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\- LIGO -

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Spatial Dependence of Force in the Initial LIGO OSEMs

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Introduction

The following report displays the results of the measurement of the magnetic force between a coil and a magnet (the same that are used at the site). The goal of the experiment was to map the magnetic force in the space. Therefore, for different position of the coil towards the magnet, a reading of the force has been made. The experiment was separated in two steps. The first set-up allowed to measure the axial force on the magnet while the second step allowed the measurement of the tranverse force.

The set-up will be described to first explain how the measurement was made. Then, a thoeric model will be developped so that it can later serve as a refenrence for the analysis of the pratical results. The data will be presented in multiple plots and since the measurment has been repeated for 6 magnets, plots of the average magnet will be displayed. Finally, the analysis of the results and the comparison with the theory will follow.

1. The Set-up

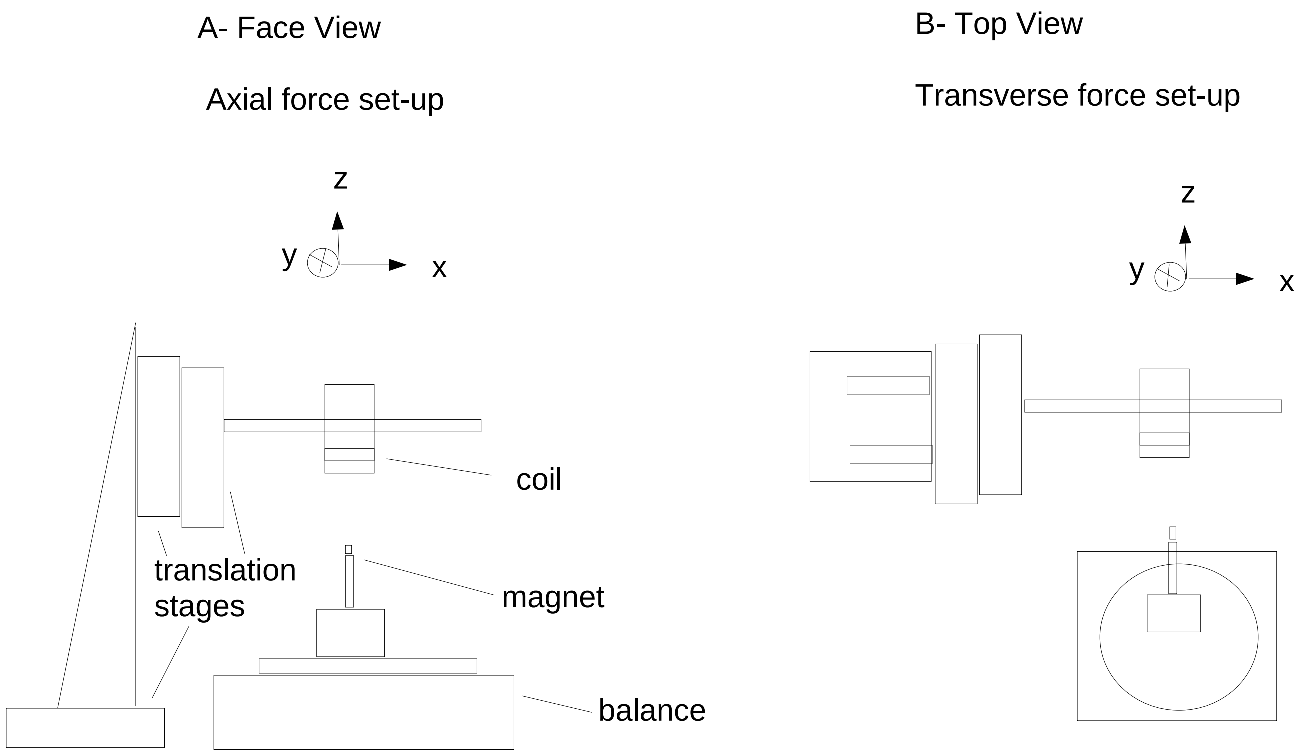
First of all, let's describe properly the set-up used to make the measurements. The magnets, made of Nd:Fe:B, are cylinders that have a 1.9 mm diameter as well as a 3.2 mm height. The coil is in fact an alumina (ceramic) cylinder that has a 25.2 mm external diameter. The coil has a rectangle hole centered on the cylinder's axis. This hole goes through the length of the cylinder. Wires enter in the cylinder by the top of this hole to be connected inside. At the bottom of this rectangle hole are sticked a led and a photodiode. So, the rectangle window in which the magnet can move is restricted to 5.4 by 7.0 mm. Also, at 3.2 mm from the bottom of the cylinder, a ring cavity allows the 400 loops of wire to be coiled up. The ring has a 5 mm thickness, a 12.6 mm outside radius and a 7.7 mm inside radius.

The basic idea remains that the magnet is sticked to a thin non-magnetic rod which itself sits on a balance. Epoxy was used to stick the magnet to the small top surface of the rod which has the same diameter as the magnet.

The support on which the coil was mounted was clamped to a table. Both that table and the balance were leveled horinzontally. The support had 3 translation stages controlled by micrometers to allow full 3D displacement of the coil. The coil was placed in some kind of a lens holder which was itself mounted to the transaltion stages. For the measurement of the axial force, the coil was fixed vertically as well as the magnet (see figure 1a). Then, in order to make the balance indicate the transverse component of the force , the coil and magnet had to be horizontal (see figure 1b).

The reference system is a direct xyz coordinates system and it stays the same for both axial and transverse forces. Any (x,y,z) coordinates express the coil's position toward the magnet, considering the center of the magnet as the origin (see figure 1). Hence, the coil is at the origin when its axis is aligned with the magnet's axis and when the middle the magnet corresponds to the middle of the thickness of the coil.

Figure1



In order to set the micrometers, the magnet was centered on the coil by eye. In fact, a camera was placed on top of the coil to facilitate the centering. The coordinates of 2 corners of the cylinder's window were taken by displacing the coil until the magnet touched the 2 edges of the corner. Then, the exact position of the center could be calculated. For the z axis zero position, the coil was placed so that the magnet's top touched the coil's bottom surface. The coil was afterwards lowered by 7.3 mm (the distance between the magnet's middle and the the coil's midlle). This is how the relative position given by the 3 micrometers were converted to absolute position in the axis system earlier described.

The balance has a 1 mg precision. Also, 30 g has been given by the balance for a 0.01 mm vertical displacement of its platter. Since the maximum weight (or force) expected should not be over 200 mg, the vertical displacement of the platter doesn't introduce errors in the z values of the data.

Finally, a simple electric circuit provided a fix DC current of 100 mA into the coil's wires. The coil's impedance was measured to be 14 ohms. So a 86 ohms resistor was placed in series with the coil as well as a 10 V DC voltage source. A current meter was added to make sure the current remained 100mA.

2. The Measurement

A complete map (275 data points) of both force components was fisrt done with one magnet and then a quicker mapping (125 data points) has been done for 5 other magnets. Since the allure of the plots would be all the same (and to save time), 6 complete maps were not considered necessary. The goal of those quick maps being to find the maximums and minimums forces over a sampling of magnets. Those quick maps allowed to have an idea of the "average magnet". The quick maps might have a lot less data points, although the data was taken on the extreme regions. As a result, the complete map plots have a smoother look than the plots generated from the quick maps. The range of displacement on the xy plane was a 2 X 4 mm window, the shorter range corresponding to the x axis and the longer one to the y axis. On the z axis, the range was limited by the wires in the upper part of the cylinder containing the coil. Hence, data was taken from 11.1 to -1.9 mm. Going further on the negative side of the z axis was physically impossible because of the wires blocking the magnet.

3. Theory

The model chosen to represent the magnetic force considers the magnet as a magnetic dipole and the coil as a single loop of current. The magnetic force on the coil is given by:

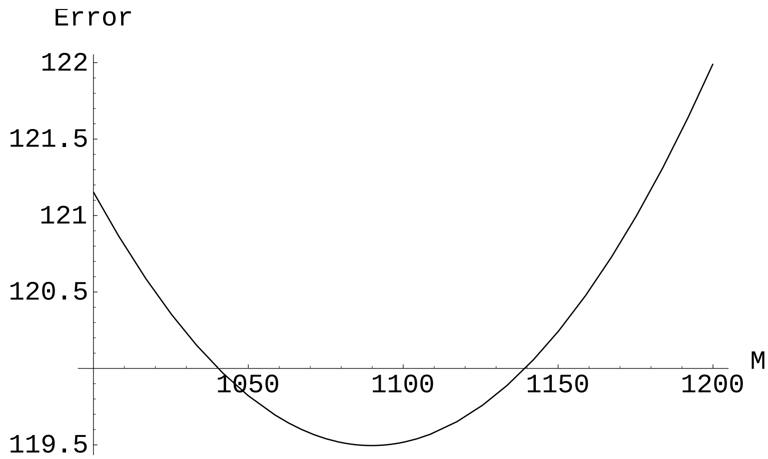
$$F = I \quad dl \quad B$$

where B is the dipole’s field. When the dipole is at the origin and the it points in the z direction, the dipole’s magnetic field is given by (Griffiths, Introduction to Electrodynamics, p238) :

$$B = r \quad A = \frac{{}_0 M}{4 \;\; r^3} \; 2\cos \; \hat{r} + \sin \;\; \hat{}$$

M being a parameter and r being the distance between the magnet and the observation point . After converting to cartesian coordinates and calculating the dl X B vectorial product, the expression of the magnetic force becomes a very heavy integral over the coil circumference. The 3 components of the force have been computed numerically since the integral has no analytic solution. The tangential force being always null, the radial and axial components only are plotted further. The only parameter of the model is M, and a fit has been made between the data and the model for different values of M. The sum of the square of the differences between the data and the model is minimum when M = 1090.

Figure 2

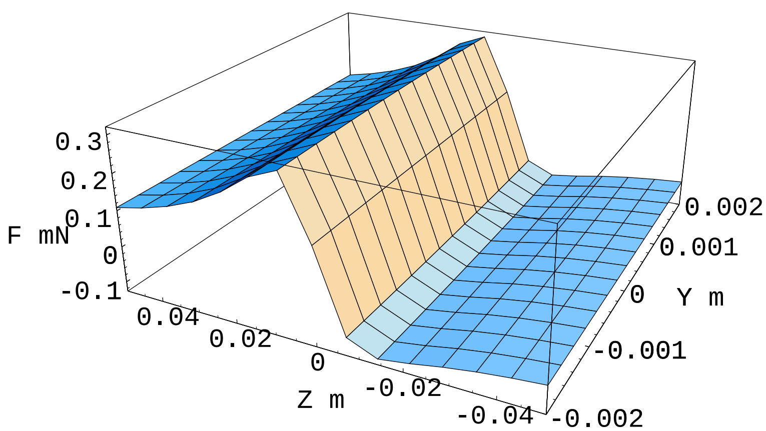


Then, with M =1090 and from the numerical calculation, we obtain plots for the transverse and axial force. Without loss of generality, the model assume that the force is symetric in the xy plane so the force will be represented as a function of z and y only (y being the radial direction). Take note that the units are meters.

a) Axial Force

The first plot shows a larger range on the z axis, for x = 0:

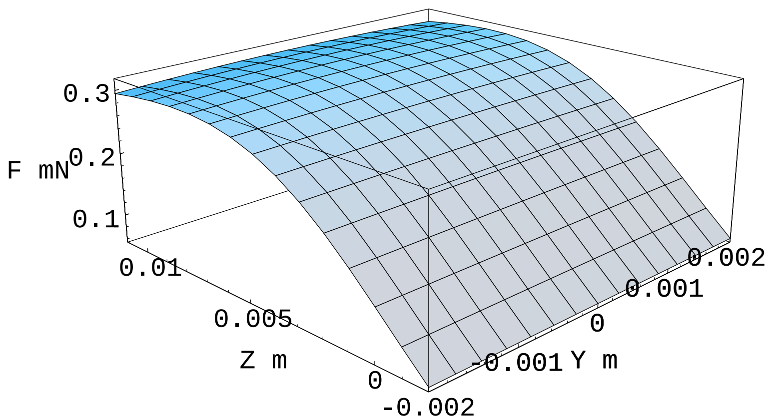
Figure 3



On this plot, we see that as long as the magnet is inside the window of the coil, there is no variation of the force as the coil moves on the y (and x by symetry) direction.We also see that the force decreases as the coil gets further in the positive and negative z direction. However, when the coil approaches the magnet, for z near zero, the force goes through its maximum then passes by zero to reach its opposite direction peak.

The second plot shows the map of the force on the same region as the data when x = 0, and it will serve for comparison later on :

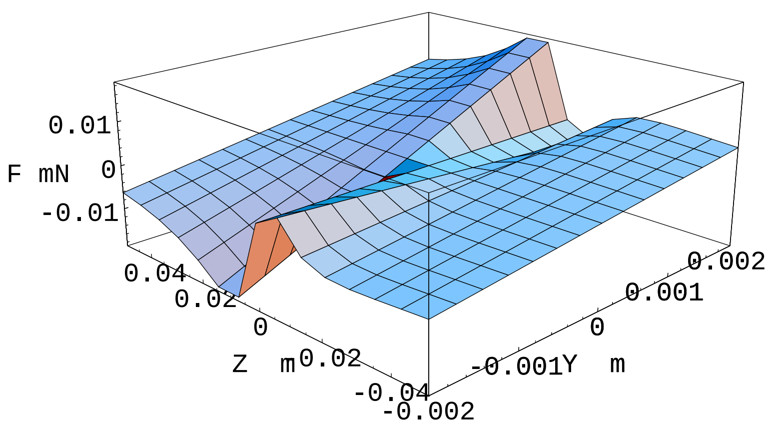
Figure 4



b) Transverse Force

Let's have an overview of the model for the transverse force.

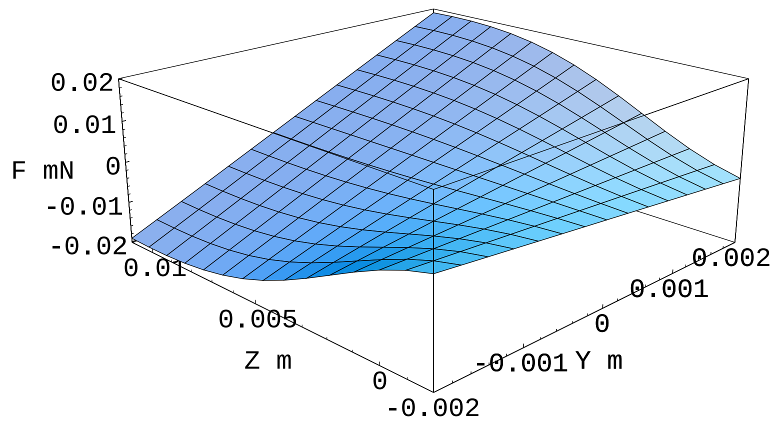
Figure 5



Like the axial force, the transverse component decreases as the coil gets far from the magnet in the z direction. There are maxima on both sides of z = 0, like we saw for the axial force. Unlike the axial force model though, the gradient is inverted around y = 0. On both Y axis extremes, the transverse component reaches its highest magnitude. Take note that the magnitude of this force is smaller than for the axial force.

The next plot displays the same range as the data for x = 0:

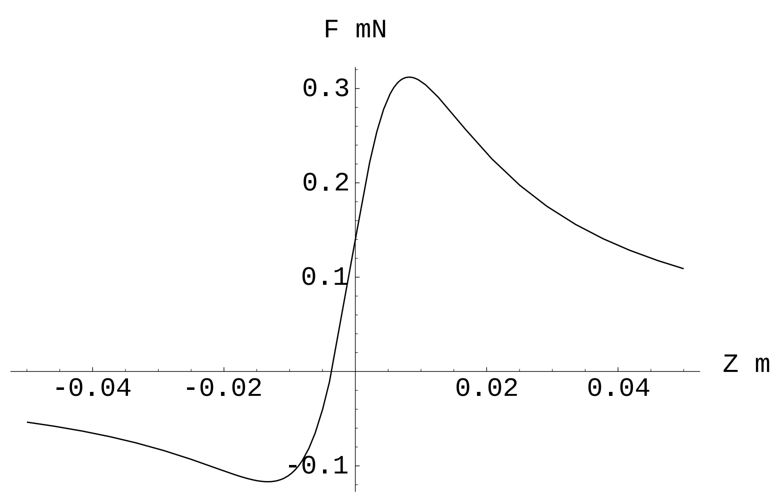
Figure 6

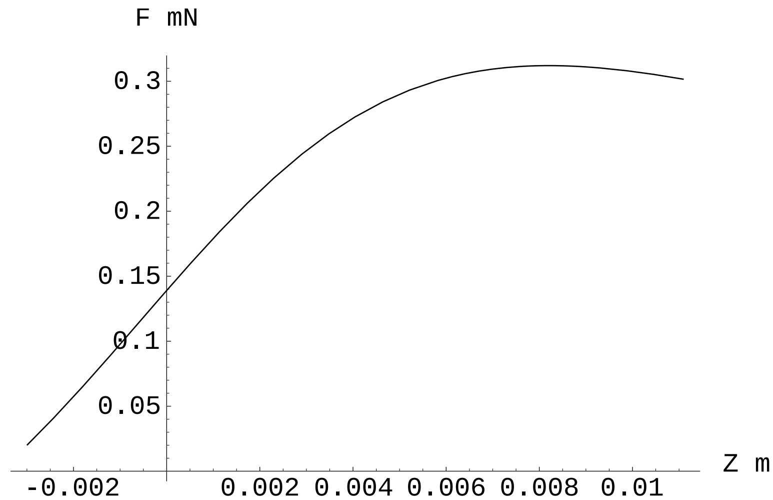


c) Axial Force as a function of z

The distance between the coil and the magnet corresponding to the maximum axial force is another important aspect (see David Shoemaker's thesis p. 100-101). Hence, here is the axial force verus z as given by the model :

Figure 7



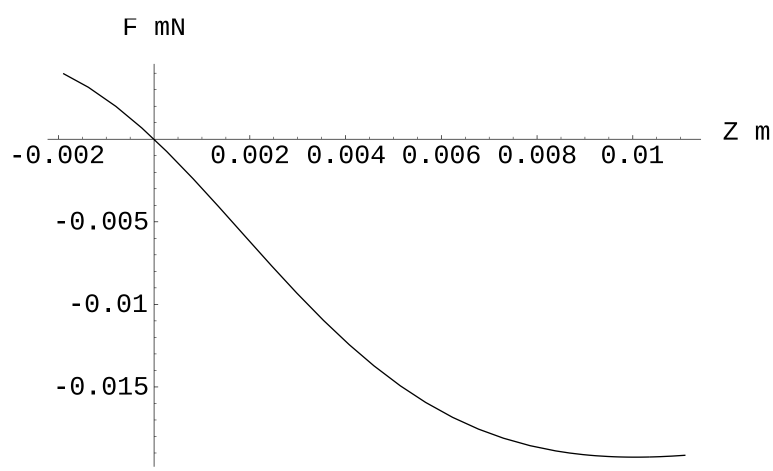


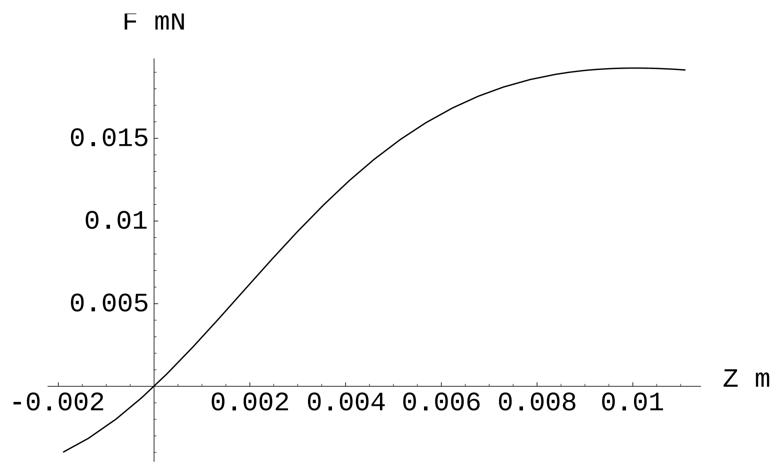
The plot shows that the theoric magnet-coil separation is 8.25 mm to reach the maximum axial force. Since the model is symetric, that characteristic distance is the same for any (x,y) point where we could plot the force vs z. However, notice that for x = 0 and y = 0, the transverse component is null. On the other hand, if x= +/-1mm and y = +/- 2 mm, those coordinates being the 4 corners of the coil's window, both the axial and transverse components of the magnetic force are maximum.

d) Transverse Force as function of z

The surface plot of the transverse force might be hard to interpret so here is the force versus z for y = -2 mm and then y = 2 mm. Note that for y = 0, the force oscillates around 10E-21 mN, so we can can considerate it flat to zero.

Figure 8





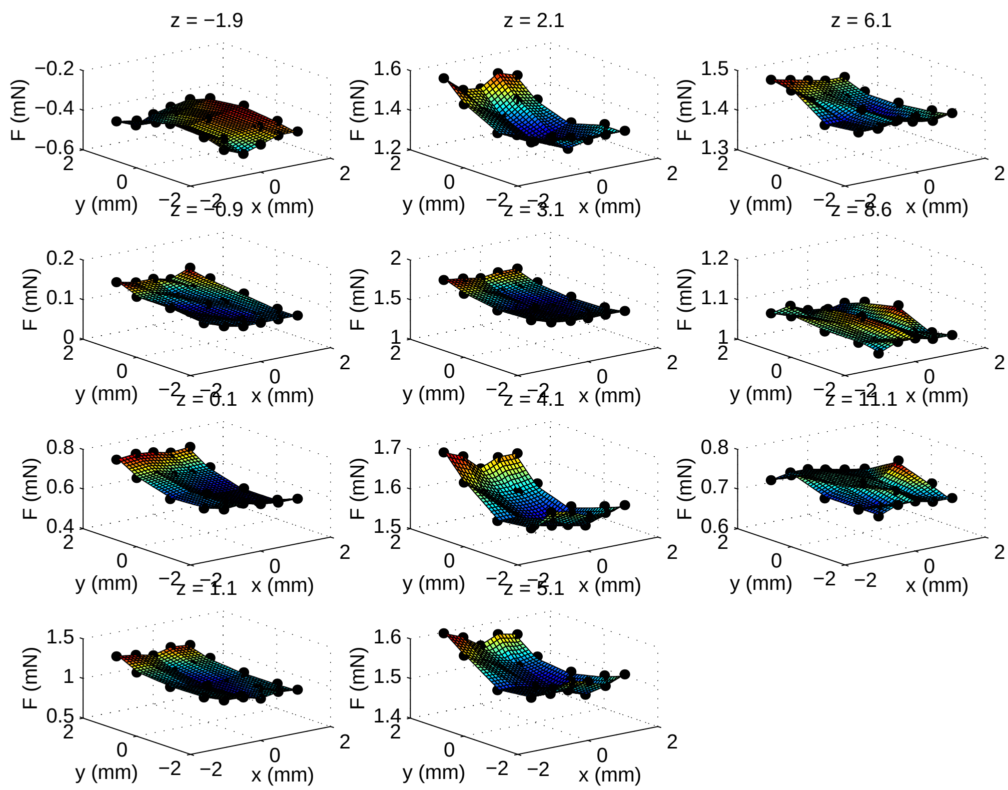
4. Experimental Results

The present section displays the most relevant data in order to be able to compare with the model. Since the range of displacement on x was a little short (+/- 1mm), the most interesting things appear on the y axis. So, surface plots of the force versus x and z are not presented here. If interested, the reader can run the M-files linked at the end of this document to have acces to all the possible ways to look at the maps. Be careful, the units are now millimeters. Also, since plots of the force versus y and z looked all the same for different constant x values, only those for x = 0 are presented so that the document is not overloaded.

a) Axial Force Complete Map

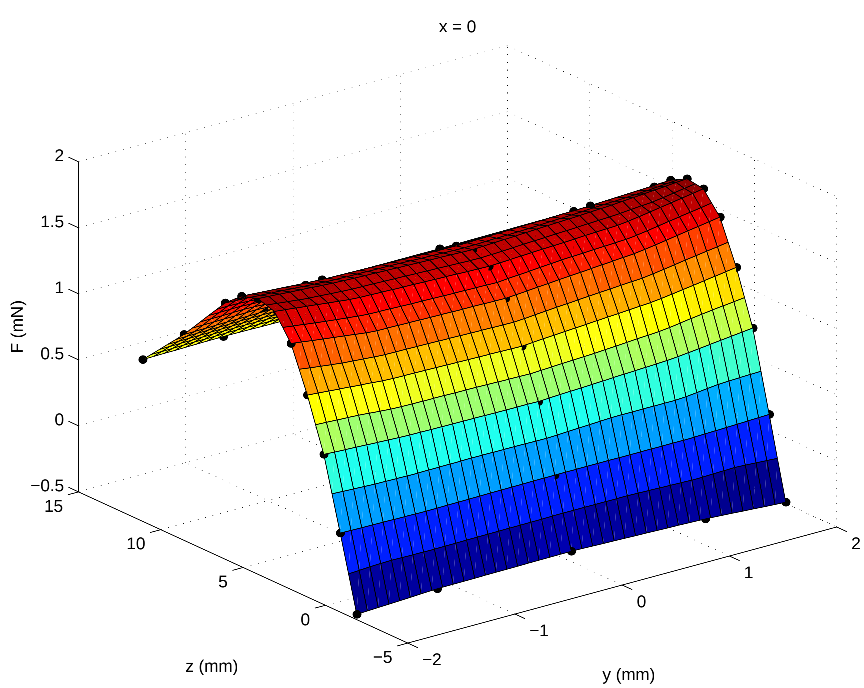
The next figure shows the force versus x and y for different z constant values. Be aware that each plot has a different scale for the force. The maximum force seems to be for z = 4.1 mm. An interesting thing is the peaks on the y axis extremes. Also, when z =-1.9 mm, the force is negative.

Figure 9



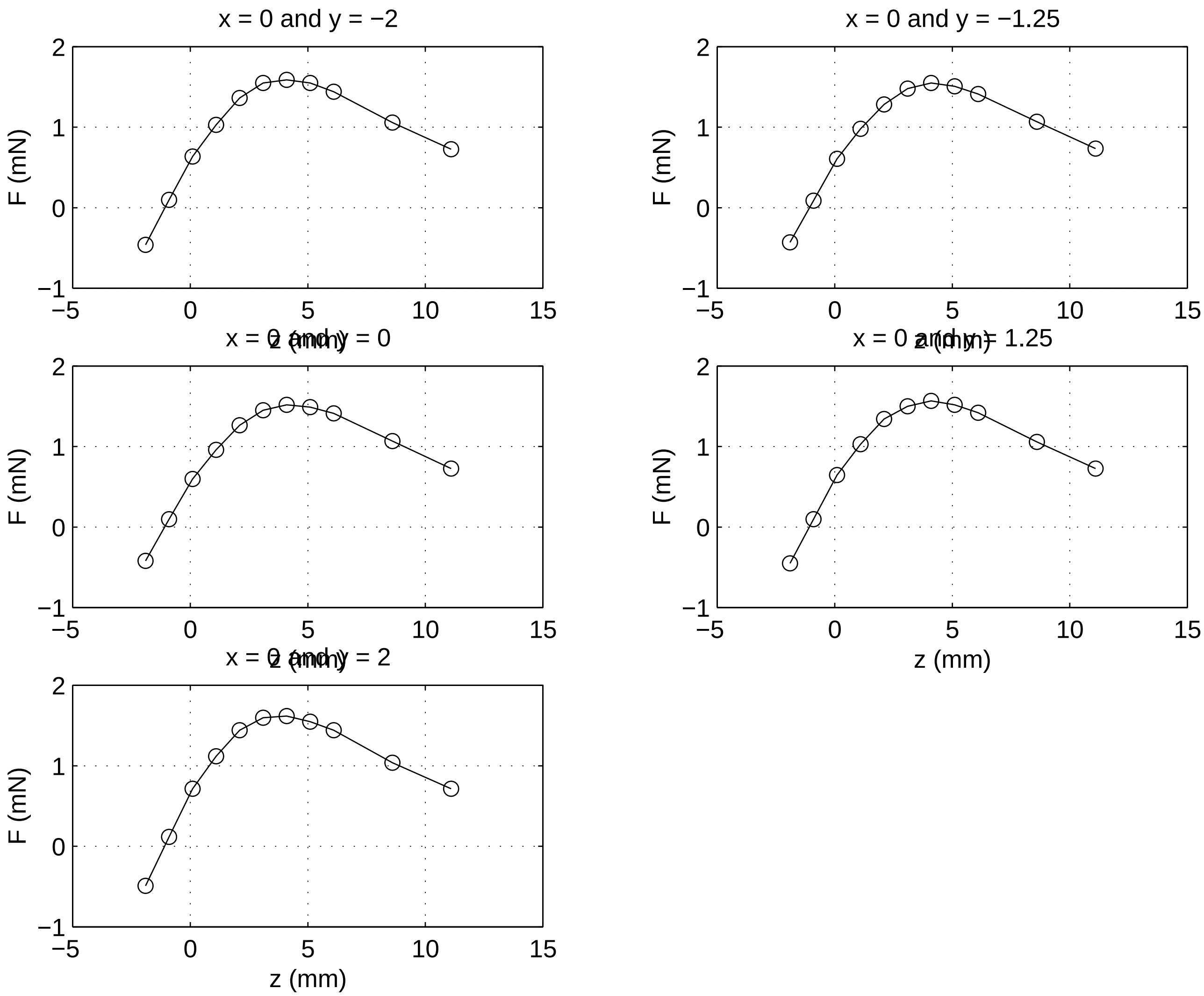
Below, figure10 displays the force as a function of z and y. It is presented the same way the model was presented. Here x = 0, so the plot should correspond to figure 4 of the model.

Figure 10



Moreover, let's look at the data as function of z only for x = 0 and different constant y values. This will allow to mesure the distance to the maximum. One can notice here again that for y = +/- 2 mm, the peak of the curve is higher than for y = 0.

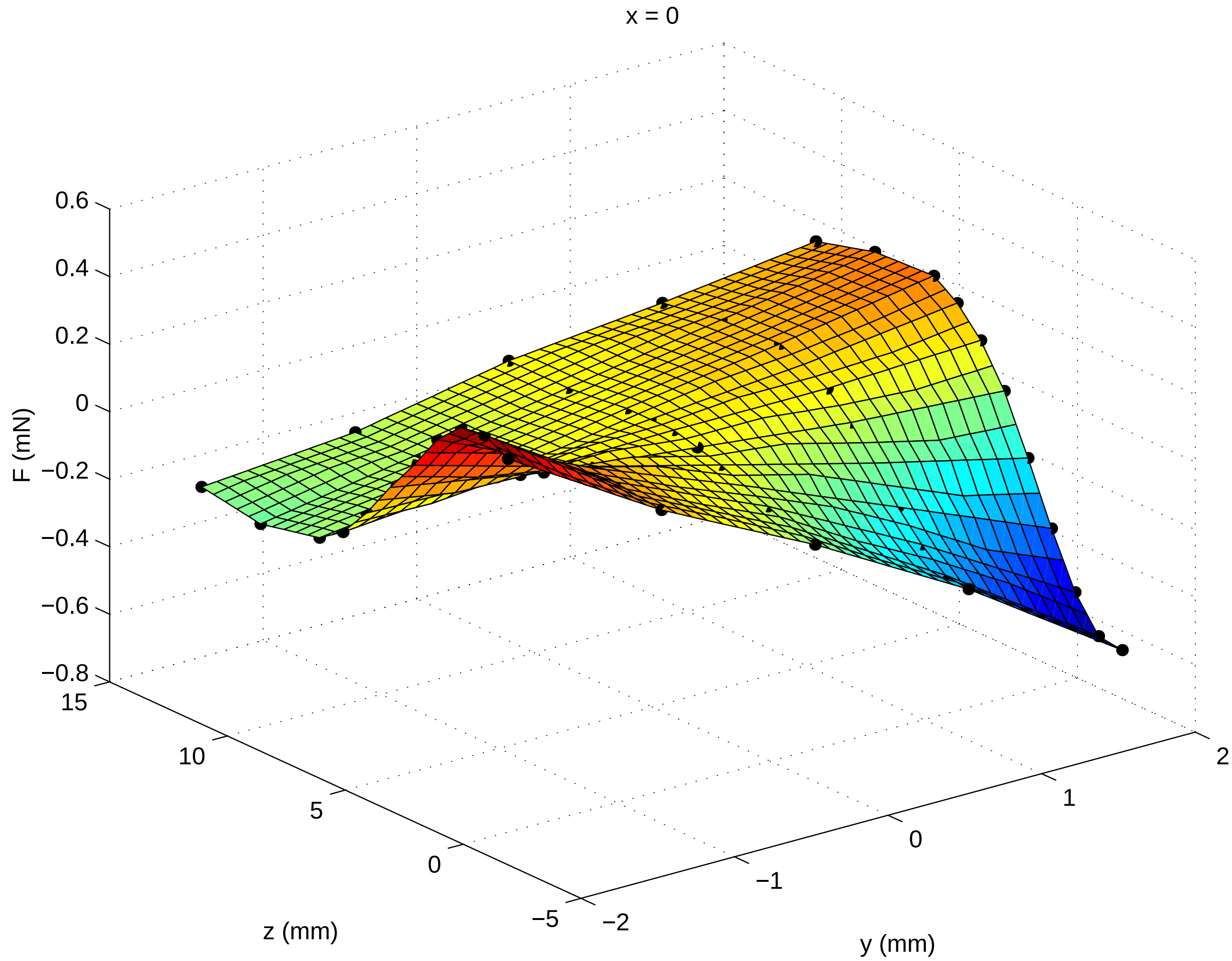
Figure 11



b) Transverse Force Complete Map

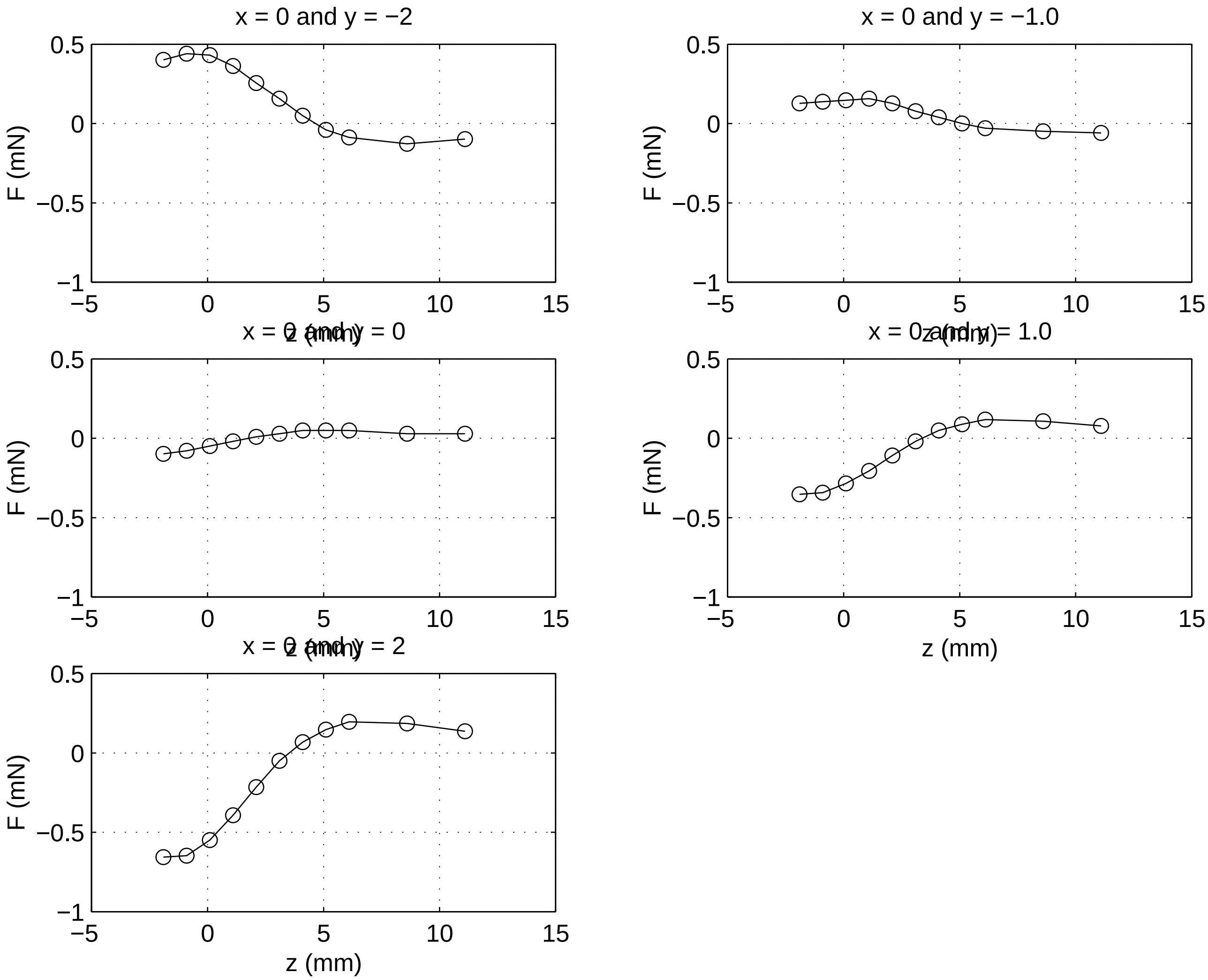
The transverse component data will be presented in two ways. The first one (figure12) shows the force as a function of z and y, for x =0. Secondly, figure 13 displays the force versus z with x and y constant.

Figure 12



In figure 12, near z = 0, the force reaches its maximum on both limits of the y direction range. The gradient changes direction near y = 0. Far away from z = 0, the magnitude of the transverse force gets much lower. Then, in figure 13, we get a better idea of how the force varies with z. First, notice that the force is nearly zero for z = 4, whatever the y value is. If you look at the evolution form the plot y = -2 mm to the plot y = 2 mm, you notice the inversion of the force direction taking place. On both side of z = 4, the magnitude of the force will decrease until the curve gets flat when y = 0 mm. Then, when y = 1 and 2 mm, the magnitude of the force increases again but the sign of the force has changed.

Figure 13



c) Quick maps

Like it’s been said earlier, less data points have been taken for the quick maps. The average force has been computed and the resulting plots are dispalyed here. The corresponding plots of figure 9 to 13 are showed below. Even if the curves are all less smooth, the quickmap for the average magnet show the same charactreristics described in the complete maps section.

Figure 14

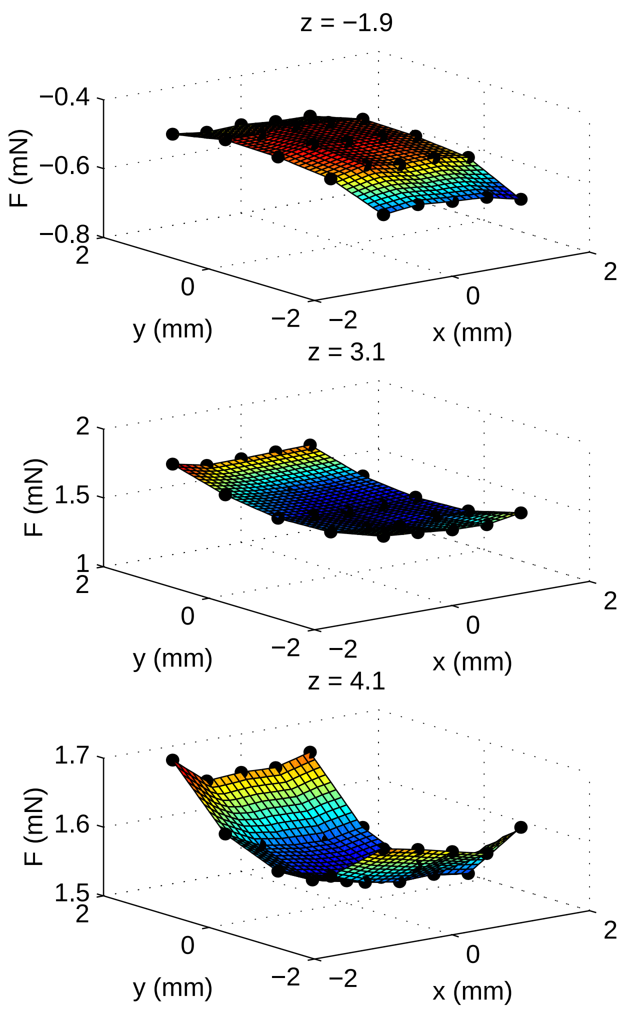


Figure 15

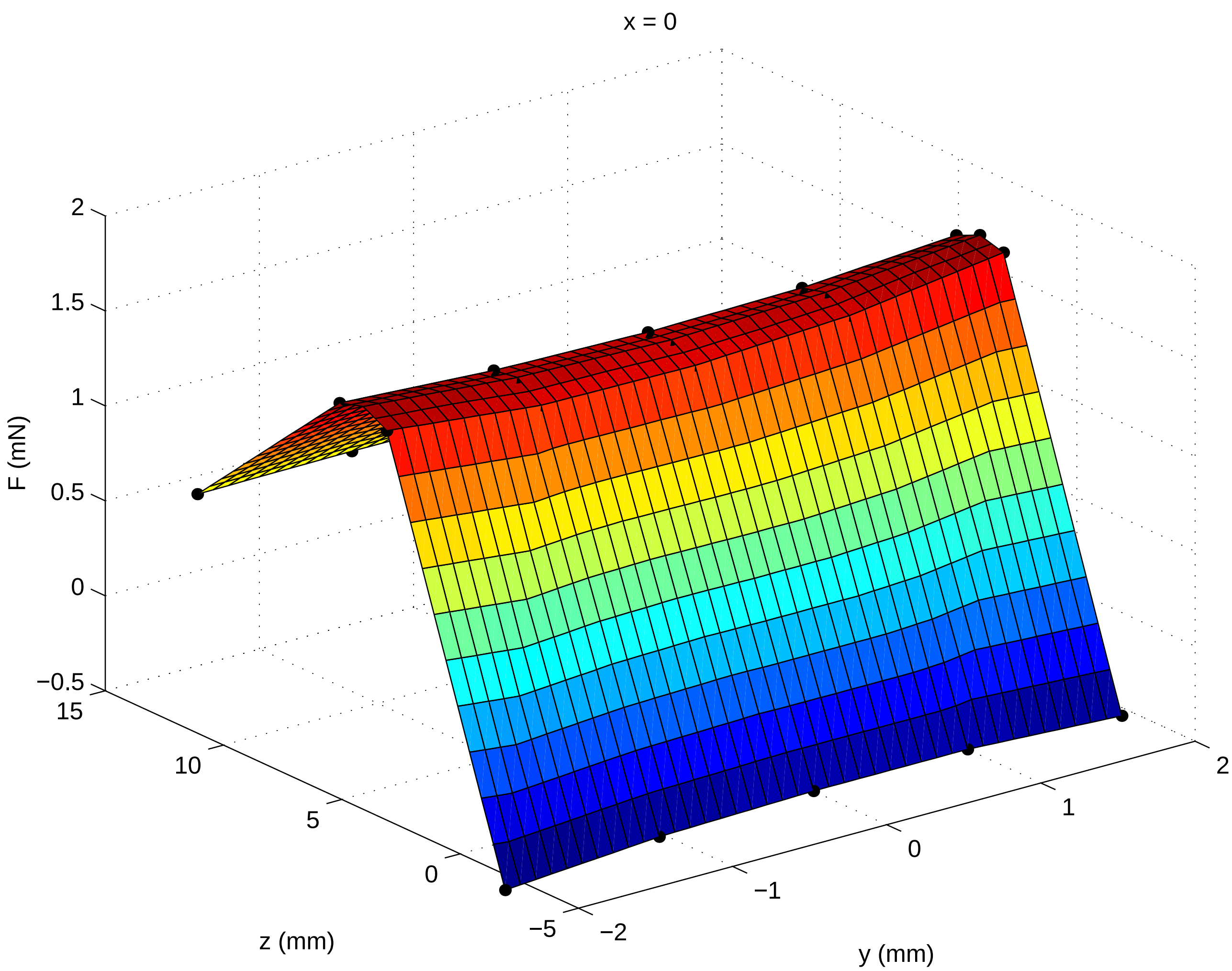


Figure 16

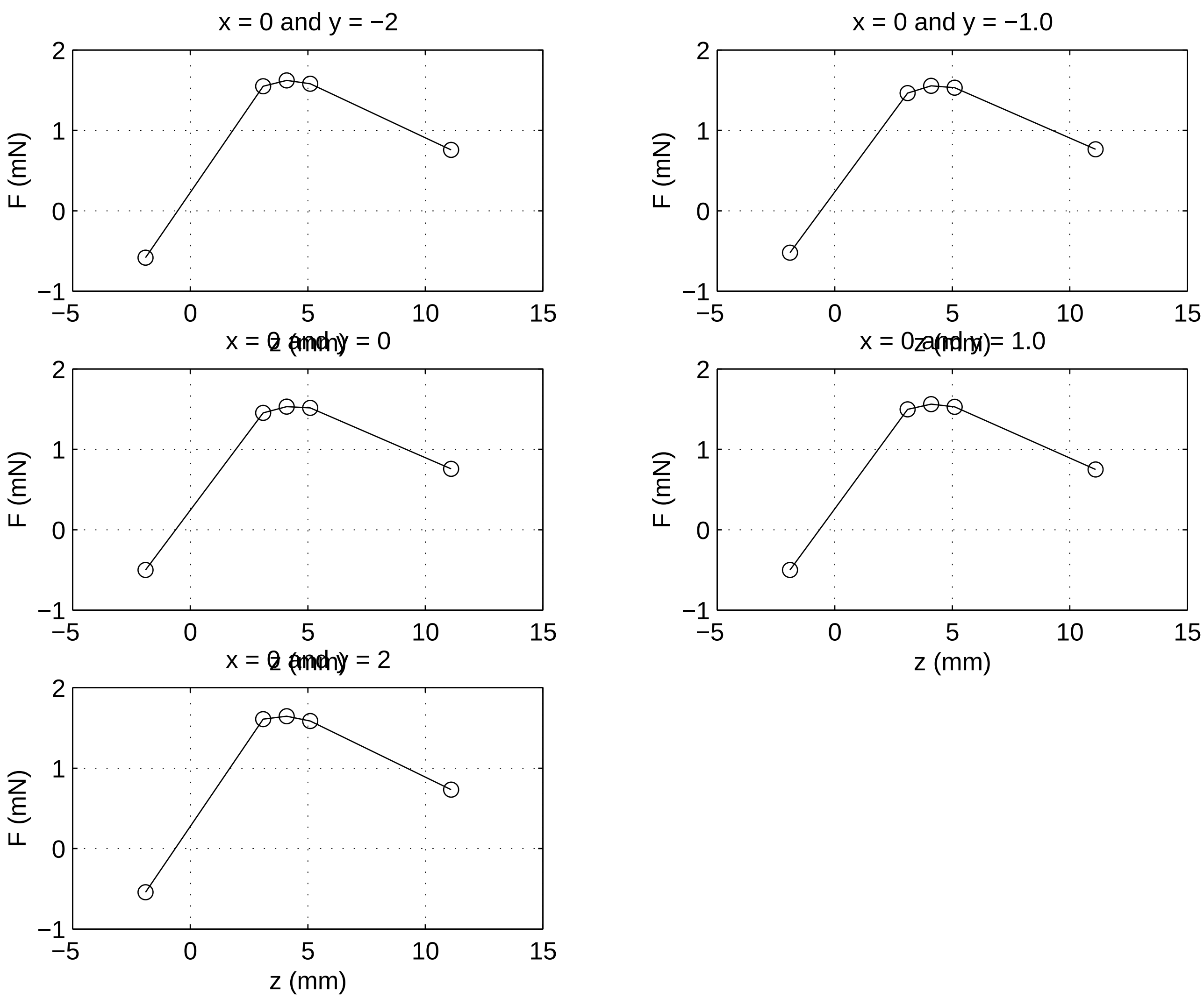


Figure 17

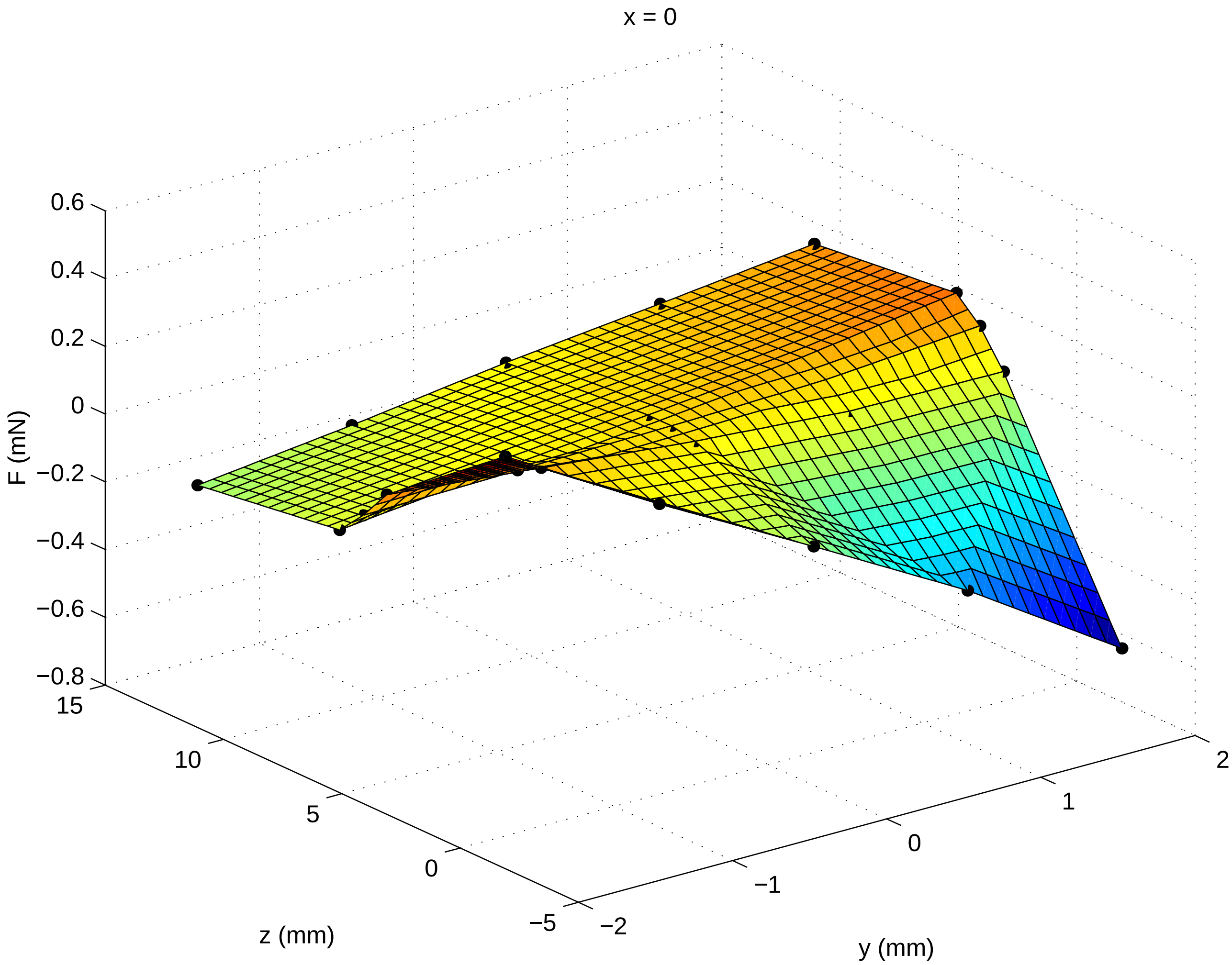
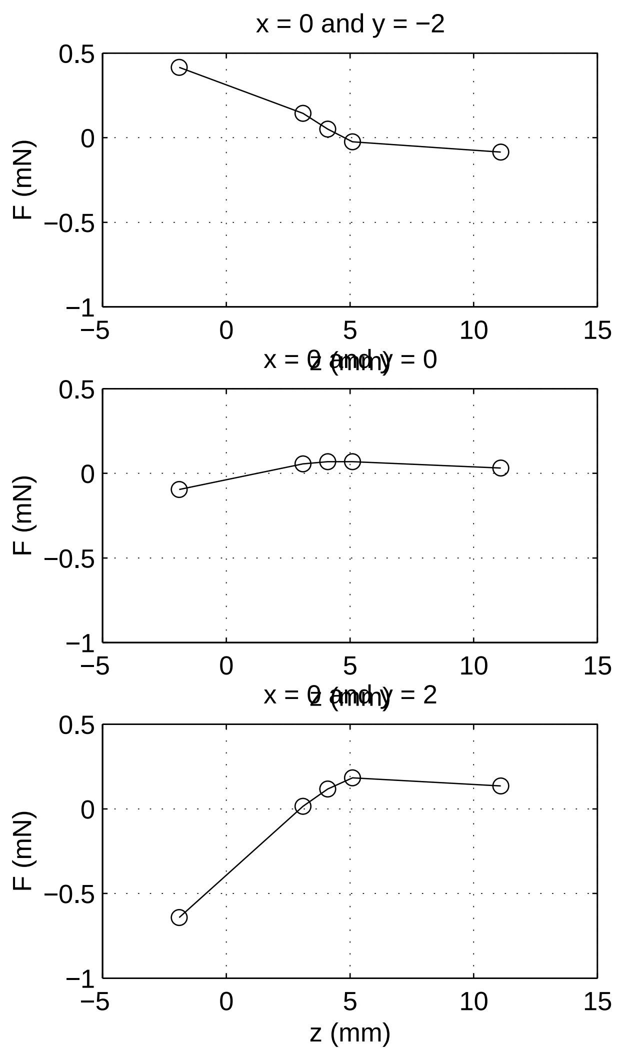


Figure 18



Of all the magnets, the highest axial force measured was 1.71 mN. Also, for all 6 magnets, this maximum force was always measured for z = 4.1 mm. Now, for the transverse force, the maximum measurement was 0.69 mN. The peaks for the transverse components were always for z = -1.9 mm and y = 2 mm. There were no major differencies between the magnets for the axial force. Unfortunately, there were more significant variation between magnets for the transverse component (see quickmaps data).

5. Analysis

a) Model versus Data

On one hand, comparing figure 4 to figure 10 and figure 7 to 11 shows that the magnitude of the force in the model is far from those of the data. However, the shape of the surface looks alike. They both increase at a high rate as z increases then they reach their peak before decreasing again. The model's peak seems to be near z = 8.25 mm while the data's peak is at z = 4.1 mm. The force seems to decrease slower in the model, the slope being smaller on both sides of the peak. The model doesn't have peaks at the extremes of the y axis like the ones of the data. The model gives a constant force for a fix z. If it would have been possible to measure the force on a long z range, the data would probably have roughly the same shape as the model given in figure 3.

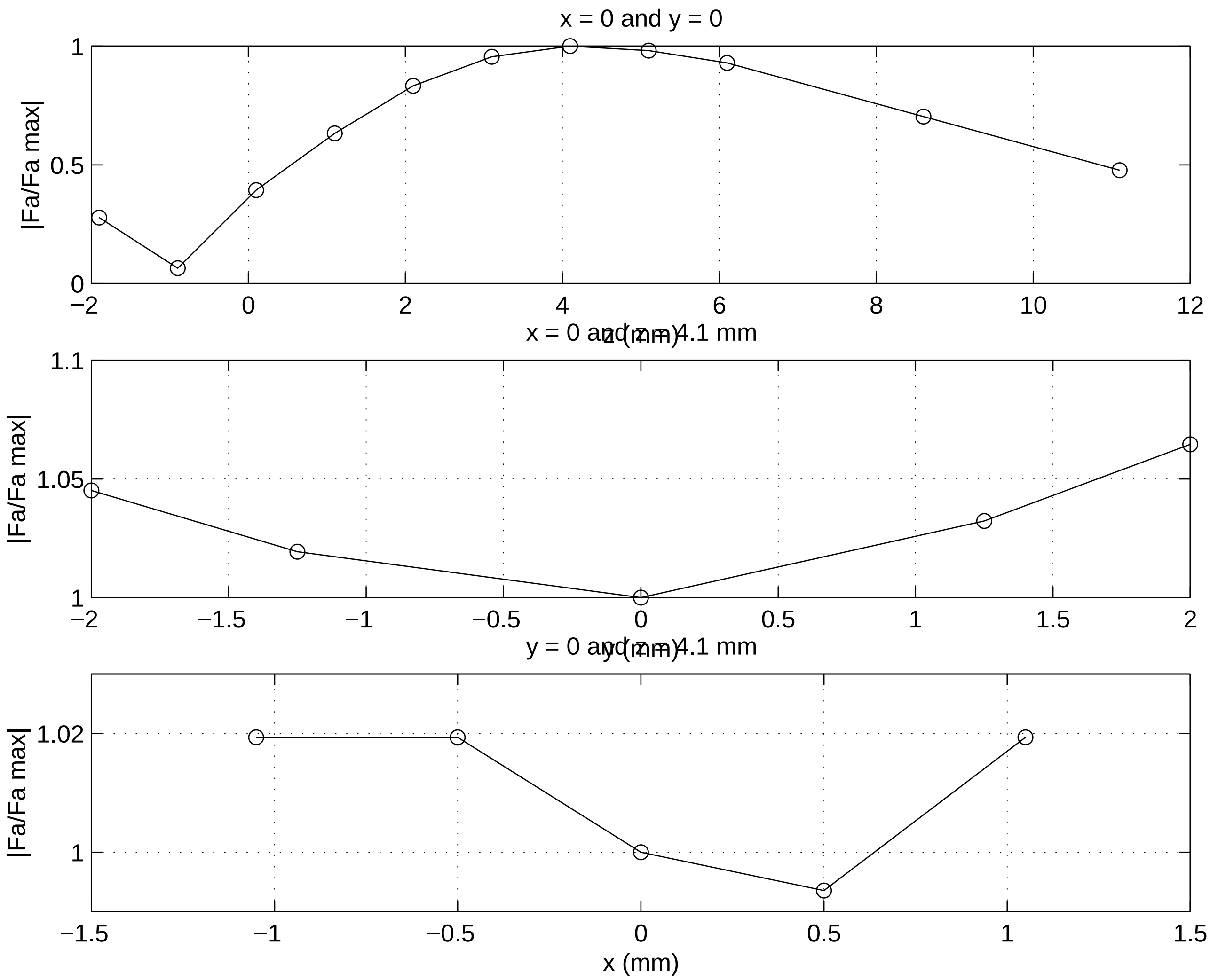
On the other hand, comparing the transverse component of the model to the data (figure 6 to 12 and figure 8 to 13), we notice once again that the magnitudes are way smaller in the model. The shape of both surfaces is similar even though the model has lower peaks than the data near z = 0 mm. The 4 edges of the the surfaces behave the same way in the model and in the experimental plots. However, for y = -2 and 2 mm, the model gives an equal but opposite force(figure 8). The data doesn't show such a behaviour on figure 13 as the magnitude of the force for y = 2 mm is higher than for y = -2 mm. It is probably fear enough to consider the shape of the surface in figure 5 as a good indication of the behaviour of the transverse force on a wide z range.

On figure 11, the maximum force is at z =4.1 mm. On figure 7, the model has its peak for z = 8.25 mm, which is almost the double. For z = 0, though, they both have a positive force. The model has a null force when z = -4 mm unlike the data where the force crosses zero for z = -1 mm. Obviously, the model is a lot more extended over the z axis than what the experiment reveals. Another thing is the fact that both the model and the data aren't having null forces for z = 0, like one would have expected.

b) Fractionnal Variation

It's also interesting to look at different ratios or fractionnal variation of the force. Considering the point (0,0,4.1) in the space, the following plot shows the proportion of the axial force compared to the maximum force measured. From the point (0,0,4.1) where the force is at its peak (1.519mN) for x = 0 and y = 0, we will know in each direction, the displacement allowed to stay within in a certain ratio of that maximum force. First, for x = 0 and y = 0, the axial force on the maximum force ratio (Fa / Fa max) will be ploted as a function of z. Then, Fa / Fa max will be presented as a function of y as x =0 and z = 4.1 mm. Finally, the same thing is done for Fa / Fa max versus x with y =0 and z =4.1 mm. In this report, we take the point (0,0,4.1) which is the center of the coil and the height where the axial force is maximum, but those curves could be computed for any other point.

Figure 19



Let’s say we want to stay with in a variation of 10 % from the maximum axial force. We can read on the graphics that between z =2.5 and z =6.5 mm, the force is at least 90% of the maximum force. Then, on the y axis, the force is always at the most 6.5 % greater than the force at (0,0,4.1) and for the x axis the force is always within 2 %. Assuming a symetry in the xy plane, we can say that if the magnet is within a prism of dimension 2 by 2 mm and a 3 mm height centered on the point (0,0,4.1), the error on the force is at most 10 %.

The next fractionnal variation presented is the ratio of the transverse force on the corresponding axial force (Ft / Fa) in the space. It’s been calculated around the same point (0,0,4.1).

Figure 20

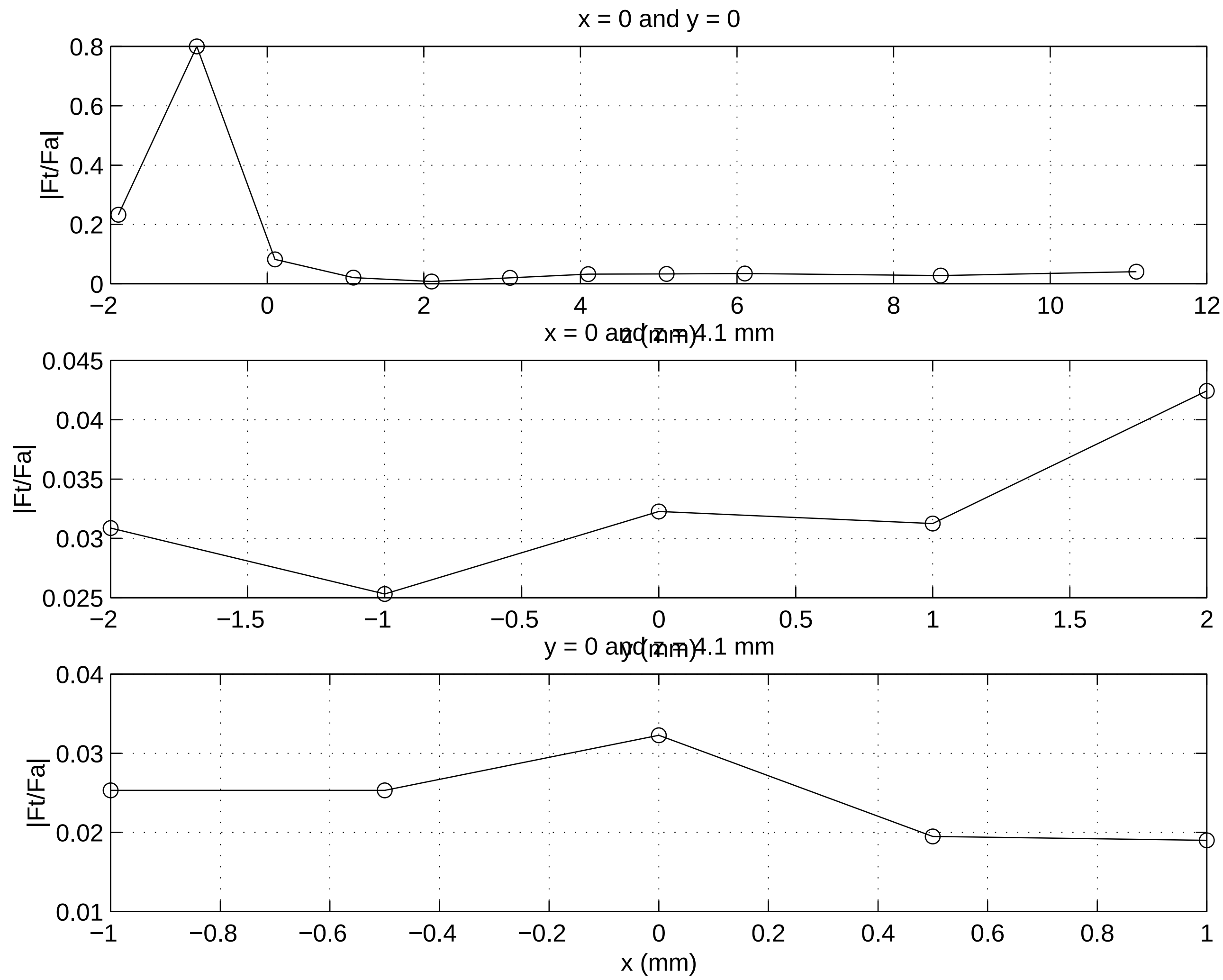


Figure 20 shows that as long as the magnet is at a z position greater than zero and inside the xy window of the coil, the transverse component of the force doesn't exceed 5 % of the corresponding axial force.

c) Initial Design

Apparently, the design was made for 0.02 N / A. That would represent a maximum force of 2 mN for a 100 mA current used in this experiment. It seems that the results are 15% lower than what was expected since the experimental maximum is 1.70 mN.

d) Error causes

Obviously, the experiment wasn't perfect. The experiment could be improved if the alignement of the coil and the magnet was more precise than by eye. Moreover, even if the magnet and coil would have been aligned, the up and down translation stage has also to provide displacement on the same axis as the coil-magnet axis. Another factor that affected the alignement was the glueing of the magnet. In fact, the magnet could have been lightly tilted due to the epoxy. The magnets were sticked so that by eye, they didn't seem tilted.

Consequently, the fact that all the magnets were aligned and sticked differently might have increased the variation of the measure from one magnet to another (that's why an average has been computed).

Another error cause might have been the current fluctuation in the coil. Throughout the measurement, the current was varying between 99.9 mA and 100.1 mA, Altough, it only represents an +/-0.1% error.

As for the thoeric model, a more realistic one could be computed for a better comparison between practice and theory. Considering the coil as a single loop of current (that has no surface) and considering the magnet as dipole was the simplest and the easiest to compute. The model of the magnetic dipole is good when the point of observation is far enough from the dipole. In this case here, the maximum distance between the coil and the magnet was 11.1 mm which is not very far considering the magnet's dimension. Furthermore, the current in the coil being distributed on a non null surface shows once again that the model used is not very realistic. Perhaps that the way the fit has been calculated is not the best way of fixing a value for the parameter M. Anyway, not a lot of time was spent on the model so it has weaknesses. Still, it gives a very good idea of the behaviour of the force in the space.

The optics table on which was sitting the whole set-up was magnetic so even though the magnet and coil were at a considerable distance of the table's surface, it might have lightly modified the magnetic field in the magnet's area. A better shielding against external magnetic fields could probably help.

A further interesting experiment that could be done is the measurement of the torq on the magnet. The model dipole with single loop model could also be computed for the torq. The main problem would be to figure out a practical way to measure the torq.

Conclusion

To conclude, the model is far from reality in terms of force magnitudes but still gives a good idea of the shape of the maps. There are still those axial force peaks on the edges of the xy window that remain unexplained. Altough, the non symetric experimental transverse force could be a result of a misalignement. Finally, we saw that the transverse component is usually smaller than 5 % of the axial force.

Note

I am also sorry for the level of written english in this document.
This experiment is incomplete, it's all I could do within 6 weeks.

Here is the link to the different files containing my work.

The report and all the images in it are in the directory:
/home/darcy/magnet/report

The matlab M-files are in:
/home/darcy/magnet/mfiles

The Mathematica files used t compute the model are in:
/home/darcy/magnet/nbfiles

The data as it was taken in the lab is in :
/home/darcy/magnet/data
Note that the data is manipulated a lot in theM-files before plottting.

Question, comments, etc

email: olivier.darcy@polymtl.ca