LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
-LIGO-
CALIFORNIA INSTITUTE OF TECHNOLOGY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Technical Note	**LIGO-T030172- 00- E**	8/1/03

SURF Progress Report - August

Jeff Jauregui (2003 SURF Student)
Mentor: Hiro Yamamoto

This is an internal working note
of the LIGO Project.

California Institute of Technology
LIGO Project – MS 51-33
Pasadena CA 91125
Phone (626) 395-2129
Fax (626) 304-9834
E-mail: info@ligo.caltech.edu

Massachusetts Institute of Technology
LIGO Project – MS 20B-145
Cambridge, MA 01239
Phone (617) 253-4824
Fax (617) 253-7014
E-mail: info@ligo.mit.edu

WWW: http://www.ligo.caltech.edu

Surf Progress Report – August 1, 2003

Jeff Jauregui, LIGO, Caltech

Advisor: Dr. Hiro Yamamoto

Project Title:

"Simulating the LIGO Laser Phase Change Resulting from Gravitational Waves"

The idea behind LIGO is that gravitational waves can effectively alter the optical path length of a laser beam. In a Michelson Interferometer with two long Fabry-Perot arms, two perpendicular lasers would undergo opposite phase shifts and produce an interference signal determined by the gravitational wave. My research has centered on developing a method to implement the physics of this phenomenon into the existing end-to-end time domain simulation software.

A month ago, I derived an approximate formula for the laser phase shift due to a sinusoidal gravitational wave (which is a special case), making extensive use of the formulation given in [1]. The round trip difference in phase for light in the "x" interferometer arm is given by:

$$\Delta\Phi_{RT}^{x}(t_0) = \frac{h_{xx} L\omega}{c}\cos\left(\Omega t_0 + Lk(1 - \frac{1}{2}\sin\theta\cos\varphi)\right)\cos\left(\frac{Lk}{2}\right),$$

where t_0 is the time the light enters the Fabry-Perot cavity, h_{xx} represents the dimensionless strain parallel to x-arm, L is the cavity length, ω is the angular frequency of the laser, Ω is the angular frequency of the gravitational wave, k is the corresponding wavenumber, and θ and φ determine the location of the source in Earth spherical coordinates [6]. A comparison in MATLAB of this approximation to a numerical integration of the exact equation showed 1% agreement up to $\Omega = 2\pi\times2800$ Hz and 5% agreement up to $\Omega = 2\pi\times6000$ Hz.

Next, I wrote a stand-alone program to generate the change in phase in terms of the several parameters. The code was written in a modular fashion to allow for easier integration with the existing simulation in the future. It soon became apparent that a few more free parameters were needed. In particular, I decided to allow for the LIGO site to be at any place on Earth, with any orientation. It was necessary to derive certain coordinate transformations to make these additions possible.

Previously, all gravitational wave signals were arbitrarily chosen for convenience. I finally implemented the equations describing the plus and cross amplitudes of gravitational radiation coming from a binary star system. This added the free parameters of star masses, orbital frequency, distance from Earth, and orbital plane tilt.

In order to show the simulated data were consistent with the physics, I performed many tests of the simulation for a number of values of the various parameters. In numerous cases, invalid results led to changes in the code. Finally, when no discrepancies were found, I wrote an extensive document to describe the tests I performed and the results produced by the simulation [7].

At last the phase change simulation was ready to be implemented in e2e, the software package for simulating LIGO. I became acquainted with the software, its source code, and the graphical program "alfi" designed for constructing systems to be simulated. After making modules to describe the gravitational wave source and LIGO interferometer arms, I quickly realized that the assumption of a sinusoidal waveform was too constricting. Thus, I returned to the equation in [1] that describes the phase shift and devised a method of approximating the phase shift for an arbitrary waveform (assuming a certain degree of smoothness). This new method of calculation was implemented into the e2e code.

As a side task, I wrote a document describing the steps necessary to create a new module for e2e, so future researchers will have an advantage [8].

Most recently, I modified the electromagnetic field propagator module in e2e to allow for full integration of my previous work into e2e. Again, a variety of validations were and will be performed. One source of difficulty was tracking down an apparent doubling of the photodetector power signal frequency relative to the gravitational wave frequency. However, when I realized that power is related to the *square* of field amplitude, the problem was resolved (since squaring a sinusoid doubles its frequency).

Probably the most important remaining part of my project is showing the simulation is valid in more extreme cases. It is possible that certain approximations will prove to have been too great, so that modifications to the numerical calculations may be necessary. Currently, there are only modules for orbiting and inspiralling binary sources. Therefore, I would like to add support for “burst” and other sources. Finally, the simulation can actually be applied to certain studies, such as differences in the signals at the two LIGO sites, and “blind spots” where the interferometer does not respond to a gravitational wave.

Throughout the course of the summer, I have met with my advisor at least once a week, and usually more. It tends to be on an “as needed” basis to discuss LIGO physics or the e2e simulation program.

[1] D. Sigg, “Gravitational Waves” Proceedings of the Theoretical Advanced Study Institute in Elementary Particle Physics, 10/23/1998.

[2] A. Abramovici et.al., “Large Scale Measurements” Science 256, 325-333 (1992).

[3] H. Yamamoto et.al., “End to End Simulation program for gravitational-wave detectors” In Seiji Kawamura and Norikatsu Mio, editors, Gravitational Wave Detection II, Frontiers Science Series No.32, pages 331-336, (2000).

[4] B.Bhawal et al., “The LIGO End-to-End Simulation Program"”to be published in proceedings of the Moriond Conference on “Gravitational Waves and Experimental Gravity” (March, 2003).

[5] Blackburn, James Kent, “The Laser Interferometer Gravitational Wave Observatory Project” in “Mathematics of Gravitation” (1997).

[6] Jauregui, Jeff. “The LIGO Interferometer Response.” LIGO-T030151-00-E

[7] Jauregui, Jeff. “Validation of the Source-Detector Simulation Formulation.” LIGO-T030152-00-E

[8] Jauregui, Jeff. “Creating an e2e Primitive.” LIGO-T030154-00-E