be typical, but it can be quite variable, and a better upper limit would be 10^{-8} m/$\sqrt{\text{Hz}}$. Thus maximum peak motion at 13 Hz would be ~1 nm-pk for the long arm cavities, and ~15-30 nm-pk for the Michelson and recycling cavity.

The result is that there is not much headroom in the coil driver for control correction at the vertical mode, and we conclude that the dewhitening filter must not significantly further restrict the range at 13 Hz.

3 DEWHITENING FILTER DESIGN

We are faced with designing a filter that has no significant attenuation at 13 Hz, and an attenuation of ~10^{-4} at 40 Hz. Furthermore, since the bandstop of the filter will be in the LSC servo bands, it must have no zeros in its response (this eliminates elliptic or Chebyshev type II filters).

The magnitude responses of two filter designs are shown in Figure 2, along with the minimum required filtering (equation (2)). One design uses Butterworth poles and zeros – where each group of poles and zeros lie on a circle in the s-plane, equally spaced in angle. The other design uses Chebyshev poles and zeros, where they lie on an ellipse; the ripple was chosen to be 0.2 dB. The roll-off of ~80 dB is achieved with 8 poles (stopped with 8 zeros) in the Butterworth filter, and with 5 poles (stopped with 5 zeros) in the Chebyshev case (Chebyshev low-pass filters in general have a steeper roll-off near the cut-off frequency than Butterworth filters; also, the Butterworth filters were not re-examined after reducing the assumed DAC noise, so it may be that a lower order would work). In each case the roll-up is made with 3 zeros (stopped with 3 poles).

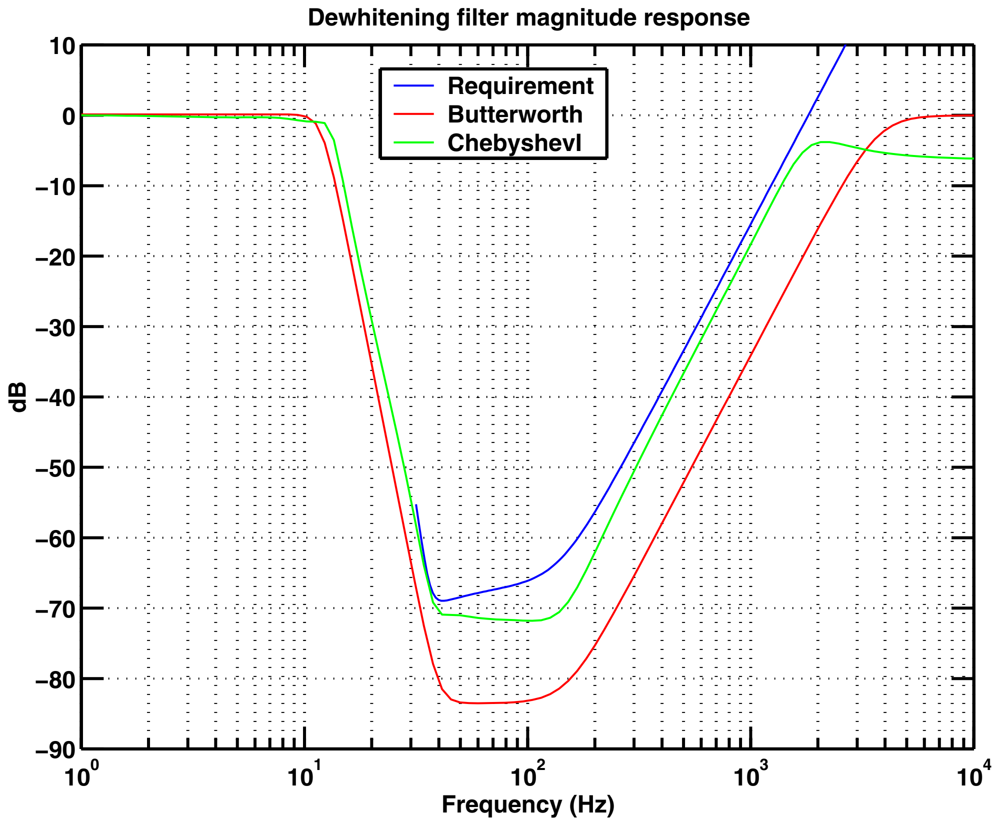

Figure 2: Dewhitening filter magnitude response. Shown are the filtering requirement (minimum filtering), and the response of two candidate realizations.

The poles and zeros are listed in Table 1.

Butterworth		*Chebyshev*	
poles at 12 Hz	-73.9 ± i14.7 -62.7 ± i41.9 -41.9 ± i62.7 -14.7 ± i73.9	poles at 12.5 Hz	-11.2 ± i82.3 -29.3 ± i50.8 -36.2
zeros at 40 Hz	-246.5 ± i49.0 -209.0 ± i139.6 -139.6 ± i209.0 -49.0 ± i246.5	zeros at 40 Hz	-32.1 ± i235.6 -168.0 ± i291.4 -544.7
zeros at 150 Hz	-471.2 ± i816.2 -942.5	zeros at 120 Hz	-307.1 ± i842.2 -614.2
poles at 3.7 kHz	-11624 ± i20133 -23248	poles at 1.5 kHz	-3839 ± i10528 -7678

Table 1: s-plane poles and zeros of the candidate dewhitening filters

4 ANTI-IMAGE FILTER

The DAC process produces images of the sub-Nyquist frequency (f_N) spectrum at frequencies above f_N. The amplitudes of the images are modulated by a sin(x)/x response, where $x = \pi f / f_s$, where f_s is the input sampling frequency (16384 Hz). These images should be filtered to avoid sending a lot of high-frequency signal into the coil driver electronics, where they could produce non-linear noise, or excite mirror resonance to high levels.

To help determine the anti-image filter, a simulation was made of the noise spectrum that is expected to be sent to the DACs. This was done a white noise generator and a set of analog filters (including an anti-aliasing filter with a 7.5kHz cutoff frequency) that produce a noise spectrum approximating that of Fig. xx of the LSC FDD (which shows the predicted DAC output spectrum for the Lm control signal). This signal – which contained about 1 V-rms from low-frequency (<20 Hz) noise, and about 0.2 V-rms from the high-frequency noise peak – was sampled at 16384/sec by a Pentek 6102, and sent directly to a Pentek DAC channel. The output spectrum of the DAC is shown in Figure 3.

The images are filtered with an elliptic low-pass filter: 4th order; 7500 Hz cut-off frequency; 4 dB passband ripple; 60 dB stopband. Since the largest imaged signal occurs around f = 16384 – 7500, a fairly low cut-off frequency is chosen to effectively reduce this component. The anti-image filter produces about 20° of phase lag at 1 kHz. The DAC output spectrum after filtering by the dewhitening and anti-image filters is shown in Figure 4.

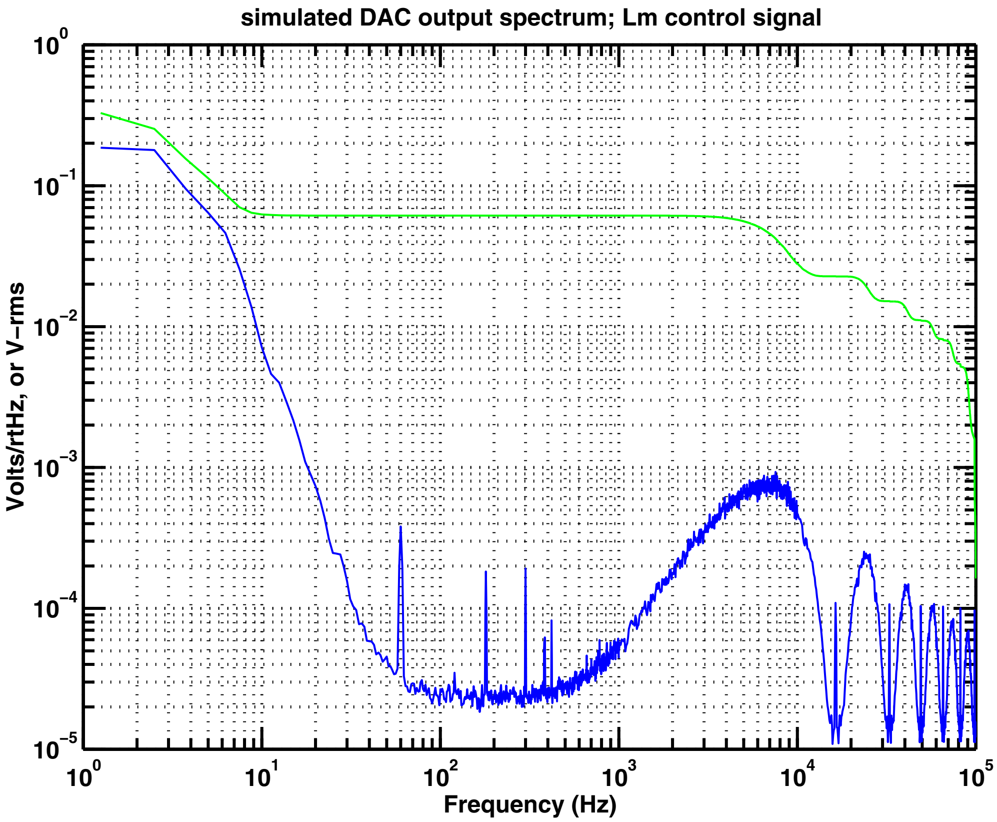

Figure 3: DAC output noise spectrum.

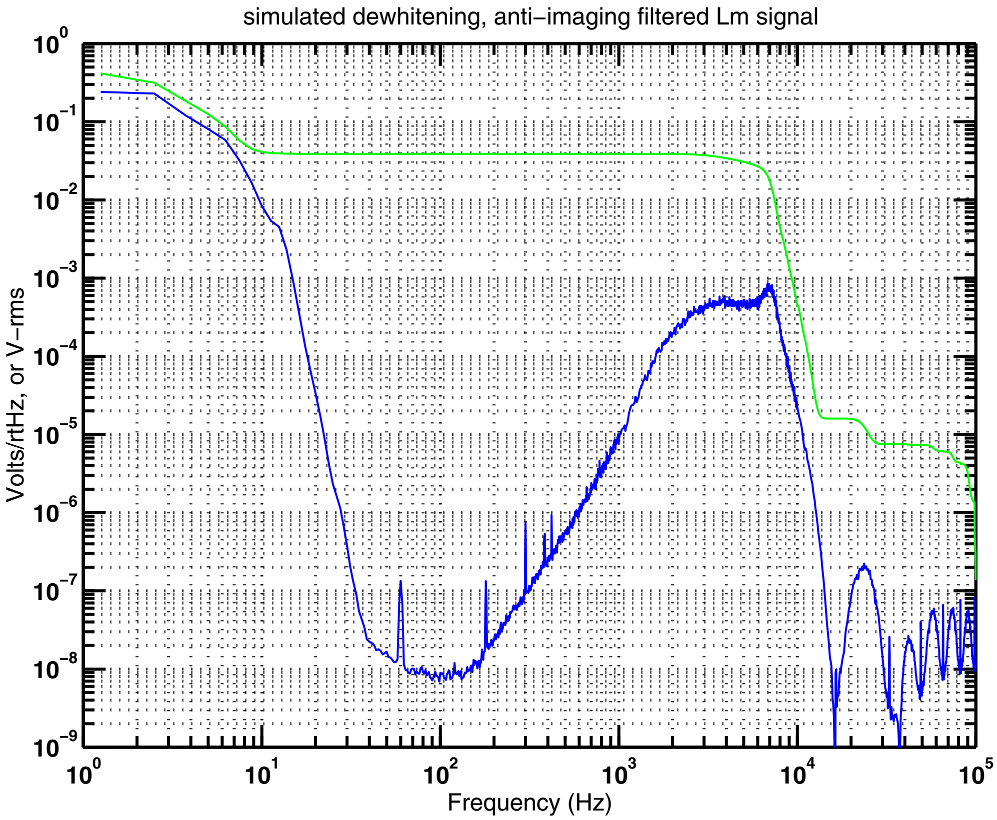

Figure 4: Dewhitened and anti-image filtered DAC output (Lm control signal model).