LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
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Approximation of the Laser Phase Change

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1 Introduction

It is difficult to write out an analytic LIGO interferometer (IFO) response function, even for the simple case of a sinusoidal gravitational wave (GW). Furthermore, we would like to deal with more complicated waveforms, such as those from burst sources. This document discusses one approach for numerically computing the response of the IFO to an arbitrary wave.

2 The Setup

Let a GW be incident on some fixed point, say a LIGO site, so that the wave amplitude is a function of time only. Then the plus and cross polarizations can each be written as an amplitude times some shape function:

$$h_+(t) = h_{+\circ} f(t) \tag{1}$$
$$h_\times(t) = h_{\times\circ} g(t) \tag{2}$$

Tranforming to the detector coordinate system and changing variables from time to position, [1] gives the round trip laser phase change in the x-arm:

$$\Phi^x_{RT}(t_\circ) = \frac{\omega}{c}\int_0^L dx \left[\sqrt{1 + h_{xx}\Big(t_\circ + \frac{x}{c}\Big)} + \sqrt{1 + h_{xx}\Big(t_\circ + \frac{2L}{c} - \frac{x}{c}\Big)}\right] \tag{3}$$

where:

$$h_{xx}(t) = -h_\times(t)\cos\theta\sin 2\phi \;+\; h_+(t)\big(\cos^2\theta\cos^2\phi - \sin^2\phi\big) \tag{4}$$

Now we may approximate (3) with a Taylor expansion:

$$\Phi^x_{RT}(t_\circ) \approx \frac{\omega}{c}\int_0^L dx \left[1 + \frac{1}{2}h_{xx}\big(t_\circ + \frac{x}{c}\big) + 1 + \frac{1}{2}h_{xx}\big(t_\circ + \frac{2L}{c} - \frac{x}{c}\big)\right] \tag{5}$$

From this point we neglect the constant terms in the integral, because we are interested only in GW-dependent phase changes of the laser. Now we make the key approximation:

> Over the time period $t_\circ$ to $t_\circ + L/c$ the phase of the GW is essentially constant.

In other words, the period of oscillation of the GW is much longer than the round trip time of light in the IFO arm. Thus, the integrands in the above equation can be taken as constants for any particular value of $t_\circ$:

$$\Delta\Phi^x_{RT}(t_\circ) \approx \frac{\omega L}{2c}\left[h_{xx}\Big(t_\circ\Big) + h_{xx}\Big(t_\circ + \frac{L}{c}\Big)\right] \tag{6}$$

Now we separate the phases acquired in the two half-trips:

$$\Delta\Phi^x_{ITM-ETM}(t_\circ) \quad \approx \quad \frac{\omega L}{2c} h_{xx}(t_\circ) \tag{7}$$

$$\Delta\Phi^x_{ETM-ITM}(t_\circ) \quad \approx \quad \frac{\omega L}{2c} h_{xx}(t_\circ + \frac{L}{c}) \tag{8}$$

The case for the y-arm is very similar: according to [1] we replace $h_{xx}(t)$ with $h_{yy}(t)$:

$$h_{yy}(t) = h_\times(t)\cos\theta\sin 2\phi \; + \; h_+(t)\big(\cos^2\theta\sin^2\phi - \cos^2\phi\big) \tag{9}$$

References

[1] Sigg, Daniel. “Gravitational Waves.” Proceedings of the Theoretical Advanced Study Institute in Elementary Particle Physics, 10/23/1998.