## LIGO Laboratory / LIGO Scientific Collaboration

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## Cantilever blade analysis for Advanced LIGO

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#### Abstract

: This report provides a historical background to the cantilever blade designs that we are currently using and summarises the results obtained from a major experimental programme carried out in the summer of 2002 at CALTECH.

\section*{Introduction:}

The cantilever spring blades are used to enhance the vertical isolation of the suspension system and their design has been adapted from designs used in the VIRGO project. The springs have a nearly trapezoidal geometry (see below) and are constructed from maraging (precipitation hardened) steel. This steel undergoes a heat treatment process in order to achieve its maximum strength.


TOP VIEW (when under load-flat profile)


## SIDE VIEW (when unloaded)



Now for a trapezoidal blade the maximal deflection at the free end is given by

$$
\begin{equation*}
\lambda=\alpha \frac{P l^{3}}{3 E I} \tag{1}
\end{equation*}
$$

where $P$ is the load supported in Newtons, $l$ is the length of the blade, $E$ is the Young's Modulus (for marval 18 steel (an $18 \% \mathrm{Ni}$ steel), with a standard heat treatment of 480 degrees C for 4 hours, most references quote a value of $186 \times 10^{9}$ Pa ), $I$ is the moment of inertia, $\alpha$ is a factor related to the ratio between the width at the tip to the width at the base (it is expected to take a value between 1.0 and 1.5)

The moment of inertia for a typical blade is given by

$$
\begin{equation*}
I=\frac{a h^{3}}{12} \tag{2}
\end{equation*}
$$

(rectangular section-axis of moments through centre)
where $a$ is the width of the blade base (at the clamp) and $h$ is the blade thickness
After substitution we obtain an expression for the maximal deflection

$$
\begin{equation*}
\lambda=4 \frac{m_{t} g l^{3} \alpha}{E a h^{3}} \tag{3}
\end{equation*}
$$

where $m_{t}$ is the total mass supported per spring

Now the spring constant of the blade is given by

$$
\begin{equation*}
k=\frac{E a h^{3}}{4 l^{3} \alpha} \tag{4}
\end{equation*}
$$

Selecting the thickness, length and width for the blade we obtain an uncoupled ${ }^{1}$ vertical uncoupled vertical frequency given by

$$
\begin{equation*}
f=\sqrt{\frac{E a h^{3}}{16 \pi^{2} m l^{3} \alpha}} \tag{5}
\end{equation*}
$$

where $m$ is the mass supported by the spring in that stage

[^0]
## Calculation of bending stresses:

The maximum stress at the support point of a cantilever blade (clamped at one end) is given by the following expression

$$
\begin{equation*}
\sigma_{M A X}=\frac{6 P l}{a h^{2}} \tag{6}
\end{equation*}
$$

The elastic limit for the grade of Marval 18 steel used is 1600 MPa , assuming a standard heat treatment. For our blade designs, thus far, we have aimed to achieve a maximum stress of around $50 \%$ of this elastic limit.

Another important consideration is the fact that the cantilever blade is not a mass less spring. This has two consequences; firstly flexural (internal) modes are observed and secondly the transfer function begins to flatten off at approximately $1 / 3$ that of the first internal mode. The internal mode frequency should ideally be as high as possible in order that the mechanical filtering action from the pendulum suspension provides sufficient attenuation at this frequency. An approximate model gives the value for this internal mode extrapolated from earlier blade designs, whose internal modes have been measured. The internal mode frequency is proportional to the thickness of the blade and is inversely proportional to the square of its length.

The deflection is strongly dependent on the blade thickness (see eqn. 3). A sheet of blade material can vary in thickness (by up to $3 \%$ ) and this would help to explain an observed mismatch in stiffness (and hence deflection) between blades cut from the same sheet of material. The supported mass can also be a variable since the mass of clamps etc. has not always been considered and this tends to increase the supported load and thus the deflection of the blade.

## Shape factor:

The Young's Modulus is not expected to vary significantly between different batches of the same maraging steel grade so the one remaining parameter is the so called shape factor

For a trapezoidal shape the shape factor predicted from simple theory is given $\mathrm{by}^{2}$ :

$$
\begin{equation*}
\alpha=\frac{3}{2(1-\beta)}\left(3-\frac{2}{1-\beta}\left(1+\frac{\beta^{2} \log \beta}{1-\beta}\right)\right) \tag{7}
\end{equation*}
$$

where $\beta$ is the ratio between the short end and wide end of the blade.
From the above expression the shape factor $\alpha$ should take a value between 1.0 as the shape approximates to a rectangle and 1.5 as the shape approximates to a triangle.

[^1]Since the blade geometry is closer to a triangular shape than a trapezoidal one we have historically used a correspondingly larger value, for the shape factor, than that calculated from the simple ratio between the short and wide end of the blade. We typically used shape factors in the region of 1.42-1.46. The blades have different shapes and so this parameter should vary a little. However from the results that were obtained, and assuming that the other parameters remained constant, it appeared that the actual value for the shape factor was higher still. This even applied when a purely trapezoidal shape was used.

In designing the blades for our new Glasgow JIF facility we load tested blades, as used in a GEO 600 triple suspension, and worked backwards to calculate what shape factor would provide the measured deflection. We then used this value for the JIF blade design and, when we subsequently tested these blades, found that the measured deflection matched our predictions to within $2 \%$.

## CALTECH results and analysis:

Clearly a more thorough investigation of all the parameters was required and this work was completed at CALTECH. Sets of blades, for the upper two stages of a modecleaner triple pendulum suspension controls prototype, supplied by two different companies were tested. The shape factors used were the same as those that were extrapolated from the GEO 600 blade measurements (the suspensions are quite similar). 8 upper blades and 16 lower blades from each company were tested (see appendix for designs) A tolerance of $+/-0.0005$ inch ( $+/-0.013 \mathrm{~mm}$ ) on the blade thickness was requested. One company (Superior Jig) used an EDM (electro discharge machining) process to achieve the required blade profile and another (Lobart) used a grinding/lapping/bending process.

The thickness at several points along the length of each blade was measured, with a rounded-tip micrometer, and an average taken. The deflection was measured with the use of a height gauge mounted on a reference plate. The results of this investigation are summarised below:

|  | Average measured <br> thickness $(\mathrm{mm})$ | Average measured unloaded <br> deflection $(\mathrm{mm})$ |
| :--- | :--- | :--- |
| Superior <br> Jig: <br> (upper) | $1.520+/-0.023$ | $139.6+/-0.9$ |
| Superior <br> Jig: <br> (lower) | $1.019+/-0.011$ | $44.2+/-1.7$ |
| Lobart <br> (upper) | $1.509+/-0.008$ | $135.4+/-1.1$ |
| Lobart <br> (lower) | $1.008+/-0.003$ | $42.7+/-0.7$ |

In terms of matching the required thickness and obtaining a small variance on this thickness the lapped/ blades were better.

The other important criteria are how close the two companies came to the desired radius of curvature and maximal unloaded deflection, and how much variation there was in this deflection. A maximal deflection of 140.0 mm was requested for the upper blades and 45.0 mm for the lower blades. From the results we can observe that the EDM process more closely matched our specification but, for the lower blades, showed greater variation in this deflection. By applying the specified load and measuring the deflection for all the blades one could observe that the lower blades made by the EDM process showed a variation of $+/-1.7 \mathrm{~mm}$ in deflection.

Finally by applying the specified load on the blades and measuring the loaded deflection it is possible, knowing the other parameters, to obtain an extrapolated shape factor for each of the blade types.

The shape factors for the upper and lower blades (averaged data) are as follows:

| Superior Jig: <br> $(\mathrm{EDM})$ | upper blade $=1.35$ <br> lower blade $=1.56$ |
| :--- | :--- |
| Lobart: | upper blade $=1.27$ |
| (lapped/bent) | lower blade $=1.52$ |

The shape factors used in the model when originally designing these blades were:
upper blade $=1.30$
lower blade $=1.55$
Overall, from this extrapolation, it would appear that the Superior Jig blades are slightly softer (higher shape factors are required to fit the model) than the Lobart blades. This is presumably related to differences in the material specification and is therefore an additional variable.

## Conclusions:

An important finding from this analysis is that the shape factors used in the current model fit reasonably closely with the extrapolated values, at least for these two designs. Of course, in practice, the blades were not flat in the prototype suspension because the thickness and maximal deflection were not exactly to specification and most importantly the actual loading used was different to that modelled. Angled clamps were used to correct for this.

Another finding is that the lapped/bent blades were closer in thickness to the specification and had a finer tolerance. However the blades from this manufacturer were not bent close to the required specification. If this process were favoured then I would recommend stating a finer tolerance on the bending. From experience gathered from the U.K. company (Accrofab), some small amount of re-shaping (to an accurate template) after the heat treatment process might be necessary. This is because a slight relaxation of the curvature has been observed with manufactured blades.

When ordering sets of blades it is advisable to order several spares in order that pairs of blades, of each type, can be matched. This is especially important in ensuring that mechanical imbalances are minimised in the final suspension.

It is also suggested that the same vendor and preferably the same sheet/block of material is used for machining all subsequent blades, for each design, to ensure uniformity of the material properties.

Addendum (August 2003):
It was decided for completeness that an accurate measure of the Young's Modulus was required for the blade material. The Young's Modulus of several samples of 1 mm thick Marval 18 blade material (that was used for GEO 600 and JIF blades and similar to the material used for the CALTECH blade prototypes) was measured, after a standard heat treatment of 480 deg. C for 4 hours, by a Sheffield testing lab. The average value was found to be equal to $176+/-2 \mathrm{GPa}$ that is somewhat lower than was previously assumed. If this were typical the shape factor would then need to be scaled down, if this value for the Young's Modulus were to be used in the calculations. It would also mean that the shape factor, for a trapezoidal shape, does indeed correlate reasonably closely with the value predicted from simple theory. Of course this does not affect the overall conclusions given by the above analysis because both the Young's Modulus and the shape factor scale linearly.

## Appendix:

MC controls prototype- upper blade schematics



MC controls prototype- lower blade schematics

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[^0]:    ${ }^{1}$ The frequency observed for a spring in a particular stage supporting only the mass of that stage.

[^1]:    ${ }^{2}$ Super attenuator vertical performance beyond the low frequency range, G. Cella, A. Vicere, VIRGO 1390-91 (1997)

