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Laser Interferometer Gravitational Wave Observatory (LIGO) Project

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Subject: **Power-Induced Angular Shifts and Instabilities**

Summary:

The angular shifts induced by the radiation pressure of the power build-up in the arm cavities can be significant. Assuming an average de-centering of the beam as good as it currently is (~1 cm on each mirror) and assuming arm powers of 10 kW the alignment shift during locking can be large enough to push the interferometer into a state where about half the arm power is lost. It is unlikely that the system stays locked much beyond the half power point. Predictably, locking will become a challenge due to the different alignment states when in and out of lock. In particular, the initial alignment determined by locking single cavities and the Michelson can be quite far off the final alignment with “loaded” cavities.

Dynamic instabilities don’t seem to be a problem at the current power levels as long as WFS 1 is engaged.

Recommendations:

Implement a dial to control the laser power incident to the interferometer/mode cleaner. The required range is about a factor of 10 down from the maximum power, i.e., 0.6 W–6 W at the recycling mirror. Start commissioning the ASC centering system.

Expected Angular Shifts:

We can write the torque introduce by a laser beam of power, *P*, that hits a mirror at a distance, *d*, away from the center as:

$$T = \frac{2Pd}{c} \tag{1}$$

The restoring torque of a torsion pendulum can be written as:

$$T = \alpha\Theta\omega^2 \tag{2}$$

where α is the angle, Θ is the angular moment along the vertical axis and ω is the angular frequency in yaw. Eliminating the torque from equations (1) and (2) and solving for the angle gives:

$$\alpha = \frac{2Pd}{c\Theta\omega^2} \tag{3}$$

For a cylinder the angular moment along the symmetry axis, z, and along the orthogonal axes, x and y, are:

$$\begin{aligned} \Theta_{zz} &= \frac{1}{2}MR^2 \\ \Theta_{xx} = \Theta_{yy} &= \frac{1}{12}Mh^2 + \frac{1}{4}MR^2 \end{aligned} \tag{4}$$

For LIGO we are using the following parameters:

Table 1: Interferometer parameters

Parameter	Description	Value	Unit
M	mirror mass (LOS/SOS)	10 / 0.25	kg
R	mirror radius (LOS/SOS)	0.125 / 0.0375	m
h	mirror thickness (LOS/SOS)	0.1 / 0.025	m
ω	yaw angular frequency (LOS/SOS)	$2\pi 0.5$ / $2\pi 0.85$	rad/s
P	laser power in the arm cavities	10	kW
P_{MC}	laser power in the mode cleaner	4	kW

Figure 1 plots the angular shift as function of the de-centering, d.

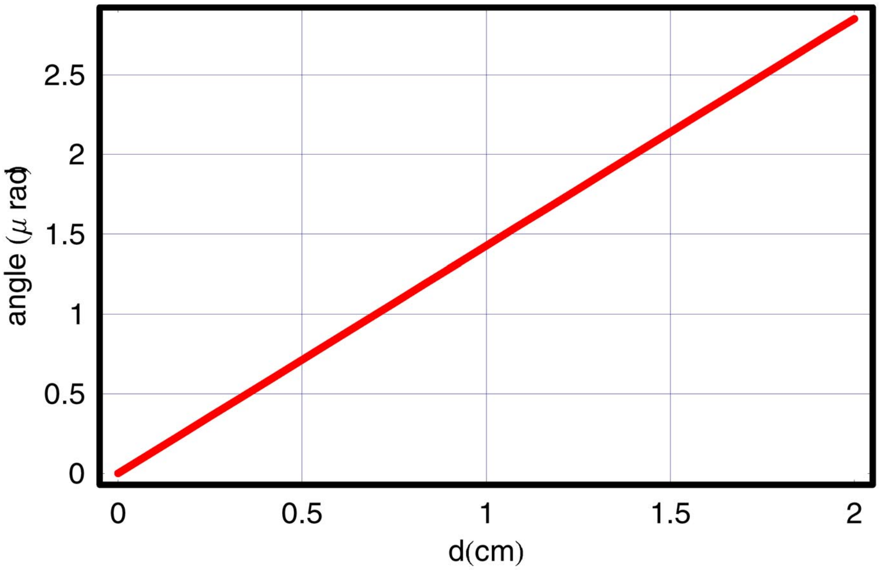

Figure 1: Angular shift as function of de-centering (large optics with 10kW laser beam).

Measured Angular Shifts:

Measured angular shifts for the LHO 4K and 2K interferometers are listed in Table 2 (LHO elogs from 2/20/03 and 2/17/03, respectively).

Table 2: Measured angular shifts for LHO 4K with 0.5 W input power and the LHO 2K with about 3.7 W input power.

Mirror	**Angular shift**		**Unit**	**De-centering**		**Unit**
	4K	**2K**		**4K**	**2K**	
ETMX pitch	–	0.6	µ rad	–	0.9	cm
ETMX yaw	0.25	0.4	µ rad	1.3	0.6	cm
ETMY pitch	–	0.8	µ rad	–	1.1	cm
ETMY yaw	–	0.5	µ rad	–	0.7	cm
ITMX pitch	0.11	0.1	µ rad	0.6	0.1	cm
ITMX yaw	0.32	0.9	µ rad	1.7	1.3	cm
ITMY pitch	0.17	0.5	µ rad	0.9	0.7	cm
ITMY yaw	0.11	1.3	µ rad	0.6	1.9	cm

The power at the recycling mirror is $P_{ifo} = T_{MC}T_{io}P_{in}$, where $T_{MC}T_{io}$ is the optical efficiency through the mode cleaner and the remaining input optics. This factor was determined to be 0.59 (see LHO elog from 12/31/03). With an input power of 0.85 W, the power at the interferometer becomes 0.5 W. With a recycling gain of 40 and an arm power build-up of 130, the power in each arm cavity is roughly 1.3kW. Using equation (3) together with the measured angular shifts we can solve for the de-centering (see column 5 and 6 of Table 2). The same calculation can be done for the LHO 2K: Assuming an optical efficiency of 0.82 (see elog from 7/27/01), a recycling gain of 20 and an input power of 4.5 W we get 3.7 W incident to the interferometer and 4.8 kW in each arm cavity.

Power Drop:

Recalling the results fromT960191, *"Modal Model Update 5 – Large Angle Regime"*: For a differential misalignment of the arm cavities (M4) the angle that reduces the arm powers by 2 is 0.6 µrad and for all other angular degrees-of-freedom it is around 5 µrad. If we take the measured 4K values from Table 2 and scale them by 12 to account for the nominal input power of 6 W, we get angular shifts between 1 µrad and 4 µrad per mirror degree-of-freedom. This is far too much for the differential arm cavity misalignment and it would be unlikely that the interferometer could stay locked without wavefront sensor 1 engaged. If we add the 4K angles of Table 2 in quadrature and again scale for 6 W input power, we get a total angle of 5.5 µrad. Even with WFS 1 engaged this would lead to a power drop in the arm cavities by about half. The 2K numbers are very similar taking into account that its recycling gain is only about half the one of the 4K.

Instabilities:

A misalignment of a mirror will change the power in the cavity. This in turn will change the radiation pressure and, if the laser beam hits the mirror off-center, it will change the torque applied to the mirror. This torque will either add to or subtract from the restoring torque of the pendulum depending on the direction of the misalignment. In the later case the cavity becomes unstable, if the change in torque induced by the radiation pressure exceeds the change in the restoring torque of the pendulum. We write the condition for instability as:

$$2\frac{d}{c}\frac{d}{d\alpha}P(\alpha) > \omega^2\Theta \qquad (5)$$

If we make the assumption that the power $P(\alpha)$ as function of angle α can be describe as the average of a gaussian and a lorentzian and if we look at the steepest gradient only, we get:

$$\frac{1.5dP_0}{c\beta} > \omega^2\Theta \qquad (6)$$

where β is the angle where the power drops by half and the factor of 1.5 comes from the average between the steepest slope of a gaussian and a lorentzian. To be precise this instability condition holds for the angle with the steepest slope in cavity power. For a perfectly aligned system a small change in alignment will not change the cavity power to first order and is therefore stable. Using LIGO I numbers we can rewrite the above equation into a more convenient form:

$$d[\mathrm{cm}] \times \frac{P_0[\mathrm{kW}]}{10} > 4.5\frac{\beta}{5\times10^{-6}} \times \frac{\omega^2\Theta[\mathrm{Nm}]}{0.468} \qquad (7)$$

The above formula indicates that the present LHO 2K interferometer ($P = 4.7$ kW) has unstable alignment states when WFS 1 is not engaged (i.e., $\beta = 0.6\times10^{-6}$ rad), but otherwise all interferometers are expected to be stable at full power.

Mode Cleaner:

For the mode cleaner the power build-up is about 500. With a input power of 8 W we get a stored power of 4 kW. The angular shift as function of de-centering is then about 10 µrad/mm (divergence angle is 200 µrad).

For the mode cleaner we obtain the following instability condition:

$$d[\mathrm{mm}] \times \frac{P_{MC}[\mathrm{kW}]}{4} > 30\frac{\beta}{200\times10^{-6}} \times \frac{\omega^2\Theta[\mathrm{Nm}]}{0.0028} \qquad (8)$$

and hence the LIGO I mode cleaners are stable for all powers and reasonable de-centering values.

Longitudinal Effect (Reference):

The longitudinal displacement of a mirror can be written as:

$$x = \frac{2P}{c\omega^2M} \qquad (9)$$

where ω denotes the angular frequency of the pendulum and M is the mass of the mirror. For the interferometer large optics we get 31 nm/kW and for the mode cleaner small optics we get 0.68 µm/kW. At full power the mode cleaner length control has to compensate for roughly 5 µm of motion introduced by the radiation pressure.