T020025-00-D
EO-Modulators for Advanced LIGO
Part I

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1 Conceptual Design

1.1 RF Modulation

1.1.1 Material

The electro-optic modulators are developed with Quantum Technology Inc. located in Lake Mary, FL.. The company specializes on high power Pockels cells for pulsed laser systems and acousto and electro optical modulators.

They studied 4 different materials: KTP, KTA, RTA (data from Crystal Associates), RTP data from Raicol. For comparison LiNb03 data from New Focus, Crystal Technology and the Handbook of Optics.

[a] :Raicol gives only one index of refraction. Because of the crystal structure, they have to be different. Nevertheless, because of the similarity between RTA and RTP, the difference will be very similar.

[b]: Number from Raicols web page.

[c]: The absorption coefficient is only a very rough upper limit. In fact, we measured already that the thermal lensing in RTA indicates a several order lower absorption coefficient. Raicol claimed that their RTP crystals have an absorption below 50ppm/cm. The largest EO-coefficient is in all 5 materials r_{33}. The optimum configuration is propagation direction y-direction, applied electrical field and polarization of the light field z-direction. The modulation depth is:

$$\Delta\phi = m = \frac{\pi L}{\lambda} r_{33} n_z^3 \frac{V_z}{d} \tag{1}$$

where L is the length of the crystal and d the thickness in z-direction. The lowest drive voltage requires LiNbO3, but as we will see later, it is not useful because of thermal problems.

RTA and RTP require a 10% larger drive voltage. Based on these data we decided to study both materials with RTP being our first choice because of the lower electrical loss

Properties	KTP	KTA	RTA	RTP	LiNbO3
Laser Damage Threshold [MW/cm^2, 10ns 1064nm]	600 (coated)	400	400	600 (coated)	280[b]
n_x @ 1064nm	1.748	1.788	1.811	1.9[a]	2.23
n_y @ 1064nm	1.756	1.796	1.815	1.9[a]	2.23
n_z @ 1064nm	1.840	1.878	1.890	1.9[a]	2.16
α[c] @ 1064 nm [1/cm]	<0.5%	<0.5%	<0.5%	<0.5%	<0.5%
EO-coefficients (pm/V) @ 633nm					
r_{33}	36.3	37.5	40.5	39.6	30.8
r_{23}	15.7	15.4	17.5	17.1	8.6
r_{13}	9.5	11.5	13.5	12.5	8.6
r_{42}	9.3	?	?	?	28
r_{51}	7.3	?	?	?	28
r_{22}					3.4
$r_{33}n_z^3$	226	248	273	272	306
Electrical (640 kHz, 22°C					
Dielectric Constant e_{33}	23	44	19	13	
Conductivity s_{33} $(1/\Omega cm)$	6×10^{-6}	5×10^{-5}	3×10^{-9}	$\approx 10^{-11}$	
Loss Tangent d_{33}	0.7	2.9	0.0004	?	

angle and higher laser damage threshold. The modulation depth of m of 0.5 requires for RTA and RTP:

$$V_z \frac{L}{d} = 620V$$

The drawback of RTA and RTP are:

- crystals only available in small sizes
- crystals are expensive
- most thermal data are unknown

The actual design requires two crystals for each modulator:

- Size of each crystal 4mm X 4mm X 15mm
- Probably rotated by 90 deg with a $\lambda/2$ plate or a quartz rotator in between

The size requires a voltage of

$$V_z = 620V \frac{d}{2L} = 83V$$

across the crystal. The low electric conductivity or huge resistivity of RTP reduces the electric power consumption inside the crystal to:

$$P = UI = \frac{U^2}{R} = \frac{83^2}{10^{11}\frac{d}{L}} W = 30nW$$

In RTA we could expect a two orders of magnitude larger heat generation. But in both cases I expect that the electric heat is dominated by rest resistivity of the leads and the electrodes and that the heat produced by the laser beam itself is more critical because of its non uniform spatial distribution.

The thermal properties of RTA and RTP are more or less unknown. But because of the similarities between RTA, RTP, KTP, and KTA, we expect that the properties are similar to KTP. All thermal problems scale with

$$\frac{\alpha}{\kappa} \frac{dn}{dT}$$

Properties	KTP	LiNb03	RTP	RTA
dn/dT_x [$10^{-6}/K$]	11^a	5.4^b	3^d	
dn/dT_y [$10^{-6}/K$]	13^a	5.4^b	5^d	
dn/dT_z [$10^{-6}/K$]	16^a	37.9^b	8^d	
κ_x [W/mK]	2	5.6^c	?	1.6^d
κ_y [W/mK]	3	5.6^c	?	1.6^d
κ_z [W/mK]	3	5.6^c	?	1.7^d

[a] :Handbook of Optics, no wavelength specified, [b] Crystal Technology, Inc. [c] only one value given, no axis specified, [d] www.cristal-laser.fr/PAGES/rtp.htm.

The dn/dT coefficients for RTP are based on formulars given on the webpage:

$$\frac{dn_x}{dT} = 1.9 \cdot 10^{-6} + 3.11 \cdot 10^{-8} T - 9.93 \cdot 10^{-12} T^2$$

$$\frac{dn_y}{dT} = 3.2 \cdot 10^{-6} + 4.21 \cdot 10^{-8} T - 7.72 \cdot 10^{-11} T^2$$

$$\frac{dn_z}{dT} = 6.30 \cdot 10^{-6} + 6.89 \cdot 10^{-8} T - 7.72 \cdot 10^{-11} T^2$$

The huge difference in the ratios of dn/dT in LiNb03 creates a huge ellipticity in the beam. We observed this in our experiments in which one direction was extremely distorted while the second direction seemed to be OK. Nevertheless, the ratio between dn/dT and κ for the bad axis in LiNb03 and the worst case in KTP is similar. The expected thermal lens can be approximated from the sag change over the beam profile of (Mansell et al.) are possible:

$$\delta s \approx 0.1 \frac{\alpha L P}{\kappa} \frac{dn}{dT} \approx 2.4 \mu m \frac{\alpha}{[10^{-2}/cm]} \frac{L}{[1.5cm]} \frac{P}{[200W]}$$

For an absorption coefficient of $\alpha = 5 \cdot 10^{-3}/cm$ the sag change is roughly in the order of one wavelength. This leads to a thermal lens of

$$f \approx \frac{w^2}{\delta s} \approx 1m \left(\frac{w}{[mm]} \right)^2 \left(\frac{[\mu m]}{\delta s} \right)$$

The amount of higher order modes that can not be compensated by changes in the mode matching can be calculated using the :

$$P_{HM} = 0.24 \frac{\delta s}{\lambda} - 0.03$$

Roughly 20% of the light would be lost in each crystal. 6 crystals and nothing is left. The absorption coefficients given in the first table include also losses that do not produce heat like scattering, 2^{nd} harmonic generation etc.. Therefor, the absorption values are only upper limits for the thermal problems we could expect. The heat producing absorption coefficient will be much lower.

According to informations from different sources is the heat producing absorption coefficient in KTP is much lower than in LiNb03. The value for RTP seems even lower. Raicol claimed on their web page that the absorption coefficient for RTP is as low as 50ppm/cm. And that still includes scattering and second harmonic generation. This is two orders of magnitude lower than the absorption coefficient used above. The subsequent sag change in the beam would be about 1% of the optical wavelength, the thermally induced lens in the 100m range in each crystal and the higher order mode content below anything we would take serious. We tried to measured the thermal lens in a 10mm RTA-crystal from crystal associates pumped with 50W laser power and failed. We could only come up with a lower limit of a about 10m which was set by our mode analyzer. More sensitive measurements are pending.

1.1.2 Stability

Besides the thermal problems and subsequent losses into higher order modes and birefringence, the main problem associated with the modulators is the stability of the sidebands. The requirements depend strongly on the used sensing scheme and the bandwidth of the various feedback loops (see [1]).

A. DC-Locking

The GW-signal is measured by beating the signal sidebands with a carrier at the dark port. This power measurement requires a small detuning in the arm cavities but does not directly involve the sidebands. In fact, the output mode cleaner will take out most of the sidebands and they won't even show up on the photodetector. The sidebands are only used to measure the auxiliary degrees of freedom and one set is used for the common Fabry Perot degree of freedom. These degrees of freedom are in first order sensitive to changes in the sidebands. The GW-signal is only sensitive because of the cross coupling between these degrees of freedom and the differential Fabry Perot degree of freedom. The specifications depend on the modulation index m. They are derived in [2]:

$$\delta m(f) < \frac{10^{-9}}{m\sqrt{Hz}} \frac{f}{[10Hz]}$$

B. RF-Sensing

The GW-signal is measured by beating the signal sidebands with RF-sidebands at the dark port. Any carrier at the dark port caused by asymmetries in the arms of the Michelson interferometer will also beat with these RF-sidebands. This makes the dark port signal sensitive to changes in the RF-sidebands. The specifications depend on the modulation index m. They are derived in [2]:

$$\delta m(f) < \frac{10^{-9}}{m\sqrt{Hz}} \frac{f}{[10Hz]} \qquad < 100Hz$$

$$\delta m(f) < \frac{10^{-8}}{m\sqrt{Hz}} \qquad > 100Hz$$

These specifications include already a safety factor of 10.

C. Summary Requirements

The requirements in both designs depend on the modulation index. Although the modulation index is not yet set, it is very likely that the modulation index in a RF-Sensing scheme has to be larger than in a DC-Sensing scheme. This and the lower requirements at higher frequencies favor again the DC-Sensing scheme. But our goal has to be to reach the RF-Sensing scheme requirements.

D. Fluctuations in the Modulation Index

The modulation index is given by eq.1

$$m = \frac{\pi L}{\lambda} r_{33} n_z^3 \frac{V_z}{d} \tag{2}$$

D.1. Voltage

The first parameter that needs to be controlled is the Voltage V_z across the crystal. The capacity of the electrodes at each crystal is:

$$C = \varepsilon\varepsilon_0 \frac{A}{d} = \varepsilon\varepsilon_0 L = 1.7 \cdot pF$$

The current that is necessary to produce the required voltage ($m = 0.5$) at a frequency f can be calculated from

$$I_c = i2\pi f C V_z \approx 160 mA$$

This is equivalent to 10^{18} electrons per sec and the shot noise is therefor

$$\Delta I_{SN} = 0.16 \frac{nA}{\sqrt{Hz}} \qquad \Rightarrow \qquad \Delta V_{SN} = 84 \frac{nV}{\sqrt{Hz}}$$

This is the shot noise without a resonance circuit.

D.2. Other Parameters

Is there any noise in the other parameters:

- Index of refraction changes (Temperature) ?
- Changes in the electro optical coefficient r_{33} (Temperature) ?
- Changes in the dimensions (Temperature) ?
- Changes in the dielectric constant ε(Temperature) ?

All these parameters will change with the temperature. This requires a good temperature stabilization to keep the modulation index constant on all time scales. Below the GW-band changes will cause drifts in offsets in each length sensor. In the GW-band it will mimic a signal because of the subsequent intensity changes in sideband and carrier. Changes of the temperature will change the index of refraction. The modulation index will change with:

$$\delta m = \frac{\pi L}{\lambda} r_{33} \frac{V_z}{d} 3 n_z^2 \delta n \approx 1.5 \frac{[0.5]}{m} \frac{\delta n}{n} \tag{3}$$

Therefor the required stability of the index of refraction is:

$$\delta n < \frac{n}{3m^2} \frac{10^{-9}}{\sqrt{Hz}} \frac{f}{[10Hz]}$$

with

$$\delta n = \frac{\partial n}{\partial T} \delta T$$

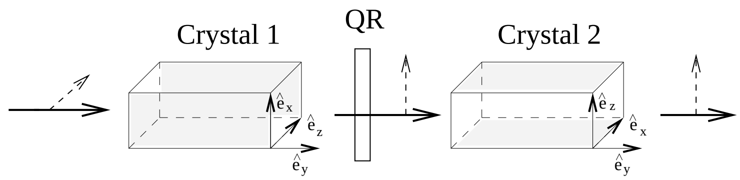

Figure 1: Two modulator crystals with a quarter wave plate should be used if the birefringence in a single crystal design is unacceptable.

the required temperature stabilization has to be:

$$\delta T < \left(\frac{\partial n}{\partial T}\right)^{-1} \frac{n}{3m^2} \frac{10^{-9}}{\sqrt{Hz}} \frac{f}{[10Hz]} \approx \frac{33\mu K}{\sqrt{Hz}} \frac{1}{m^2} \frac{f}{[10Hz]}$$

or about $0.13mK/\sqrt{Hz}$ for a modulation index of 0.5.

The temperature dependence in the other parameters are not well known, but I expect it to be in the same $10^{-5}/K$ range as is the index of refraction. This would lead to similar requirements for the temperature stability.

1.1.3 Design

A. Optical Layout

Each modulator will consist of two crystals (see Fig. 1). The direction of propagation is the y-axis, the applied electric field will be parallel to the polarization of the laser field and points in the z-direction. Both crystals are rotated by 90° with respect to each other and between the crystals is either a quartz rotator (QR) or a $\lambda/2$ plate to rotate the polarization by 90°.

The decision to go for two crystals instead of one for each modulator was driven by the fact that the manufacturer do not produce crystals with sufficient length. But if we have to use two crystals we can also try to reduce the thermally induced birefringence with this configuration.

Each crystal will have a surface of 4mm X 4mm. A $w =$1mm Gaussian beam radius will encounter losses due to clipping below 500ppm.

The intensity distribution in a Gaussian mode with total power P_o is:

$$P(r) = P_o \frac{2}{\pi} \frac{e^{-2\frac{r^2}{w^2}}}{w^2}$$

The intensity in the center is therefor:

$$P(0) = 12.7 \frac{kW}{cm^2}$$

About 50000 times below the damage threshold quoted for 10ns pulses. Although this still needs to be verified, it seems unlikely that this power level even at CW damages the coatings. Raicol quotes a damage threshold for the uncoated RTP crystals of $1GW/cm^2$.

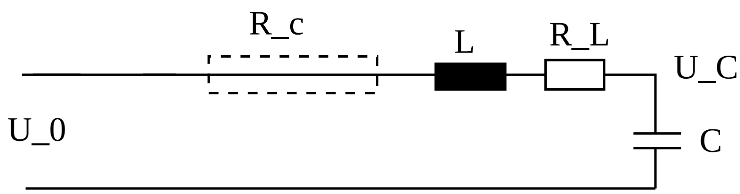

Figure 2: The source voltage is connected through the cable to the load. This cable is a $Z_c = 50\Omega$ cable. The load consists of an inductor L, a spurious resistiance $R_L < 1\Omega$, and the capacitive load of the crystal C. The voltage U_C across the crystal and the power dissipation in the crystal and the rest of the tank circuit is important.

B. Temperature Stabilization

It is planned to achieve the temperature stability with an active temperature control circuit. The temperature can be measured with standard low voltage temperature sensors like the TMP35/36/37 which has a sensitivity of 10mV/K. The temperature will be stabilized by feeding the error signal to peltier elements that can cool the crystal from all four sides. If additional heat sinks are needed to be used is TBD.

1.2 Resonant Circuit and Power Dissipation

The tank circuit is shown in Fig. 2. The current through the full circuit can be calculated from the total impedance Z_t:

$$Z_t = Z_c + i\omega L + R_L + \frac{1}{i\omega C}$$

The resistance of the cable is

$$Z_c = \sqrt{\frac{L}{C}} = 50\Omega \qquad \text{resistive, L : inductance/m, C : capacitance/m}$$

The current is

$$I = \frac{U_0}{Z_t}$$

The circuit is supposed to be on resonance at the resonance frequency of $f = 180\text{MHz}$:

$$\omega = \frac{1}{\sqrt{LC}}$$

and the current is then limited by the resistors:

$$I = \frac{U_0}{Z_c + R_L} \approx \frac{U_0}{50\Omega}$$

The voltage accross the capacitance is:

$$U_C = \frac{I}{i\omega C} = \frac{U_0}{i\omega(Z_c + R_L)C}$$

The capacitance across the crystal is somewhere between $1.7pF$ and $3pF$. Raicol claims it $3pF$, our small calculation gave $1.7pF$. Lets continue with $2.5pF$ (about what Larry measured).

$$U_C = -i7.08 \cdot U_0 \qquad \text{at } 180\,\text{MHz}$$

The measured build up was about 4.75. This indicates a capacitance of about $3.75pF$ in the tank.

The half wave voltage accross the crystal is 165V if we use only one crystal. A build up of 4.75 would reduce the required drive voltage to 35V (all peak values). The total current for such a drive voltage would be:

$$I = \frac{35V}{50\Omega} = 0.7A$$

and the power consumption would be:

$$P = U_{rms}I_{rms} = \frac{35V \cdot 0.7A}{2} = 12.25W$$

Nearly all of it would go in the cable and the source resistor. We are mainly interested in the power consumption in the crystal and the tank circuit.

1.2.1 Power Dissipation

The power dissipation in the crystal is

$$P = U_{rms}I_{rms}\sin\phi$$

where ϕ is the total loss angle (0.0004 for RTA and even less for RTP):

$$P_C = U_{C,rms}I_{C,rms} \cdot 0.0004 = 12.25W \cdot 0.0004 = 5mW$$

The power dissipation in the inductor and the leads is

$$P = 12.25W \cdot \frac{R_L}{\omega L} = 5.4mW \qquad L = \frac{1}{4\pi^2 \cdot f^2 \cdot C} = .2\mu H$$

1.2.2 Laser Power Dissipation

The total laser power is supposed to be 200W. The absorption coefficient of RTA and RTP is small but we don’t have very reliable informations. I found one day something in the order of 50ppm/cm for RTP. The total absorbed power is then:

$$P_a = 200W \cdot 5 \cdot 10^{-5}\frac{1}{cm} \cdot 1.5cm = 15mW$$

References

[1] Advanced LIGO Systems Design LIGO-T010075-00-D, LSC ed. Peter Fritschel

[2] *Sideband Requirements for Advanced LIGO, Part I* (LIGO-T020021-00-D) Guido Mueller