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Notes on thermal noise, with a bibliography

Eric D. Black

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of the LIGO Project.

California Institute of Technology
LIGO Project - MS 51-33
Pasadena CA 91125
Phone (626) 395-2129
Fax (626) 304-9834
E-mail: info@ligo.caltech.edu

Massachusetts Institute of Technology
LIGO Project - MS 20B-145
Cambridge, MA 01239
Phone (617) 253-4824
Fax (617) 253-7014
E-mail: info@ligo.mit.edu

WWW: http://www.ligo.caltech.edu/

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Contents

Abstract

This document serves as my collection of references on thermal noise, with a sketch of the story of how we have come to understand it so far.

Keywords

thermal noise; substrate; pendulum; coating; bulk; brownian; fluctuation-dissipation

1 Introduction

The sensitivity of interferometric gravitational-wave detectors is expected to be dominated by seismic, thermal, and shot noise in three distinct frequency bands [1, 2]. There is a lot of interesting physics in thermal noise, particularly mirror thermal noise. This document gives a very brief outline of what we know about mirror thermal noise, especially coating noise, and how we came to know it.

What's left out: I have listed here a number of references on pendulum thermal noise, but for the first draft I will not cover the story of that subject in the text. I have also not listed or covered the extensive materials-related work on microscopic damping mechanisms, such as the dislocation-damping study done by Cagnoli [3]. I'll get around to covering both these subjects eventually, either in a later draft of this document or a separate note.

2 Ancient history

Thermal motion in mechanical systems was observed long before it was understood. The botanist Robert Brown is considered the first to report it, having observed irregular motion of pollen grains and bits of dust in warm water under his microscope [4]. Brown apparently thought, at first, that the particles were moving under their own power. In an attempt to test this hypothesis, he did everything he could think of to kill them, including boiling them before placing them under his microscope. The motion persisted, and the mysterious source of the "Brownian motion" went unexplained for decades.

The first real breakthrough came when it was recognized that the motion of the particles was related to the thermal motion of the molecules in the surrounding bath [5, 6]. As is so often the case in modern physics, a precise, quantitative understanding of the phenomenon came along when Einstein took an interest in it [7, 8, 9, 10, 11], at the time apparently unaware of most of the previous quantitative work on the subject. Einstein developed a complete and rigorous theory of Brownian motion based on kinetic theory, made the first truly accurate determination of Avogadro's number N, and convinced, once and for all, even the most recalcitrant of the reality of the atom [12]. (Einstein's papers on Brownian motion have been translated into English and collected into a convenient and accessible Dover book [13].)

2.1 Thermal noise as a fundamental limit to the precision of measurements

Early in the last century, Johnson and Nyquist identified voltage fluctuations across a resistor as being thermal in origin, and therefore the electrical equivalent of Brownian motion [14, 15]. Together, they formulated the theory of what we now call Johnson noise, which says that the RMS voltage across any resistor is, intrinsically,

$$V_{rms} = \sqrt{4k_BTR(\Delta f)},$$

where k_B is Boltzmann's constant, T is the temperature of the resistor (in Kelvins), R is the resistance, and Δf is the bandwidth over which the measurement is made. This is a fundamental relationship that depends only on two physical properties of the system being studied, the resistance R (the amount of loss in the system), and the temperature T. It does not depend on the physical construction of the resistor. A cheap, metal film resistor has the same Johnson noise as an expensive, wire-wound resistor.

Some time later, Callen, Welton, and Greene developed a generalized theory of loss and fluctuation, embodied in their fluctuation-dissipation theorem [16, 17]. This work was somehow related to the enormously important Onsager relations [18, 19], but I don't remember how right now. (Don't forget to look this up and fill it in for the final version.)

With the development of ever more precise measurement techniques as the twentieth century progressed, thermal noise began to be recognized as a limiting factor in the resolution of mechanical instruments, as well as electrical measurements. It was long appreciated that thermal noise would limit the precision of high-sensitivity galvanometers [20] and resonant-mass gravitational-wave detectors [21]. Each of these systems is designed to operate on resonance, and so it was the thermal noise at the systems' mechanical resonant frequencies that was studied. Little attention was paid to the frequency distribution of the thermal noise, except to note that the root-mean-square excitation, integrated over all frequencies, had to correspond to an energy of $k_B T/2$ for a system in thermal equilibrium with its surroundings.

3 The frequency dependence of thermal noise in mechanical systems

The frequency distribution of mechanical thermal noise depends on the frictional losses in the system. Nyquist (?) recognized that as the losses in a torsional pendulum were reduced, the thermal noise would become more and more concentrated around the resonant frequency of the pendulum. The fundamental requirement that the thermal energy of oscillation correspond to $k_B T/2$, integrated over all frequencies, naturally leads to the conclusion that the off-resonance thermal noise spectral density can be reduced if the losses are made small. This fact was appreciated early in the development of interferometric gravitational-wave detectors [2], but the frequency-dependence of the damping was not well understood in the community. They knew about the fluctuation-dissipation theorem; they just didn't know what kind of damping to put into it. Viscous damping, where the frictional force is proportional to frequency (or velocity) was almost universally assumed (see for example [22]).

The study of thermal noise as it relates to interferometric gravitational-wave detection appears to have been established as a field in its own right by Peter Saulson in 1990 [23]. Peter pointed out two things that were to be of major importance:

1. The level and frequency dependence of mechanical thermal noise depends strongly on the frequency dependence of the damping, and
2. viscous damping, i.e. losses proportional to frequency, may not be an appropriate model for the vacuum-isolated suspensions and test masses of an interferometric gravitational-wave detector.

We may express the fluctuation-dissipation theorem this way. If the variable x represents the position of some part of a mechanical system, with $x = 0$ as equilibrium, then the mean-squared fluctuations in x caused by finite temperature are, as a function of angular frequency ω,

$$x^2(\omega) = \frac{4k_B T}{\omega^2} \Re \{ Y(\omega) \} ,$$

where $Y(\omega)$ is the *mechanical admittance*, defined as the ratio of the velocity of the point x divided by the magnitude of the force applied there to produce that velocity.

$$Y(\omega) \equiv \frac{v(\omega)}{f}$$

The real part of the admittance describes the loss in the system. For a generalized simple harmonic oscillator, we may collect both the restoring force and the damping term together into a complex effective spring constant.

$$k_{eff}(\omega) = k\left(1 + i\phi(\omega)\right),$$

where the damping term $\phi(\omega)$ is known as the *loss angle*. The mechanical admittance of this system is then

$$Y(\omega) = \frac{i\omega}{\left[-m\omega^2 + k\left(1 + i\phi(\omega)\right)\right]}.$$

The mean-squared thermal noise in a simple harmonic oscillator is just $4k_BT/\omega^2$ times the real part of this, or

$$x^2(\omega) = \frac{4k_BT}{\omega}\frac{k\phi(\omega)}{\left[\left(k - m\omega^2\right)^2 + k^2\phi(\omega)^2\right]}.$$

For the familiar case of viscous damping, covered in most freshman-level textbooks, with a frictional force proportional to velocity ($f_{friction} = -\gamma v$), the loss angle is proportional to frequency.

$$\phi(\omega)_{viscous} = \frac{\gamma}{k}\omega$$

Peter pointed out that the loss angle could have other frequency dependences, including

$$\phi(\omega)_{structural} = \text{constant}$$

for losses dominated by internal friction or *structural damping*, and

$$\phi(\omega)_{thermoelastic} = \Delta\frac{\omega\tau}{1 + \omega^2\tau^2}$$

for thermoelastic damping. (The parameters Δ and τ are material- and geometry-dependent.)

Basically, we may paraphrase Peter's conclusion as this:

> In air, losses in the test masses and suspensions will be viscous and dominated by the properties of the surrounding gas. We may reduce these losses, and hence reduce the off-resonant thermal noise, by reducing the pressure of the surrounding gas. However, we may not reduce the total losses indefinitely. While we may make the losses due to air friction arbitrarily small, at some point these losses will become comparable to intrinsic losses in the materials the test masses and suspensions are made from, and further reduction of the surrounding gas pressure will not further reduce the thermal noise.
>
> The losses due to internal friction are expected to be frequency-independent, which gives rise to a characteristic frequency-dependence of the thermal noise that is different from that caused by viscous damping.

4 Early developments

Following Saulson's seminal work, there was much early activity on pendulum thermal noise [24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50], which I will not cover in detail here. I would like to point out one interesting result, however, that Peter Saulson and his graduate student, Gabriela González, obtained after Peter had won a faculty appointment at Syracuse.

That the loss angle ϕ due to structural damping in most materials is independent of frequency was well established as early as 1927. In an aesthetically beautiful series of measurements on spinning, weight-loaded rods, Kimball and Lovell showed that ϕ was constant over a broad range of frequencies [51]. However, when Saulson made his groundbreaking predictions, no one had yet measured the broad-spectrum Brownian motion in a system with losses dominated by internal friction. This frequency spectrum for Brownian motion in this condition was distinctly different from the case of viscous damping, and so a direct measurement of the broad-spectrum thermal noise should be able to distinguish between the two damping types. The frequency-dependence of the loss angle should tell you what kind of thermal noise the system will exhibit, and *vice versa*. Saulson and González set about performing just such a measurement in a very sensitive, but highly-lossy, system: a torsional pendulum with nylon monofilament (fishing line) used as the suspension fiber [35].

First, they measured the loss angle as a function of frequency in their torsional pendulum. In their system the loss angle wasn't quite independent of frequency, but no matter. If they knew what the frequency dependence was, they could use the fluctuation-dissipation theorem to predict the thermal noise spectrum. They then performed this calculation, and once they had the frequency-dependence of the loss angle and a prediction for the thermal-noise spectrum, they placed their torsional pendulum under vacuum and set about measuring the Brownian motion with the losses dominated by internal friction. Using an optical lever and a shadow sensor, they were able to measure the thermal noise in the torsional mode with remarkable precision, achieving angular displacement sensitivities as low as $3 \times 10^{-9} rad/\sqrt{Hz}$. [1] The results agreed with their theoretical predictions and provided a beautiful confirmation of the fluctuation-dissipation theorem in that system. It remained to be seen if the result would extend to compound systems or low-loss materials. The fishing-line torsional pendulum displayed a loss angle that was never much less than 10^{-2} at any frequency studied, whereas in the mirrors and suspensions for gravitational-wave detectors, loss angles on the order of 10^{-7} were expected.

The torsional pendulum had one other significant aspect, it was essentially a one-dimensional system with a single degree of freedom. The suspensions and test masses for a gravitational-wave detector have many degrees of freedom across all three dimensions, including the (for all practical purposes) infinite degrees of freedom in the continuum modes of the test mass. Now that they understood the fluctuation-dissipation theorem in a simple system, the lossy torsional pendulum, it remained to extend the model to the more complicated and subtle case of a real suspension and mirror system. As I mentioned before, much of the early work that followed Saulson's 1990 paper involved pendulum suspensions, but here I will concentrate on the thermal noise in the mirrors themselves.

[1] Even more impressive, they achieved this resolution at a frequency of $.4Hz$. Their measurement went down to about $2 \times 10^{-3} Hz$, where their signal was as high as $3 \times 10^{-7} Hz$.

5 Early estimates of mirror thermal noise

Saulson made a preliminary estimate of mirror thermal noise in his 1990 paper [23]. He considered the normal modes of a cylindrical test mass as independent, simple harmonic oscillators in one dimension, summing the noise contributions of each oscillator in quadrature, at measurement frequencies well below the lowest normal-mode frequency.

$$x_{th}^2 \approx \frac{8k_BT}{\omega} \sum_{n=1}^{\infty} \frac{\phi_n(\omega)}{M\omega_n^2}$$

For his quantitative estimate, he approximated the infinite sum by the first two terms. This approximation was more or less justified by the weighting of each term by ω_n^{-2}, since the resonant frequencies ω_n increase with increasing n by definition. (You can calculate the mode frequencies analytically [52], but it's hard. Most people just use a commercial finite-element-analysis package.) Gillespie and Raab carried out the calculation in more detail. They looked at the question of how many modes must be included and found that this number depended on the laser spot size used to sample the surface of the mirror. Their conclusion was that "all modes with acoustic wavelengths greater than the laser spot diameter must be considered for an accurate estimate of the thermal noise" [53]. They did a careful evaluation of the mode-sum expression and found that the additional terms added a factor of six to Saulson's original estimate. In a similar, but independent, effort, Bondu and Vinet developed a numerical code to evaluate the mode-sum expression and optimize the mirror and beam sizes [54].

The assumption that the mode-summation technique would yield the correct result appears to have been just that, an assumption. It seemed like a reasonable assumption at the time, but on closer examination there it has potentially serious flaws.

Majorana and Ogawa examined the mode-sum procedure and compared it to rigorous direct application of the fluctuation-dissipation theorem. They looked at the two methods in the simplest system with more than one normal mode, a pair of coupled, simple harmonic oscillators. They concluded that it is possible to get the right answer by summing over normal modes, but only if a great deal of care in the orthogonalization of those modes is taken [55]. This doesn't sound so bad, until you realize that the orthogonalization depends on both the Q's and the frequencies of the individual modes, which makes treatment of a general system quite difficult. Moreover, as they point out,

> In undergraduate textbooks . . . it is properly said that even for a simple system like the double damped oscillator the diagonalization of the energy matrix is not trivial and it is not possible in general, but only in particular cases. [55].

Clearly, there are serious potential problems with the mode-sum approach, and we would have much more confidence in a direct application of the fluctuation-dissipation theorem.

6 Experimental observation of mirror thermal noise

6.1 40-meter: An early claim

By March of 1994, Caltech's 40-meter prototype team had achieved, in their interferometer, a displacement sensitivity better than $10^{-18} m/\sqrt{Hz}$. Their test masses were, at the time, compound objects with actuator magnets glued directly to the backs of the mirrors, and the quality factors were, not surprisingly, poor. Nevertheless, the team made a prediction of the mirror thermal noise, based on the modal-expansion method, and compared this with their observed noise spectrum. Their conclusion was,

> The measured noise matches the predicted slope of the model over a wide frequency band and agrees with the predicted value, within the rather large uncertainties [59].

The assertion is made in the text of the paper, but no supporting analysis or graph is provided.[2] By October of the same year, the team had made a number of substantial improvements to their instrument, including replacing the test masses with monolithic mirrors, separated the magnets from the mirrors by the use of mechanically isolating dumbell standoffs, and significantly improving the seismic isolation. The result of this activity was a reduction of the lowest noise level in the instrument to $3 \times 10^{-19} m/\sqrt{Hz}$ over most frequencies between $400Hz$ and $1kHz$.

6.2 Hongo group: A definitive observation in lossy mirrors

In 2002 a group in Tokyo, led by Kimio Tsubono, built a small interferometer to directly measure the thermal noise in lossy mirrors. Other groups had attempted similar experiments, typically focusing on measuring thermal noise in high-Q (low-loss) mirrors with large spot sizes. The idea was to push the thermal noise down as far as possible and then measure it at that low level. Such a measurement was expected to be relevant because it would be performed using the same low-loss mirror materials as were used in the "big" interferometers. Taking a different approach, the Hongo group decided that, since mirror thermal noise had never been conclusively observed in any interferometer, it would be better to measure thermal noise in lossy mirrors as a validation of the fluctuation-dissipation theorem. Instead of high-purity, synthetic fused silica, they used common BK7 for their substrate material, and instead of pushing their spot size to the largest practical value, they made it as small as they could. These two innovations combined to bring their thermal noise up to the relatively easily measured level of nearly $10^{-16} m/\sqrt{Hz}$ at $100Hz$. They made definitive observations of structural-damping-dominated thermal noise in BK7 and thermoelastic-damping noise in CaF_2 over three decades of frequency, from $100Hz$ to $100kHz$ [60].

7 The fluctuation-dissipation theorem applied to mirrors

González and Saulson were quick to apply the fluctuation-dissipation theorem directly to calculate pendulum thermal noise [56], but a similar treatment of mirror thermal noise was a little longer in coming.

Aware that there were potential problems with the modal-expansion method, but apparently unaware of Majorana and Ogawa's work, Nakagawa, *et al.* developed a two-point-correlation-function method of evaluating the mirror thermal noise based on elastic Green's functions. They developed a

[2] The noise curve given shows the total instrument noise as it was in June of 1992, June of 1994, and October of 1994, along with a projected requirement for an initial LIGO noise curve.

procedure and outlined the method for carrying out a calculation of mirror thermal noise in an early paper [57], and they published the results of their calculation in a later article [58].

Yuri Levin used a different technique to apply the fluctuation-dissipation theorem directly and produced a simple formula for the thermal noise in the case where the spot size is small, compared with the diameter of the mirror. More important, he pointed out that how the losses were distributed inside the test mass is critically important. Losses far from the beam spot contribute little to the total thermal noise, whereas losses near the spot, in the dielectric coating for example, contribute a lot. This does not seem surprising, in retrospect, considering the Langevin model of thermal noise, where lossy regions in bulk materials are sources of fluctuating stress. Levin's result is, for homogeneous losses, [61]

$$x_{th}^2 = \frac{4k_BT}{f}\frac{1-\sigma^2}{\pi^3 Yw} I\phi(f)\left[1+O\left(\frac{w}{R}\right)\right],$$

where $f = \omega/2\pi$ is the measurement frequency, σ is Poisson's ratio, Y is Young's modulus, w is the laser spot size, and R is the characteristic size of the test mass. The numerical factor $I = 1.87322\ldots$.

The idea that coatings could significantly affect thermal noise stimulated a burst of activity that continues up to the time of this writing.

8 Coating thermal noise

Following Levin's initial work on inhomogeneous losses, a large community assembled to address the issue of coating thermal noise in interferometric gravitational-wave detectors. Nakagawa, *et al.* looked at the theory of coating thermal noise in the restricted case where the coating has the same elastic properties as the substrate but greater loss. Their conclusion for the scaling of thermal noise with coating thickness, laser spot size, and coating loss was that, "the excess noise scales as the ratio of the coating loss to the substrate loss and as the ratio of the coating thickness to the laser beam spot size" [62].

In a separate work, a Japanese group numerically calculated the thermal noise in a mirror with inhomogeneous losses, treating the cases of losses concentrated a) on the front surface of a mirror (*i.e.* an HR coating), b) on the back surface (*i.e.* an AR coating), and around the cylindrical edge (the barrel). They also considered losses concentrated at points where magnet are conventionally glued. In each case they verified the expectation that losses far from the laser spot contribute relatively little to the total thermal noise, whereas losses on the front surface contribute a lot [63]. They did not, however, produce an analytical formula or scaling law.

In a pair of companion papers, published in the same issue of Classical and Quantum Gravity, two very large collaborations (with some overlapping members) reported both a comprehensive theory of coating thermal noise and a set of preliminary measurements of the coating loss.

One coalition was led by Peter Saulson's Syracuse group. The formula they derived is somewhat complicated, and it depends explicitly on the values of Poisson's ratio and Young's modulus for both the substrate and coating. We may gain considerable physical insight into the thermal noise, however, by neglecting Poisson's ratio, *i.e.* setting it to zero for both the substrate and the coating. The error introduced by such an approximation is expected to be about 30%, so we can even use this for rough estimates of the final thermal noise. In this approximation, the total thermal noise at the surface of a mirror is [64],

$$S_x(f) = \frac{2k_BT}{\pi^{3/2}f}\frac{1}{wY}\left\{\phi_{substrate} + \frac{1}{\sqrt{\pi}}\frac{d}{w}\left(\frac{Y'}{Y}\phi_{\|} + \frac{Y}{Y'}\phi_{\perp}\right)\right\},$$

where $\phi_{substrate}$ and Y are the loss angle and Young's modulus in the substrate, respectively, Y' is the coating's Young's modulus, w is the laser spot size, and d is the coating thickness. The two loss angles $\phi_{\|}$ and $\phi_{\perp}$ are coating loss angles for strains parallel and perpendicular to the coating layer, respectively. Since the coating is not homogeneous, but rather a collection of layers, we have no reason to expect these loss angles to be the same. (Note that the scaling law with respect to both of these loss angles, and with respect to the ratio of coating thickness to laser spot size, is the same as that obtained by Nakagawa, *et al.* [62].)

According to this result, the coating thermal noise should not be completely independent of the substrate properties. The relative contributions of the parallel and perpendicular losses depend on the ratio of the Young's moduli of the substrate and coating. Different substrate materials require different coatings to minimize the total thermal noise.

The Syracuse group also measured the Q's of several samples with and without coatings and looked at the Q's before and after the coating process. Notable by its absence in this work was a control sample, a mirror that was handled and annealed in the same way as the coated samples. This is potentially an important omission, since the substrate Q's may change dramatically if the samples are annealed, and the coating process involves considerable elevated temperatures. Nevertheless, they reported values of $\phi_{\|}$ of $4.2 \pm 0.3 \times 10^{-4}$ and $1.0 \pm 0.3 \times 10^{-4}$ for rectangular- and disk-shaped fused-silica samples, respectively [64].

The Glasgow-led group did not try to measure Q's before and after the coating process. Instead, they developed a clever model that accounted for how the coating losses would affect the Q's of different vibrational modes of a mirror, based on the amount of strain each mode induces at the coated surface, and they only measured Q's of already-coated samples. They did not, however, distinguish between $\phi_{\|}$ and $\phi_{\perp}$. Their results for a generalized $\phi_{coating}$ were $6.4 \pm 0.6 \times 10^{-5}$ and $6.3 \pm 1.6 \times 10^{-5}$ for Corning 7940 and 7980 substrate materials, respectively [65]. These results seem to lend credence to those of the Syracuse group, indicating that the omission of a control sample in that study may not be a problem.

8.1 Where in the coating is the loss?

Dielectric coatings are made of alternating layers of materials with different indices of refraction, and we may expect some layers to be mechanically more lossy than others. As of this writing, the most recent development is the identification of tantalum pentoxide (Ta_2O_5) as the lossy material in a Ta_2O_5/SiO_2 coating. A group of sixteen authors reported in a single paper, building on the results of both the Glasgow-led and the Syracuse-led papers, that the relative loss angles for silica and tantala layers were [66]

$$\begin{aligned}\phi_{silica} &= (0.5 \pm 0.3) \times 10^{-4}, \text{and} \\ \phi_{tantala} &= (4.4 \pm 0.2) \times 10^{-4}.\end{aligned}$$

(These values are presumably for $\phi_{\|}$.)

9 Braginsky's unexpected sources

While much attention was focused on structural damping and its many effects, one group at Moscow State University quietly pointed out that, at the extremely low noise levels required for gravitational-wave detection, many other fundamental noise sources are likely to contribute. The first of these is related to the thermal expansion of the mirror.

When we think of temperature, we often think of a uniform property, evenly distributed throughout the interior of an object. In reality, the temperature of an object is merely an averaged value. In any real object the temperature will always exhibit local fluctuations, even when the object is nominally in thermodynamic equilibrium [67]. These fluctuations cause the material to expand or contract, depending on the sign of the fluctuation, through the coefficient of thermal expansion. These fluctuations are fundamental. You can't get rid of them by improving the material's Q, for example. Braginsky estimated the size of the surface deformations in a half-infinite slab of material, and showed that in some cases they can be larger than the noise due to structural damping [68]. Most worrisome, these thermodynamic fluctuations are, all else being equal, much larger in Sapphire than in Fused Silica. This is potentially problematic, since it had been proposed to use Sapphire in an advanced detector, instead of Fused Silica, because of its lower structural damping [69, 70]. (Note to self: Wasn't calcium fluoride also a candidate [71]? Why didn't that go anywhere? Maybe its thermoelastic damping noise is too big.)

On a more prosaic level, any time a photon gets absorbed on the surface of a mirror there is a small amount of thermal expansion due to the local heating of the material. Braginsky also estimated the size of this effect, assuming the intensity of the incident beam is shot noise limited, and found it to be smaller than Brownian motion in both Sapphire and Fused Silica.

One thing Braginsky pointed out in his paper was that this thermodynamical-temperature-fluctuation-based noise is not based on a new effect. This noise source can be thought of as ordinary Brownian motion, as predicted by the fluctuation-dissipation theorem, with thermoelastic damping as the loss mechanism, instead of internal friction. Thermoelastic damping has been known and reasonably well understood for many years [72, 73, 74]. Peter Saulson treated thermoelastic damping in his groundbreaking 1990 paper [23], and its effects on pendulum thermal noise have been widely recognized and studied [27, 30, 31]. Nevertheless, Braginsky seems to have been the first to point out that this loss mechanism could be important in determining the mirror thermal noise.

Yuk Tung Liu and Kip Thorne verified the results of Braginsky, *et al.* and extended their analysis to mirrors of finite size. They found the finite size of the mirror, compared with the laser spot size, increased the thermoelastic contribution to the thermal noise by a "modest amount," something on the order of 10% [75].

Very shortly after pointing out the connection between temperature fluctuations and physical motion of a mirror's surface, Braginsky and company looked at the changes in the reflectivity of dielectric coatings caused by such fluctuations. The index of refraction of a material often depends strongly on temperature, and if the temperature inside the dielectric coating on the surface of a mirror fluctuates, this will lead to an apparent noise source. Again considering both intrinsic, thermodynamic temperature fluctuations and fluctuations due to absorbed photons, they found that thermo-refractive noise could contribute to the displacement noise in an advanced interferometer at a level "comparable with other known noises at frequencies $\sim 1kHz$" [76].

Turning their attention from passive Fabry-Perot cavities to lasers, Braginsky and Vyatchanin found that thermoelastic-damping noise, photothermal noise, and thermorefractive noise could also contribute substantially to the noise level in a laser [81].

Finally, at least as of this writing, Braginsky and company looked at thermoelastic-damping noise (temperature fluctuations generating mechanical fluctuations) in dielectric coatings. They found that the thermoelastic-damping noise in coatings can be larger than the analogous effect in substrates, due to the larger coefficient of thermal expansion often found in coating materials and the smaller effective volume [82]. Unfortunately, the thermal expansion coefficients of commonly-used coating materials are not well known, having been reported by different researchers as having not only different magnitudes, but different signs [83, 84]. Braginsky and Samoilenko performed their own measurement of the thermal expansion coefficient of Tantalum Pentoxide (Ta_2O_5). The value they obtained yields a relatively low thermoelastic-damping noise prediction for coatings, compared with other published values [85].

Thermoelastic-damping noise in coatings was also investigated independently by Shiela Rowan and Marty Fejer [86].

10 Miscellaneous experimental work

In addition to González and Saulson's initial demonstration of the connection between mechanical admittance and thermal noise [35], a few other groups have experimentally verified the predictions of the fluctuation-dissipation theorem. One group did their experiment with a phosphor-bronze cantilever [77], instead of a torsional pendulum. A second group looked at a two-mode system, using a measurement of the admittance at the anti-resonant frequency [78], and shortly afterward looked at the effects of inhomogeneous loss in a leaf spring [79] and a hollow Aluminum drum [80], exploring Levin's earlier prediction [61] in simple, well-controlled systems.

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