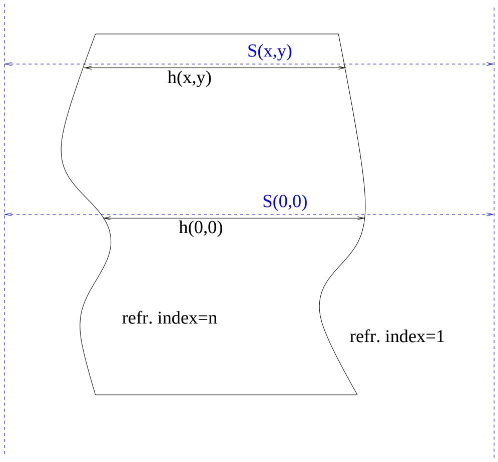
LIGO-T030047-00-D

Thermal lensing
Pasadena, March 2003

Erika D'Ambrosio
Ligo Project, Caltech

My personal interpretation of thermal lensing



I will start by defining the optical path-difference upon a transversal plane

$$S(x,y) - S(0,0) = nh(x,y) - nh(0,0) - (h(x,y) - h(0,0))$$

as I am not interested in any common phase, since it can be compensated for by a longitudinal adjustment of the position of the mirror. For the same reason I am not interested in any common variation of the optical path-difference. When the mirror is heated the temperature distribution is no longer uniform and it affects the optical path-difference as the index of refraction changes inside the bulk and the medium does also expands. The resulting change in the optical path-difference is evaluated by subtracting any common phase shift since I am not interested in it:

$$\begin{aligned}
\Delta\Phi(x,y) &= \Delta S(x,y) - \Delta S(0,0) \\
&= \int_0^{h(x,y)} \frac{\mathrm{d}n}{\mathrm{d}T}\Delta T(x,y,z)\mathrm{d}z - \int_0^{h(0,0)} \frac{\mathrm{d}n}{\mathrm{d}T}\Delta T(0,0,z)\mathrm{d}z \\
&+ \alpha(n-1)\int_0^{h(x,y)} \Delta T(x,y,z)\mathrm{d}z - \alpha(n-1)\int_0^{h(0,0)} \Delta T(0,0,z)\mathrm{d}z
\end{aligned}$$

where I can define

$$\Delta S(x,y) = \left[\frac{\mathrm{d}n}{\mathrm{d}T} + \alpha(n-1)\right]\int_0^{h(x,y)} \Delta T(x,y,z)\mathrm{d}z$$

for the variation in the optical path-difference. The above formula takes into account the change in the wavefront of the electromagnetic field when it goes through the mirror. When it is reflected from the mirror the thermal expansion causes a similar variation

$$\Delta\Phi_R(x,y) = \Delta S_R(x,y) - \Delta S_R(0,0)$$

with the following definition

$$\Delta S_R(0,0) = -2\alpha \int_0^{h(x,y)} \Delta T(x,y,z)\mathrm{d}z$$

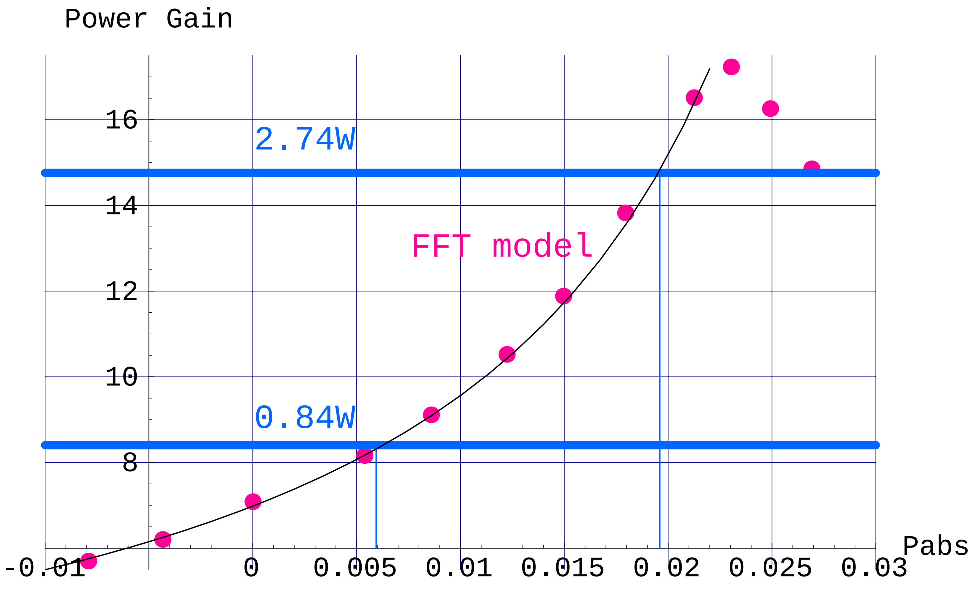
where any common phase is neglected. Once we know $T(x,y,z)$ the integrals can be evaluated and the accumulated distortion computed. When the diameter of the mirrors is comparable to their thickness and the radii of curvature are large $h(x,y) \simeq h(0,0)$. Moreover $T(x,y,z)$ depends weakly on z when the spot size of the beam is small compared to the diameter of the mirror. In this case the distribution $T(x,y,z)$ can be approximated by a very simple formula that is equivalent to a variation in the radius of curvature of the beam wavefront

$$\Delta T(x,y) - \Delta T(0,0) \simeq -\frac{P_{abs}}{2\pi K_{cond}} \frac{x^2+y^2}{w^2} \tag{1}$$

with K_{cond} the thermal conductivity and w the spot side of the beam. In the special circumstances where the above approximation is valid, the physical result is a thermal focusing that makes the wavefront flatter as we can expect because of the minus sign in the formula above. Therefore the recycling mirror is flatter than it should be, if there were no absorbed power in the two input mirrors of the Fabry-Perot cavities. As the power that is planned to be stored in the Fabry-Perot cavities for Advanced Ligo will reach $\sim 1MW$, scientists are developping more sophisticated techniques to understand and deal with the distortions due to so large power loads. Since I am interested in this problem as well, I started from reading several papers and considering the fundamental issues. My conclusions are conceptually different from the model that is being used for the evaluation of thermal lensing, since I predict there is no effect due to thermal expansion if the medium has index of refraction $n = 1$. In other words the expansion of a medium, having same index of refraction as vacuum has no impact on the wavefront of a traversing electromagnetic field. I developped this model in January and used it in my numerical simulations for Ligo I, with substrates made out of fused silica. Since the contribution due to α is small compared to the term $\sim \frac{dn}{dT}$ the disagreement is conceptual.

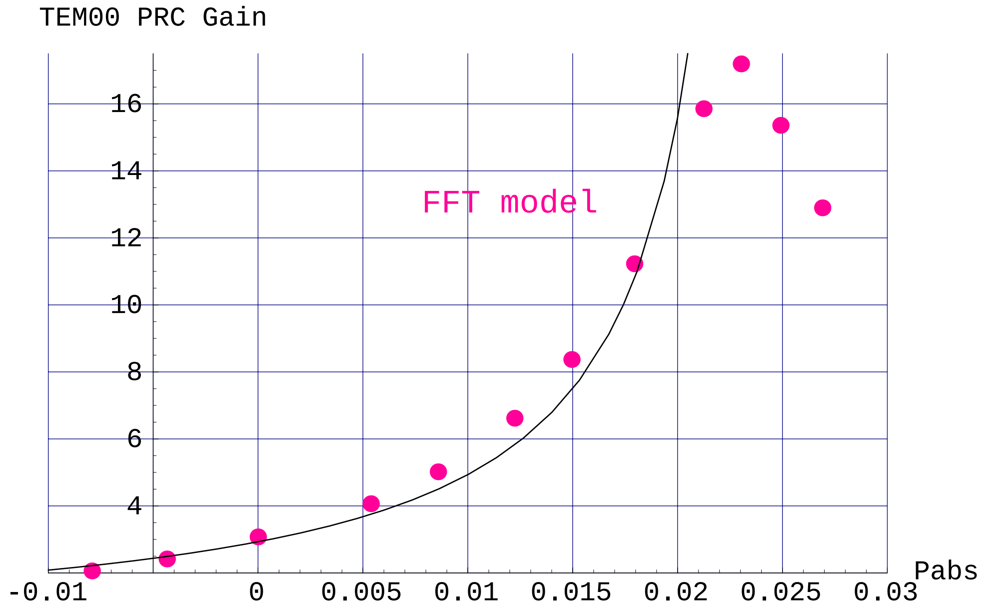
Simulations for H1

An interesting investigation has been carried out by William Kells on January 18^{th} and Peter Fritschel on Januray 21^{st} on thermal lensing. After a stretch of full lock, the sideband power in the recycling cavity decays from a maximum value corresponding to the thermal lensing when the carrier resonates in the two Fabry-Perot, to a minimum value that corresponds to the cold system with no absorbed power.



I compared the data reported by Bill and Peter with numerical simulations. The sideband power corresponding to the cold interferometer is scaled, in order to make it the same value predicted by the FFT-code for $P_{abs} = 0$. The sideband power corresponding to the hot interferometer has been scaled accordingly, for the two input powers.

Although the values for P_{abs} we read from the intercepts are in the same ratio as the input power, if this comparison were reliable it would entail the matched configuration would occur at an input power that is about half of its designed value. We cannot infer anything from this yet but the power gain that the FFT-model predicts for the sidebands when the ETM are misaligned (they are totally transmittive) seems to be a factor of 2 higher than what is observed by the experimental measurements.



The amount of sideband power contributed by the fundamental mode decreases as the mismatch increases; the beam propagated out of the long arm cavity through the internal test mass mirror has a curvature that depends on the temperature distribution of that medium [1].
For the geometrical characteristics of the mirrors used in Ligo, the variation of the optical path length due to the changes in the bulk and the distortions of the coated surface is numerically very similar, after having divided by the amount of power absorbed [2].

For $P_{abs} = 0$ which corresponds to the cold interferometer, not even half of the total power is due to the TEM_{00} contribution. When $P_{abs} = 23mW$ the TEM_{00} component accounts for the whole electromagnetic field. The excited modes in the latter case contribute $\sim 0.2\%$ of the total power. If the two ITM mirrors were identical the field would consist entirely of the TEM_{00}.

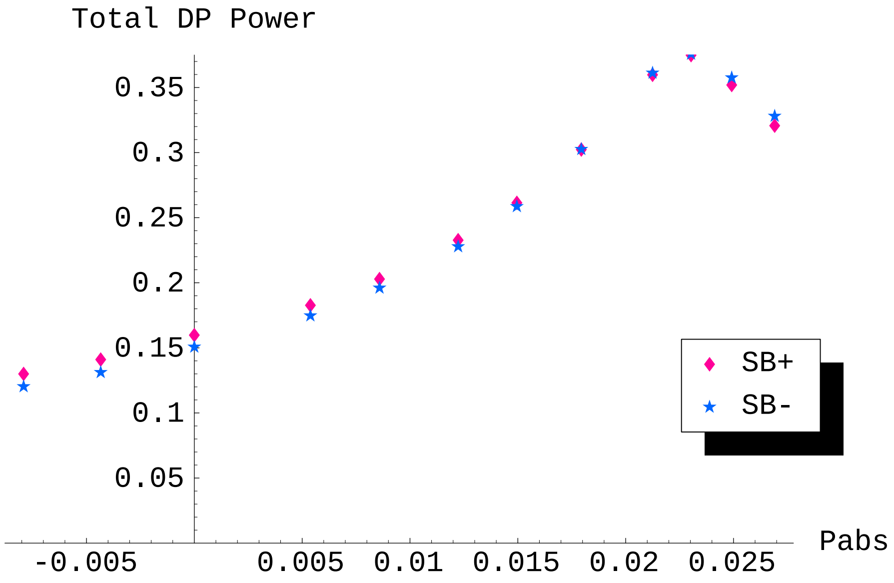
The absorbed power P_{abs} is the sum of the power absorbed in the bulk and in the coated layer.
I reported the average power of the two sidebands. Because of the asymmetry of the interferometer the sidebands have a different composition and gain.

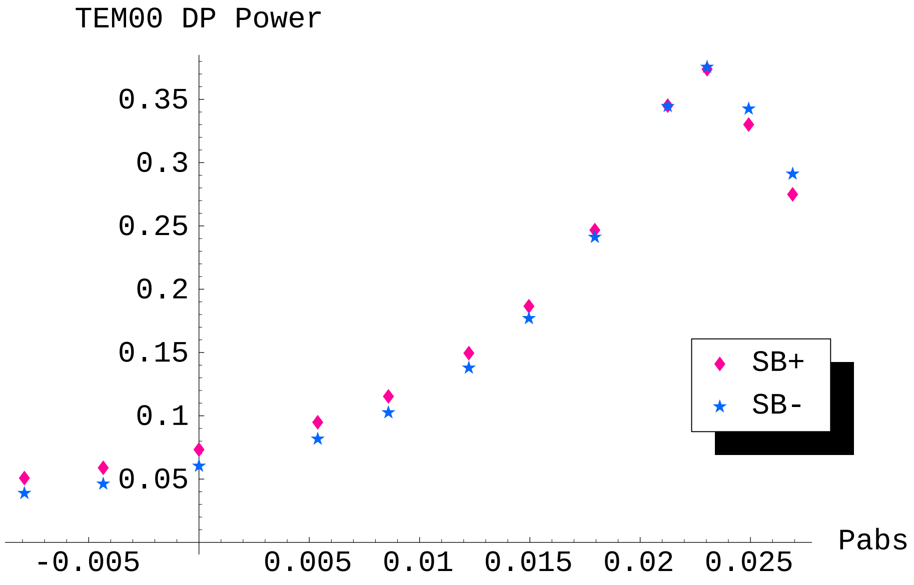
---

[1] Winkler et al. *Phys. Rev. A* 44 (1991) 7022-7036

[2] P.Hello and J.-Y.Vinet *J. Phys. France* 51 (1990) 1267-1282

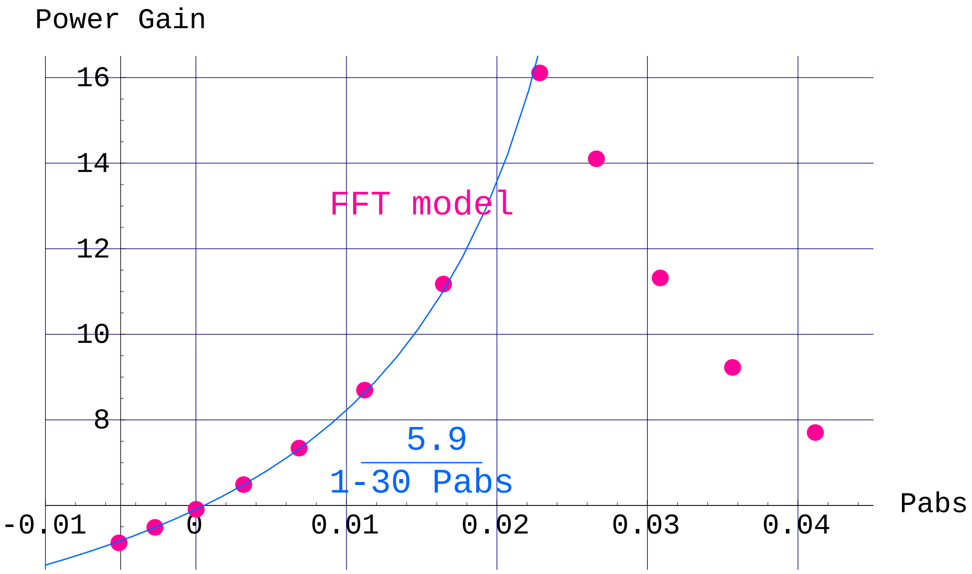
At the dark port, the power carried by the TEM_{00} mode differs for $\sim 20\%$ for the two sidebands when the interferometer is cold that is $P_{abs} = 0$. For the optimal P_{abs} the difference in the sideband power is reduced to $< 0.5\%$.





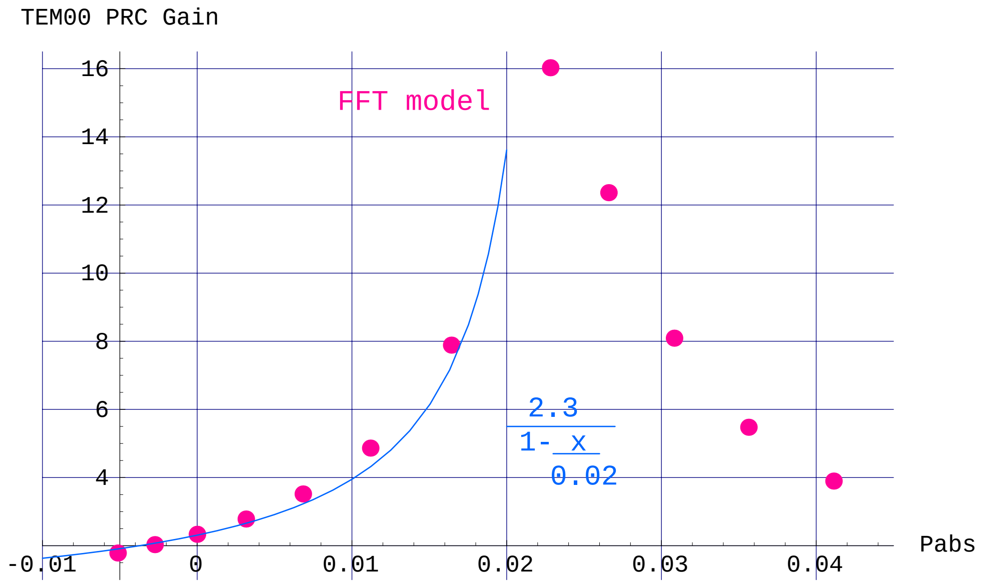
Simulations for L1

The resonant curve of the sideband power in the recycling cavity, when the two Fabry-Perot cavities are not locked, is qualitatively very similar for the two interferometer. The source of P_{abs} can be either the carrier power if the arms were in lock immediately before, or an external device that is introduced to stabilize the recycling cavity by thermal heating. Since the sideband power that is currently observed suffers from fluctuations, that are of the order of a factor of three in the last few months, an absolute comparison with the real data cannot be done.



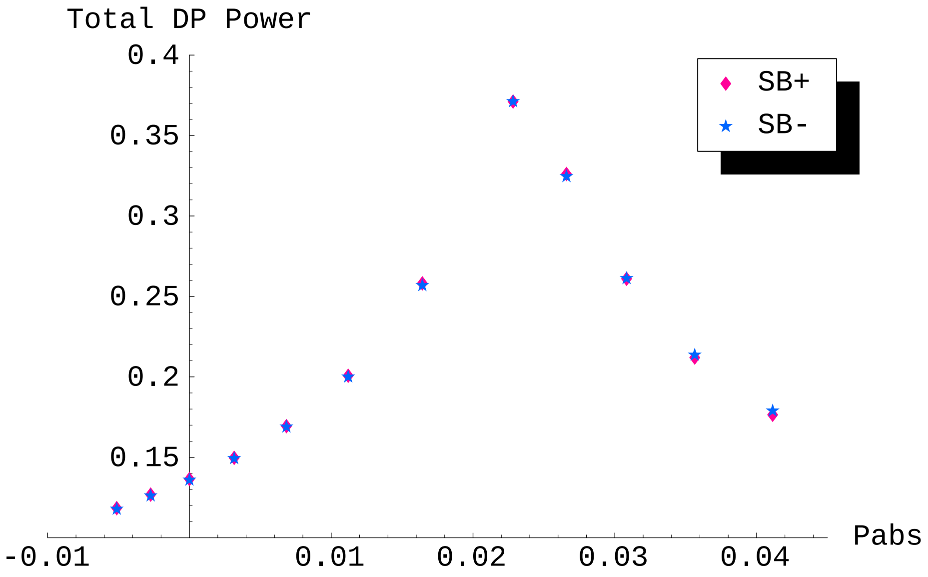
At the peak of the resonance the power is entirely carried by the fundamental mode, while in the cold configuration more than half of the total power is due to higher order modes.

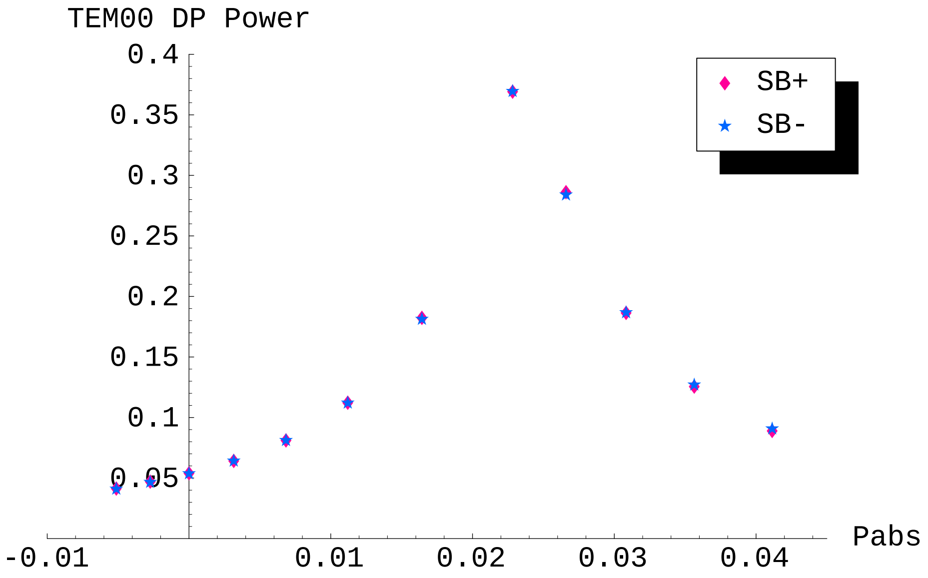
The behaviour of the two interferometers is the same in this regard. The light that is diffracted out of the fundamental mode is only partially lost, because of the quasi-degeneracy of the recycling cavity. As a consequence the shape of the circulating field is largely modified. The average of the power of the two sidebands has been plotted for both the interferometers. There is a difference regarding the geometrical symmetry that is L1 is more balanced. What we expect is that the sidebands would interact in a more consistent way with L1 and the numerical results show this.



Even in the cold configuration, when the electromagnetic field is perturbed by a large mismatch of its wavefront with the recycling mirror surface, the power associated with the two sidebands is about the same and the amount due to the fundamental mode is also the same. Although these conditions make L1 closer to ideal, no clean evidence of thermal lensing has ever been observed that I know while for H1 the drop of the sideband power, after the lock of the two arms is broken, has been correlated with the cooling of the internal test mass mirrors and considered as a proof of the physical effect. What makes real measurements difficult to interpret is that the normalization of the sideband power is not clearly understood yet. In H1 there is likely a correlation with the alignment of the beamsplitter that is currently under investigation. Other problems seem to affect L1 as the sideband power doesn't increase when the arm cavities get locked. Moreover both the interferometers are working with an input power that is far less than the design value and the percentage of that which enters the interferometer is somehow uncertain.

More sophisticated tools are needed in order to precisely estimate the effect of thermal lensing and real measurements must be done to determine the actual amount of power that is being absorbed in the bulk and on the coating surface.


Total DP Power
0.4
0.35
0.3
0.25
0.2
0.15
-0.01
0.01
0.02
0.03
0.04
Pabs
♦ SB+
★ SB-


TEM00 DP Power
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
-0.01
0.01
0.02
0.03
0.04
Pabs
♦ SB+
★ SB-