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Analysis of thermal noise of newly proposed design and material for the Advanced LIGO suspensions

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Abstract
For the suspensions of Advanced LIGO, a new model of low noise, flex joint is being investigated. In order to optimize the design of the joint we need a theoretical prediction of the thermal noise, depending on the shape and on the material of the suspension. The analytical formula is found for the thermal noise spectrum but it depends on the mechanical properties of the material that are investigated experimentally with stress-strain measurements here. Due to their superior mechanical properties and low loss factor, two materials, the MoRuB alloy and Monocrystalline Silica, are tested for this application.

Introduction

Thermal noise is one of the most important factors limiting the sensitivity of interferometers for the detection of gravitational waves and is playing a fundamental role in the design of the mirror suspensions and of the test masses. A crucial point in the suspension system is the flex joint connecting the mirror to the seismic attenuation multiple pendulums. The design of a new joint for Advanced LIGO includes an analytical prediction of thermal noise and the experimental determination of mechanical properties of the materials chosen for this application.

Thermal Noise for a Pendulum

The system composed of the joint and the test mass can be considered as a fluctuating pendulum, where thermal noise is generated by the internal friction in the material of which the suspension is made. The energy dissipation in a pendulum is represented by the quality factor Q, defined as $Q = Q_{mat}\frac{S+G}{S}$, where S is the strain energy, G is the gravitational energy and Q_{mat} is the quality factor of the material.
The Fluctuation-Dissipation Theorem [1] predicts a thermal noise spectrum for the pendulum of

$$x^2(f) = \frac{4K_B Tk\phi}{2\pi f[(k - m(2\pi f)^2)^2 + k^2\phi^2]},$$

where K_B is the Boltzmann constant,

T is the temperature, k is the stiffness of the flex joint, $\phi = Q^{-1}$ and m is the mass each joint has to sustain.
Since thermal noise spectrum sets the operative range of frequencies of a gravitational interferometer, realistic values for k and Q are investigated through theoretical and experimental methods.

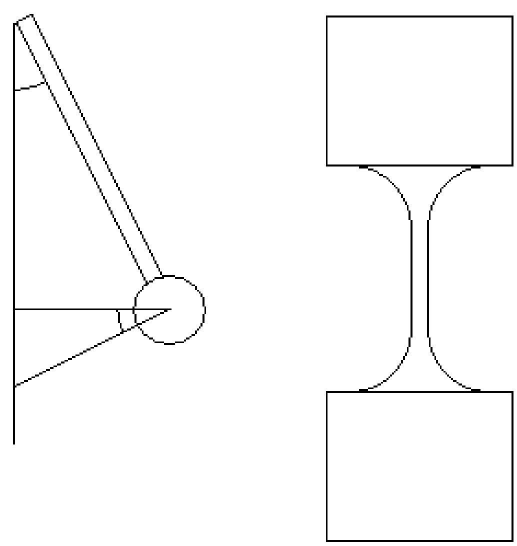

Figure 1.Flex Joint and Ideal Pendulum

Analytical derivation for Q and k

As seen before, for a pendulum the quality factor is given by $Q = Q_{mat}\frac{S+G}{S}$. The gravitational potential energy, for small angles, is given by

$$G = Mgy = Mgx\tan\alpha \cong Mgx\sin\alpha = \frac{Mgx^2}{h}$$

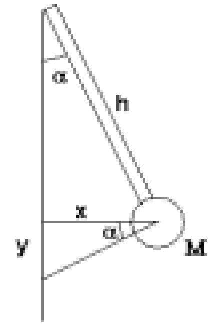

Figure 2.Pendulum composed of the central part of the joint and the test mass

The strain energy for the real joint is still an open problem and is being investigated in these days; the most probable formulation is the following:

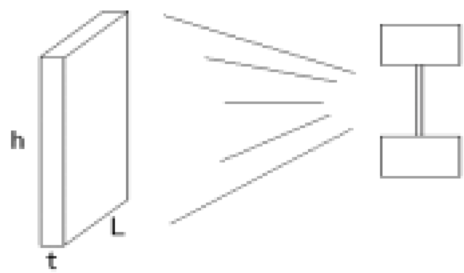

Figure 3.Dimensions of the central beam

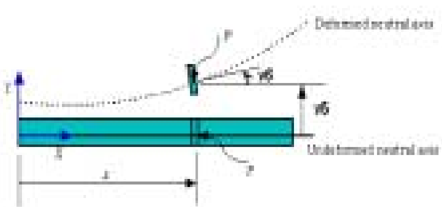

Figure 4.Bending beam

If we consider the central part of the joint, the problem is that of a bending beam free at one end. A beam has six degrees of freedom [see Fig n]: v2 and v5 are the transversal deviation from equilibrium, v1 and v4 are the linear deformation due to the load, v3 and v6 are the angular deflections from the undeformed axis.

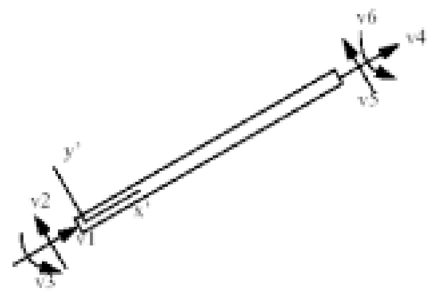

Figure 5.Degrees of freedom for a bending beam

The Strain energy is given by $S = \frac{1}{2}u^T Ku$, where u=(v1,v2,v3,v4,v5,v6) and K is the stiffness matrix:

$$k = \begin{bmatrix} \frac{AE}{L} & 0 & 0 & \frac{-AE}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ \frac{-AE}{L} & 0 & 0 & \frac{AE}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

Where L is the length of the beam, A is the beam cross-section area, I is the

second moment of area and E is the Young's modulus of the material.
Since our beam is fixed at one end, we have only three degree of freedom, our deformation vector is u=(0,0,0,v4,v5,v6). For the Strain energy we have

$$S = \frac{1}{2}E\left[\frac{A{v_4}^2}{h} + \frac{4I(h^2{v_6}^2 - 3hv_5v_6 + 3v_5^2)}{h^3}\right]$$

For small angles v5=-hv6, so the strain energy is

$$S = \frac{1}{2}E\left[\frac{A{v_4}^2}{h} + \frac{4I(v_5^2 + 3{v_5}^2 + 3v_5^2)}{h^3}\right] = \frac{1}{2}E\left(\frac{Av_4^2}{h} + \frac{28Iv_5^2}{h^3}\right)$$

The force applied to the free end, in the direction of v5, is given by

$$F = (0,0,0,0,1,0)Ku = \frac{6EI(2v_5 - hv_6)}{h^3} = \frac{18EIv_5}{h^3}$$

So a possible formula for the stiffness we need is $k = \frac{dF}{dv_5} = \frac{18EI}{h^3}$.

The second moment of area I is a geometrical property of the beam, we have I=Lt3/12. The relation between load and v4 is given by

$$Load = (0,0,0,1,0,0)Ku = \frac{AEv_4}{h}$$

So we can write the Strain energy as

$$S = \frac{1}{2}E\left[\frac{A{v_4}^2}{h} + \frac{4I(v_5^2 + 3{v_5}^2 + 3v_5^2)}{h^3}\right] =$$

$$= \frac{1}{2}E\left(\frac{A\left(\frac{Load * h}{AE}\right)^2}{h} + \frac{28Iv_5^2}{h^3}\right) = \frac{1}{2}E\left(\frac{Load^2 h}{AE^2} + \frac{28Iv_5^2}{h^3}\right)$$

This could be the Strain energy if we ignore the effect of the fillets between the central beam and the rest of the joint. Since the effect of the fillets is difficult to be found analytically, next month the joint will be simulated with ANSYS, in order to obtain a more realistic value for S.

Experimental measurements

The mechanical properties of the material, such as the Young's modulus E and the quality factor Q_{mat} will be investigated next month through stress-strain measurements.
Due to their superior mechanical properties, the two materials tested are the amorphous MoRuB alloy and the Monocrystalline Silica.

Making an amorphous MoRuB alloy

Amorphous metals are characterized by big Q factor, about 10^7 and more, and superior hardiness. In order to produce an amorphous material, the melted alloy must be quenched before the crystallization process starts. An ultra rapid quenching is obtained through a machine composed of a melting coil and two pistons activated by a laser beam. The alloy sample is placed in the middle of the coil and levitates, due to the magnetic field. The sample also melts because of the heat generated by the coil. When we switch off the current circulating in the coil, the sample falls down and interrupts the laser beam that activates the pistons. The melting alloy is mashed between the pistons and cool off so quickly that the atoms can't reach a crystalline configuration. All the process happens in a vacuum chamber at a pressure of –5Torr, in order to avoid the contamination of the sample caused by dust and other impurities.
The samples obtained by rapid quenching are analyzed through X rays in order to certify their amorphousness.

Achievements and future developments

After some calibrations, he quenching machine now works properly and I obtained ten good samples that will be sent to the X rays test in order to start the stress-strain measurements as soon as possible.
As concerns the theoretical derivation of the thermal noise spectrum useful information will come from the ANSYS simulation, I'm still looking for an

analytical expression for the stiffness of the joint.

[1] *Peter R. Saulson, Fundamentals of Interferometric Gravitational Wave Detectors*