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Longitudinal and angular alignment requirements for Advanced LIGO mode cleaner

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The requirements for intensity noise [1]in Advanced LIGO below 100Hz are

$$RIN(f) = \frac{\delta I(f)}{I_o} < \frac{2\cdot 10^{-9}}{\sqrt{\mathrm{Hz}}}\frac{f}{[10\,\mathrm{Hz}]} \qquad f < 100\,\mathrm{Hz}$$

'RIN" refers to relative intensity noise. Above 100Hz the requirements depend on the sensing scheme.
For **DC-sensing** the requirements at higher frequencies decrease with the same frequency law than below 100Hz:

$$RIN(f) = \frac{\delta I(f)}{I_o} < \frac{2\cdot 10^{-9}}{\sqrt{\mathrm{Hz}}}\frac{f}{[10\,\mathrm{Hz}]}$$

For **RF-sensing** the requirements above 100Hz remain more or less constant at a level of

$$RIN(f) = \frac{\delta I(f)}{I_o} < \frac{2\cdot 10^{-8}}{\sqrt{\mathrm{Hz}}}$$

Frequency Noise

Changes in the laser frequency convert into changes in the intensity transmitted through the mode cleaner (MC):

$$I(t) = I_o\left|\frac{t_1t_2}{1-r_1r_2e^{i\Phi(t)}}\right|^2 = I_o\frac{T_1T_2}{1+R_1R_2-2r_1r_2cos(\frac{2\pi\nu(t)}{FSR})}$$

Here FSR is the free spectral range of the MC. The r_i's and t_i's are amplitude reflectivities and transmissivities; R_i and T_i refer to intensity reflectivity and transmissivity.

In the case of the MC, we can assume $R_1 = R_2$ and $T_1 = T_2$. Any deviations from this ideal case do not change the requirements.

Close to resonance the cosine can be approximated by

$$cos(\frac{2\pi\nu(t)}{FSR}) \cong 1 - \frac{2\pi^2\nu^2(t)}{FSR^2}$$

and the transfer function becomes quadratic in frequency. In this situation, the conversion from frequency noise into intensity noise requires a static detuning $\Delta\nu$of the laser frequency from the MC resonance.

Frequency noise can then be expressed as frequency modulation:

$$\nu(t) = \nu_c + \Delta\nu + \int \delta\nu(f)\sin(2\pi ft)df$$

This leads to

$$I(t) \approx I_o\left(1 - \frac{4\pi^2}{T^2FSR^2}\left(\Delta\nu^2 + 2\Delta\nu\int \delta\nu(f)\sin(2\pi ft)df\right)\right)$$

The linewidth of the mode cleaner is:

$$FWHM = \frac{FSR}{Finesse} \approx \frac{T \cdot FSR}{\pi} \approx 4.5\,\text{kHz}$$

which gives

$$I(t) \approx I_o\left(1 - \frac{4}{FWHM^2}\left(\Delta\nu^2 + 2\Delta\nu\int \delta\nu(f)\sin(2\pi ft)df\right)\right)$$

The spectral density of the relative intensity noise is then:

$$RIN(f) = 8\frac{\Delta\nu}{FWHM}\frac{\delta\nu(f)}{FWHM}$$

The frequency noise requirements in the mode cleaner in the frequency range 10 to 1000 Hz are [1]:

$$\delta\nu(f) < 3 \cdot 10^{-3}\frac{\text{Hz}}{\sqrt{\text{Hz}}}$$

This leads to

$$RIN(f) \approx \frac{10^{-9}}{\sqrt{\text{Hz}}} \times \frac{\delta\tilde{\nu}(f)}{[3 \cdot 10^{-3}\frac{\text{Hz}}{\sqrt{\text{Hz}}}]} \times \frac{\Delta\nu}{[\text{Hz}]} < \frac{2 \cdot 10^{-9}}{\sqrt{\text{Hz}}}\frac{f}{[10\text{Hz}]}$$

The requirement that the intensity noise be < $\frac{2 \cdot 10^{-9}}{\sqrt{\text{Hz}}}$ at 10Hz leads one to conclude $\Delta\nu < 2$ Hz is sufficient. The frequency noise includes the 'LIGO safety factor' of 10.

Pointing Noise

In the paraxial approximation the spatial part of the solution to the wave equation may be expressed in the orthonormal basis of Gaussian modes: $U(x,y) = \sum c_{nm} U_{nm}$. By design the laser input to the mode cleaner should match the lowest order eigenmode, U_{00}, of the cavity. Mismatches result from misalignments of the input beam, such as translations of the mode axis or tilt of the input phase front. Small pointing problems cause some of the laser intensity to be distributed to the higher-order U_{10} and U_{01}modes.

We use the Hermite Gaussian basis:

$$U_{nm} = U_n(x)U_m(y)$$

and calculate only for x, later to assume that half the pointing error may be attributed to each of the transverse axes. For small misalignments the laser input is:

$$U_0(x) = \left(\frac{2}{\pi}\right)^{\frac{1}{4}} \sqrt{\frac{1}{\omega}} e^{-\frac{(x-\Delta)^2}{\omega^2}} e^{i\alpha x}$$

where Δ denotes a translation from the axis of the cavity modes and $\alpha = \frac{2\pi\theta}{\lambda}$ with θ a tilt of the phase front. Expanding to second order, and including all terms proportional to Δ^2, α^2, and $\Delta\alpha$, the new spatial mode can be expressed as a linear combination of the cavity U_0 and U_1 modes:

$$U_0^{'} = aU_0 + bU_1$$

We find that

$$|a|^2 = 1 - \frac{\Delta^2}{\omega^2} - \frac{\alpha^2\omega^2}{4} - \alpha\Delta$$

and

$$|b|^2 = 1 - |a|^2$$

The coefficient b will contain both static misalignment and dynamic pointing terms:

$$|b|^2 = |b_o + \delta b|^2 \cong |b_o| + |2b_o\delta b|$$

Half the allowed relative intensity noise is allotted to the x-direction:

$$4 \cdot |b_o\delta b(f)| < RIN(f) < \frac{2 \cdot 10^{-9}}{\sqrt{\text{Hz}}} \frac{f}{[10\text{Hz}]}$$

After the mode cleaner the beam jitter ($\delta b(f)$) must be less than $\frac{4 \cdot 10^{-10}}{\sqrt{\text{Hz}}}$. With a finesse of ~2000, the mode cleaner will reduce the input jitter by three orders of magnitude, thus the current MC input beam jitter specific is $\delta b(f) < \frac{4 \cdot 10^{-7}}{\sqrt{\text{Hz}}}$. The permissible static misalignment of the input beam is:

$$b_o < \frac{RIN(f)}{\delta b(f)} = 10^{-3}\text{rad}$$

Comment

The analysis does not include the fact that the transmitted intensity will be stabilized by the intensity feedback loop to the laser. In a closed loop configuration, the changes of the intensity caused by FM or pointing will be compensated by the feedback. So far, we haven't identified a process in which this coupling between FM and AM and Pointing and AM causes problems. Details TBD.

References

[1] Advanced LIGO Systems Design (LIGO-T-010075-00-D, June 27, 2001, LSC ed. Peter Fritschel)