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Upper limits for externally triggered searches using signal injections

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1 Statement of problem

Let the waveform used for injection be $h(t;\psi)$ where ψ is a parameter of the waveform on which we want an upper limit (e.g., ψ could be $h_{\rm rms}$ or peak amplitude).

I assume we follow the steps below in the signal injection study.

1. Fix a value for ψ and generate the signal $h(t;\psi)$.
2. Inject it into data from each IFO. Repeat N times with different data segments but same value of ψ.
3. Follow the same analysis procedure with each segment to obtain a single number $z =< x, y >$. The procedure includes data conditioning, using off-source data for normalization etc. Let the number from the $i^{\rm th}$ segment be z_i, $i = 1, \ldots, N$.

For a given set of z_i, let the cumulative distribution function [3] constructed from the set be $\widehat{F}(z;\psi)$. The cumulative distribution function is labelled by the parameter ψ used for injection and the $\widehat{}$ marks it as a quantity obtained from a sample as opposed to the true underlying population distribution function $F(z;\psi)$. Let $p = p(\psi)$ be such that $\widehat{F}(p;\psi) = 1 - \alpha$ where α is the confidence value for the desired upper limit (e.g., $\alpha = 0.95$ for a 95% confidence upper limit).

Finally, one takes the segment corresponding to the external trigger and, without any injection, obtains a value z_0 for z.

2 Upper limit

The α confidence upper limit on ψ is the value ψ_α such that $p = p(\psi_\alpha) = z_0$.

The meaning of this upper limit is the following. The upper limit ψ_α is the value of waveform parameter ψ such that if there were a signal present with $\psi \geq \psi_\alpha$, then the probability of seeing a value of $z > z_0$ would be more than α. Thus, if $\alpha = .99$, there is a $\geq 99\%$ chance that we would have obtained a value of z larger than what we actually got ($= z_0$). (The interpretation is actually simpler if one follows the standard definition of interval estimates as a generalization of point estimates [1].)

The procedure described above is strictly an upper limit approach where one is not interested in detection, unlike the Feldman-Cousins prescription [2]. No matter what the value of z_0, one will always get an upper limit. If one had a cumulative distribution function constructed from *off-source* data *without* injections, then the observed value z_0 can be used to decide between detection and non-detection by calculating the significance of obtaining z_0 from the off-source injection-free distribution. In such a case, one might want to set the injection based upper limit only when there is a detection.

Instead of scanning the ψ space, one could use a bisection type approach to make the search for ψ_α faster. This would also help is for some reason we do not want to use the same data segments for different values of ψ but the total amount of useable data is limited.

3 Statistical issues

Injections are a way of estimating the true cumulative distribution function $F(z;\psi)$ and are essential if there is no analytic procedure for obtaining $F(z;\psi)$ or we do not know what functional form to use for $F(z;\psi)$. However, as with all estimators, the estimate $\widehat{F}(z;\psi)$ will suffer from both random and systematic errors. Random errors will come from the finite sample size N. Systematic errors could come from the fact that the data used for injections was not stationary so that $\widehat{F}(z;\psi)$ does not really pertain to any one specific segment. There is probably a trade off between the desire to reduce random errors by increasing sample size and reduction in systematic errors by only using data that is "close" in distribution to the on-source data. Besides, because of finite sample size, the value of $p(\psi_0)$ cannot be pinned down with arbitrary accuracy. There are ways around the last problem that can be tried if needed.

References

[1] "Note on frequentist confidence intervals for a continuous wave GW source", Soumya D. Mohanty, internal working note of LSC-PULG (Oct 10,2002).

[2] G. J. Feldman, R. D.Cousins, arXiv:physics/9711021 (*Also* Phys. Rev. D (1998)).

[3] For a random variable X, the cumulative distribution function $F_X(x)$ is defined as $F_X(x) = \text{Prob}(X \le x)$ and $\widehat{F}_X(x)$ can be obtained by accumulating the histogram of sample values of X.