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Convenient and Reliable Method for Measuring Frequency Noise of the Pre-Stabilized Laser

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ABSTRACT

A convenient and reliable method for measuring the frequency noise of the light coming out of the Pre-stabilized Laser using the mode cleaner is introduced. It only requires taking a spectrum of the feedback voltage of the mode cleaner servo to the voltage controlled oscillator, measuring a transfer function in the voltage-controlled-oscillator path of the mode cleaner servo, and dividing the obtained spectrum by the obtained transfer function. This method is not only convenient but also very reliable and not subject to variation of some optical parameters of the system such as the light power, the finesse of the mode cleaner cavity, and the visibility of the cavity.

1. Introduction

The frequency noise of the light coming out of the Pre-stabilized Laser (PSL) is measured by locking the light to the mode cleaner (MC). The method which has been used in the 40m was to calibrate the error signal of the MC servo in terms of frequency by sweeping the MC length and measuring the slope of the error signal. This method requires performing the calibration of the error signal each time because the calibration depends on various optical parameters of the system such as the light power and alignment of the MC. Moreover, measuring the slope of the error signal does not provide an extremely precise calibration of the error signal. Here we introduce a very convenient, but yet very precise and stable method for measuring the frequency noise of the PSL.

2. Block Diagram

Figure 1 shows the simplified schematic diagrams of the PSL and MC system. First, the Laser light is stabilized by the reference cavity (RC) servo. Then the error signal of the MC servo is filter/amplified and fed back to the voltage controlled oscillator (VCO) and one of the MC suspensions (SUS). The VCO drives the acousto-optical deflector (AOD) to change the frequency of the light entering the RC. The pre-mode cleaner is omitted from the diagram since it does not play any significant roll in the following discussion.

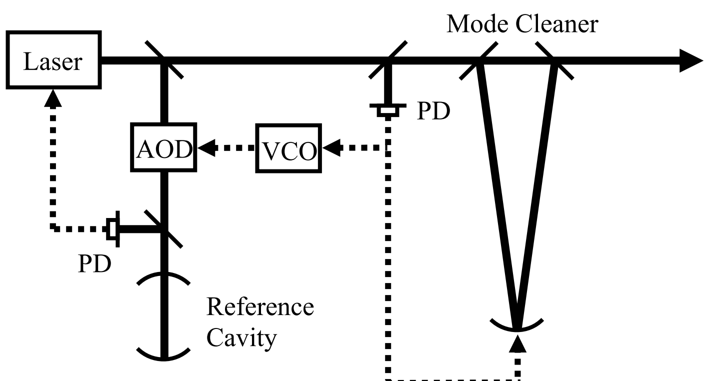

Fig. 1 Simplified schematic diagrams of the PSL and MC system.

The corresponding block diagram of the PSL and MC system is shown in Fig. 2. Two typical noise sources in the RC system are added to the diagram. Here, each symbol represents the following physical quantity:

ν_L: Frequency of the Laser
ν_{RC}: Resonant frequency of the RC determined by the cavity length of the RC
ν_{MC}: Resonant frequency of the MC determined by the cavity length of the MC
ν_{out}: Frequency of the light coming out of the PSL (after addition by Noise Source 2)
ν_{fb}: Feedback frequency of the VCO path in the MC servo
ν_{N1}: Noise Source 1 (e.g. shot noise and electronic noise of the RC servo)
ν_{N2}: Noise Source 2 (e.g. noise produced by vibrating folding mirrors between the PSL and MC)
L_{RC}: Low pass filter due to the cavity pole of the RC

$$\left(L_{RC} = \frac{\omega_{RC}}{s+\omega_{RC}} \right.$$, s: Laplace variable, ω_{RC}: cavity pole frequency of the RC)

L_{MC}: Low pass filter due to the cavity pole of the MC

($L_{\mathrm{MC}} = \dfrac{\omega_{\mathrm{MC}}}{s+\omega_{\mathrm{MC}}}$, s: Laplace variable, ω_{MC}: cavity pole frequency of the MC)

A_{RC}: Gain of the sensor/filter/amplifier/actuator of the RC servo
A_{SUS}: Gain of the sensor/filter/amplifier/actuator of the SUS path in the MC servo
E_{VCO}: Gain of the sensor/filter/amplifier of the VCO path in the MC servo
H_{VCO}: Gain of the actuator of the VCO path in the MC servo
V_{fb}: Feedback voltage of the VCO path in the MC servo

A triangle with + and – represents a discriminator.

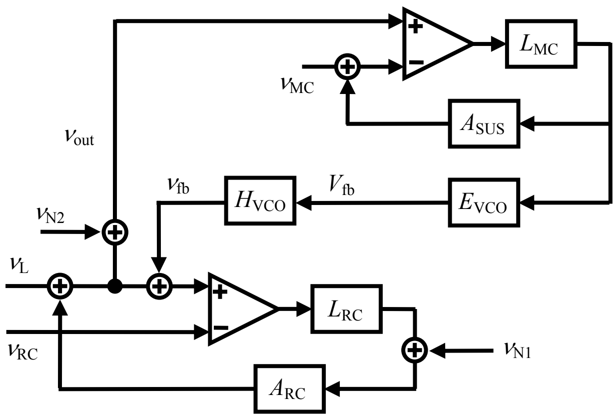

Fig. 2 Block diagrams of the PSL and MC system.

3. Method for Measuring Frequency Noise of PSL

What we are trying to measure is the frequency noise of the PSL, that is, the frequency of the light entering the MC[1]. This noise, ν_{PSL}, is defined to be ν_{out} (in Fig. 2) when there is no MC servo system as shown in Fig. 3. Thus,

$$\nu_{\mathrm{PSL}} = \nu_{\mathrm{out\,(No\,MC)}} = \frac{\nu_{\mathrm{L}} - L_{\mathrm{RC}} A_{\mathrm{RC}} \nu_{\mathrm{RC}} + A_{\mathrm{RC}} \nu_{\mathrm{N1}}}{1 - L_{\mathrm{RC}} A_{\mathrm{RC}}} + \nu_{\mathrm{N2}} .$$

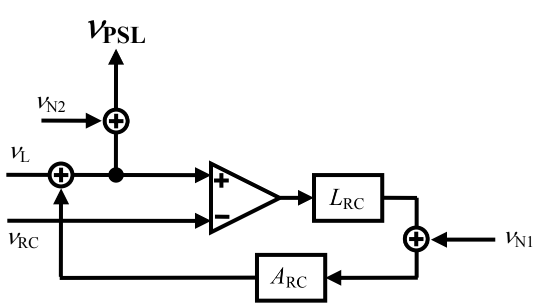

Fig. 3 Frequency noise of the PSL.

[1] This includes the noise ν_{N2}.

Now we assume that the loop gain of the RC servo, $L_{RC} \times A_{RC}$, is much larger than unity at all the interesting frequencies. This is the case for the 40m RC servo at frequencies below 100 kHz. Under this condition, the transfer function from x_1 to x_2 (See Fig. 4) in the PSL system (without the MC system) is equal to -1.

$$T_{x1\rightarrow x2} = \frac{L_{RC} A_{RC}}{1 - L_{RC} A_{RC}} \approx -1 .$$

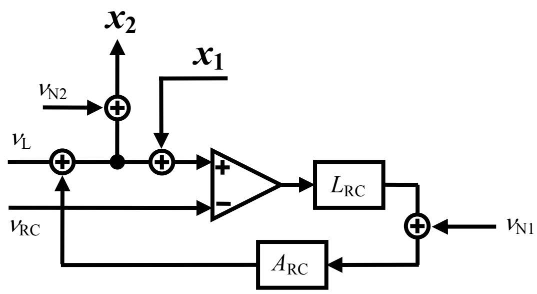

Fig. 4 RC loop as a frequency actuator.

This approximation will make it possible to simplify the block diagram of the RC and MC system shown in Fig. 2 to the one shown in Fig. 5.

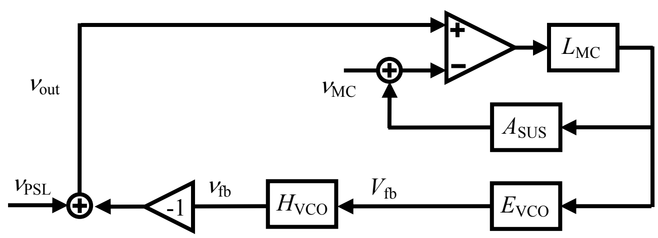

Fig. 5 Simplified block diagrams of the PSL and MC system.

If there were no SUS path, and the loop gain of the VCO path in the MC servo were much larger than unity, that is,

$$A_{SUS} = 0$$

$$L_{MC} E_{VCO} H_{VCO} >> 1 ,$$

we could obtain v_{PSL} just by measuring V_{fb} and multiplying it by H_{VCO},[2] because

$$v_{PSL} = v_{fb} = H_{VCO} V_{fb} .$$

However, this is not the case for the 40m. In reality there is a SUS path and the loop gain of the VCO path in the MC servo is less than unity above 60 kHz. In this case, the relationship between v_{PSL} and v_{fb} is

[2] In the following discussion, v_{MC}, is assumed to be zero. In reality, of course, the measured V_{fb} is limited by v_{MC} especially at low frequencies. However, this does not affect the validity of the method described in this note.

$$\nu_{\mathrm{PSL}} = \frac{1 + L_{\mathrm{MC}} A_{\mathrm{SUS}} + L_{\mathrm{MC}} E_{\mathrm{VCO}} H_{\mathrm{VCO}}}{L_{\mathrm{MC}} E_{\mathrm{VCO}} H_{\mathrm{VCO}}} \nu_{\mathrm{fb}} .$$

This coefficient is more than unity at frequencies where the SUS path is dominant in the MC servo or the loop gain of the VCO path is less than unity.

Fortunately there is a convenient way of measuring this coefficient, that is, to measure the transfer function from y_1 to y_2 in Fig. 6. It is exactly equal to the reciprocal of the coefficient, that is,

$$T_{y1\to y2} = -\frac{L_{\mathrm{MC}} E_{\mathrm{VCO}} H_{\mathrm{VCO}}}{1 + L_{\mathrm{MC}} A_{\mathrm{SUS}} + L_{\mathrm{MC}} E_{\mathrm{VCO}} H_{\mathrm{VCO}}} .$$

Therefore

$$\nu_{\mathrm{PSL}} = \frac{-1}{T_{y1\to y2}} \nu_{\mathrm{fb}} = \frac{-1}{T_{y1\to y2}} H_{\mathrm{VCO}} V_{\mathrm{fb}} .$$

Incidentally H_{VCO} consists of two parts: the VCO as a voltage-to-frequency converter and the AOD as a frequency actuator. Since the voltage-to-frequency coefficient of the VCO requires only electronic measurement, the result should be precise and stable. It was already measured carefully. As for the AOD, since the frequency shift of the light is caused by the Doppler shift of the light due to the sound wave propagating in the AOD crystal, the frequency shift is exactly equal to twice (round-trip) the RF frequency of the signal applied to the AOD. Therefore H_{VCO} can be determined precisely and stably.

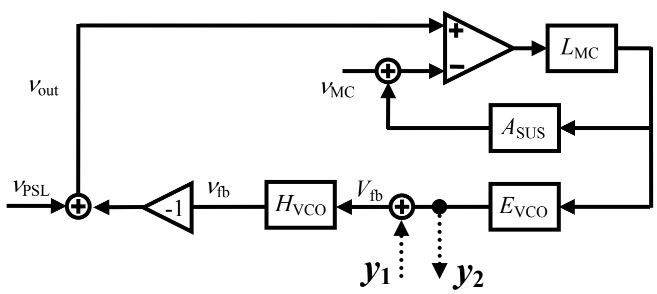

Fig. 6 Transfer function to obtain the coefficient.

In summary, in order to obtain the spectrum of the frequency noise of the light coming out of the PSL and entering the MC, we should
(1) take a spectrum of V_{fb}, that is, δV_{fb},
(2) measure a transfer function, $T_{y1\to y2}$, and
(3) Calculate $H_{\mathrm{VCO}}\, \delta V_{\mathrm{fb}} / T_{y1\to y2}$.

This method is significantly simple and stable compared with the previously-used method.