LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
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Testing the Laser Phase Calculation

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Introduction

Provided in [1] is an exact formula for the phase acquired by light in a LIGO interferometer arm in the presence of incident gravitational waves (GWs). However, the integral is difficult to evaluate, so approximations must be made. Furthermore, the formula is limited to sinusoidal incoming waves. The derivation given in [2] handles these issues and provides an alternative calculation method. In this document we investigate the validity of this approximation when implemented in the e2e simulation.

Analysis

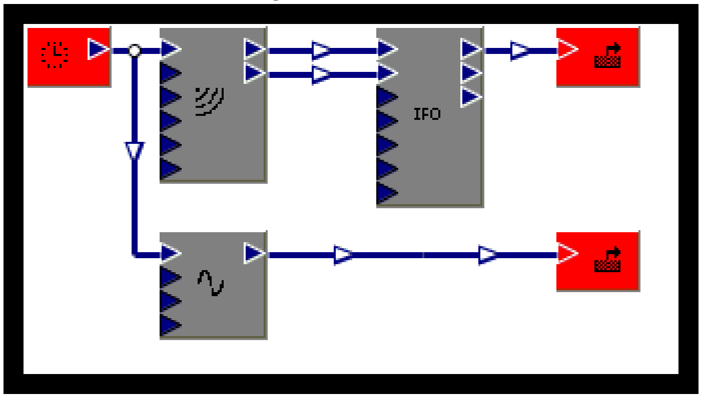
The approximation will be tested in two ways. First, it must give accurate values for the laser phase in a GW frequency band from 10 Hz to 10 kHz. Second, we should witness a delay in the response of the interferometer between when the GW arrives and when the signal exits the Fabry-Perot cavity.

Validity in LIGO frequency band

Let's show that the approximation gives valid values of the laser phase for GW frequencies detectable by LIGO. This is done using a simple box file, depicted in figure 1. One output is the laser x-arm phase resulting from a sinusoidal GW source, calculated using the approximation in [2]. The other output is the control; it is simply a cosine wave generated at the same frequency. (We assume the inteferometer response to a cosine wave is a cosine wave.) Now we analyze the output of this simulation for a variety of GW frequencies, using a time step of 1.33×10^{-5} s[1]

[1]This time step is half the round-trip time of light in the Fabry-Perot cavity. Since it is the largest value allowed by the LIGO simulation, it is actually a worst-case scenario of the accuracy of the approximation.

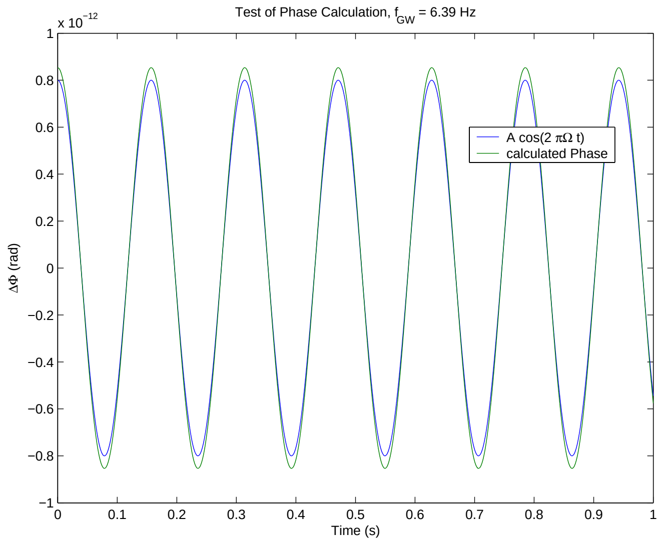
Figure 1: Box File



Shown here is a graphical representation of the ALFI *.box file used in the simulation described in this section. The clock runs to a cosine wave generator and to the binary system GW source. The GW amplitudes are passed to the detector module, which performs the phase calculation.

First, let's test the low-frequency case. In figure 2 below, the calculated laser phase change from a 6.4 Hz GW is plotted next to a cosine wave of the same frequency. They agree quite well; in fact, if the cosine amplitude were set to match the laser phase shift amplitude, the two plots would be visually indistinguishable.

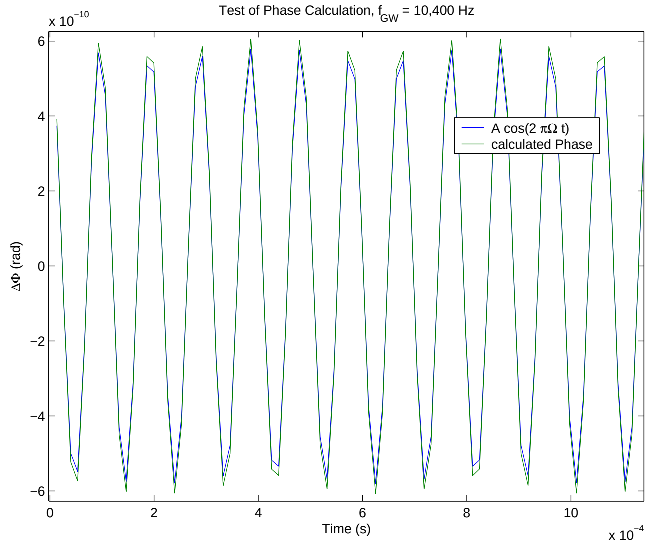
Figure 2: Test at Low Frequency



The output of the module that calculates the laser phase is compared to a cosine wave. The two agree extremely well, showing that the phase calculation is valid for low frequencies.

Next, consider the high frequency case. The same procedure as above was performed for 10.4 kHz waves. As shown in figure 3, the phase calculation still works. When the cosine amplitude was chosen to be a best-fit, the maximum disagreement between the two was a miniscule 4×10^{-15} rad.

Figure 3: Test at High Frequency



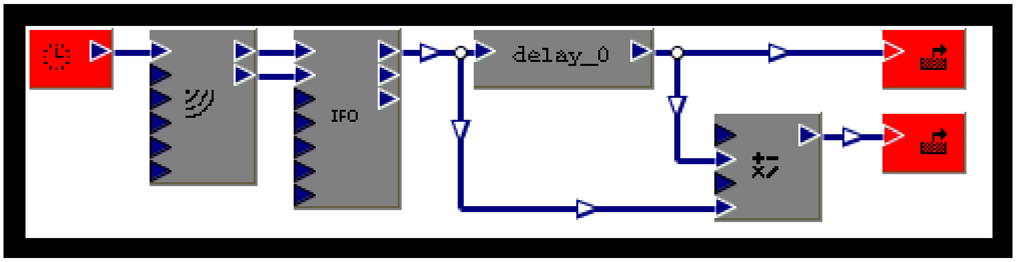
The output of the module that calculates the laser phase is compared to a cosine wave. The two agree extremely well, showing that the phase calculation is valid for high frequencies as well.

Signal Phase Shift Check

First, it must be established why exactly the interferometer output signal is shifted with respect to the GW signal. To do so, consider how the effect of GWs is implemented in e2e. At each time step, the instantaneous laser phase acquired over half a round trip is passed to each propagator (there are two propagators for each interferometer arm). Thus, light that enters the cavity at time t_0 acquires phase $\Phi_x(t_0)$ for the ITM to ETM path. At the start of the return trip (ETM to ITM), a time $\tau = L/c$ has elapsed, so the light acquires additional phase $\Phi_x(t_0 + \tau)$. The fact that the GW phase is different on the two trips gives rise to a slight phase shift of the interference signal.

To test whether the simulation exhibits this phase shift, let us use the box file in figure 4.

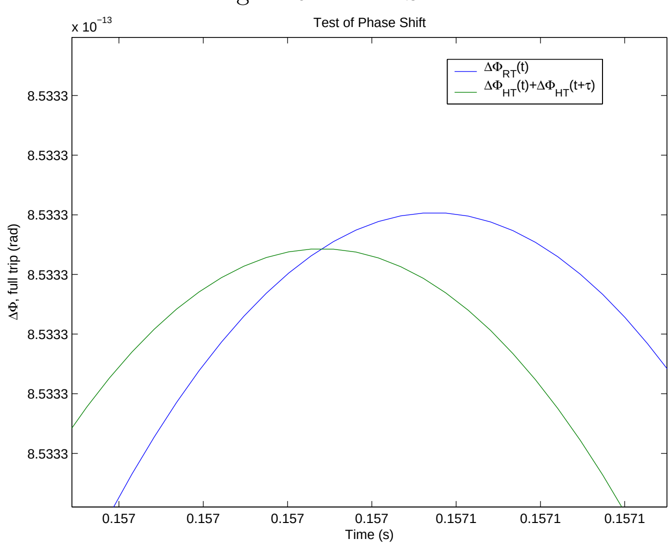
Figure 4: Box File



Shown here is a graphical depiction of the ALFI *.box file used in the simulation described in this section. The upper output module is simply the laser phase, delayed by τ. The lower output module is the average of the delayed laser phase and the instantaneous laser phase. This awkward implementation was necessary, because the simulation only has access to information at the current time, but we need to know the phase at a future time, $t + \tau$.

One output of the box file is the instantaneous round-trip phase shift $\Delta\Phi_{RT}(t_0)$ of the light entering the cavity. The other output is the total phase of light acquired in both half trips: $\Delta\Phi_{HT}(t_0) + \Delta\Phi_{HT}(t_0 + \tau)$. Now we compare these two outputs with the assitance of MATLAB, as shown in figure 5 below.

Figure 5: Phase Shift



Here we see that by accounting for the GW phase changing between the two half-trips, a small phase shift in the laser phase occurs.

The above plot was generated with an e2e time step of $\tau/10$, or 1.33×10^{-6} s. Corresponding maxima and minima of the plot were each separated by 5 time steps, or $\tau/2$. We see that the phase, when calculated using different values for the two half-trips, is shifted to the "left" with respect to the GW signal. Note that this time shift is a constant, irrespective of such factors as incidence angle.

References

[1] Sigg, Daniel. "Gravitational Waves." Proceedings of the Theoretical Advanced Study Institute in Elementary Particle Physics, 10/23/1998.

[2] Jauregui, Jeff. "Approximation of Laser Phase Change." LIGO-T030161-00-E.