# Investigation of Suspension of Compensator Plate in ITM Reaction Chain

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## 1. Introduction.

It has been agreed as a working baseline that the Compensator Plate (CP) of the Active Thermal Compensation (ATC) system should be the ultimate mass of the Input Test Mass (ITM) reaction chain to save space and design effort. This agreement has been captured in RODA M040005-00-Y. The working assumption is that the CP will be the same size as a sapphire ITM, but made of silica. Thus its mass is 22 kg. To keep the overall design of the ITM reaction chain similar to the quadruple suspension of the ITM itself, it is further proposed that the penultimate mass (directly above the CP) should be 58 kg in mass, and thus the total load on the blades is kept the same. (ref. P Willems, Design of a suspended compensator plate 1.doc). In this way the design of the top two masses in the chains, and all the blade parameters, can be common.

In this document we address the resulting behaviour of the reaction chain in terms of mode frequencies, coupling and damping.

## 2. Suspension parameters.

This analysis was carried out starting with the current ETM/ITM (henceforth ETM) quadruple suspension design, which reflects recent changes in length as captured in "Investigation of Wire Lengths…" T040028-00-R, 26th Jan. 2004. The wire thicknesses have also been updated since that document was produced. In addition the spacing of the fibres in the longitudinal direction in the final stage (wires in the case of a reaction chain), i.e. between penultimate and test mass, was considered a variable, since that spacing is currently under investigation while the ear design is developed for the ETM. The default setting for this parameter was taken as 0.015 m for the half-spacing – since this is the working value which the ear designers are currently considering. The full parameter set used in the MATLAB model is given in Appendix A.

The behaviour of the current ETM model in terms of frequencies, coupling of modes and damping was used as a baseline against which to compare the behaviour of the reaction chain model with its 58 kg penultimate mass and 22 kg ultimate mass. The parameters which were changed between the two models were the two lowest masses and also the material (silica to steel) and thickness of the "wires" in the final stage. The 58 kg mass has been taken to have the same dimensions as the CP itself, and thus a “fictitious” average density has been chosen for the purposes of the analysis, around 5800 kg $m^{-3}$, corresponding for example to steel with suitable holes. If the behaviour with such a mass is shown to be acceptable, further refinement of the details can be done at a later stage and the model rechecked, using the placement of holes to get close to the moments of inertia of this simplified mass.

## 3. Results

### 3.1 Frequencies of modes

As expected, the highest vertical and roll modes are significantly raised due to changing from silica to steel wire in the final stage. This of course would also be the case for the reaction chain in the ETM, and it does not materially affect the coupling or damping of other modes. In general other modes are changed slightly, the most significant being the two middle pitch modes which are raised in the CP model. A full summary of the modes (as calculated using the MATLAB model) is given in table 1 below. The pitch modes are highlighted in bold.

| |
|---|
| longpitch1: [**0.383** 0.441 0.987 **1.43**] |
| longpitch2: [**1.84** 1.99 3.42 **3.63**] |
| yaw: [0.666 1.40 2.42 3.07] |
| transroll1: [0.447 0.841 1.00 2.02] |
| transroll2: [2.78 3.56 3.96 12.5] |
| vertical: [0.685 2.73 4.40 8.76] |

| |
|---|
| longpitch1: [**0.373** 0.462 0.823 **1.70**] |
| longpitch2: [1.90 **2.39** 3.41 **3.63**] |
| yaw: [0.685 1.31 2.17 2.96] |
| transroll1: [0.469 0.832 0.838 1.94] |
| transroll2: [2.80 3.56 3.97 19.5] |
| vertical: [0.687 2.75 4.42 13.7] |

Table 1: Mode frequencies in Hz for the current ETM design (above) and the proposed CP design (below).

### 3.2 Coupling of modes and damping.

The coupling and damping behaviour was investigated by running the ETM model with the standard active control law and gains set to give settling times less than 10 seconds, and then changing the parameters as described in section two and comparing the resultant transfer functions and impulse curves.

In general coupling stays good and for all directions settling times remain under 10 secs without changing gain. The most significant difference is seen in pitch behaviour. Figure 1 shows the pitch transfer function for ETM and CP models. We see that for the CP the third pitch mode at around 2.4 Hz is not well damped compared to other modes (the second mode at 1.7 Hz is heavily damped and not visible). The settling times corresponding to these two curves are 5.2 s and 9.5 s for the ETM and CP models respectively. Thus even with this high Q and relatively poorly coupled mode the requirement on settling time can be satisfied. The isolation is also significantly less for the CP – however its isolation requirements are expected to be orders of magnitude less than for an ETM.

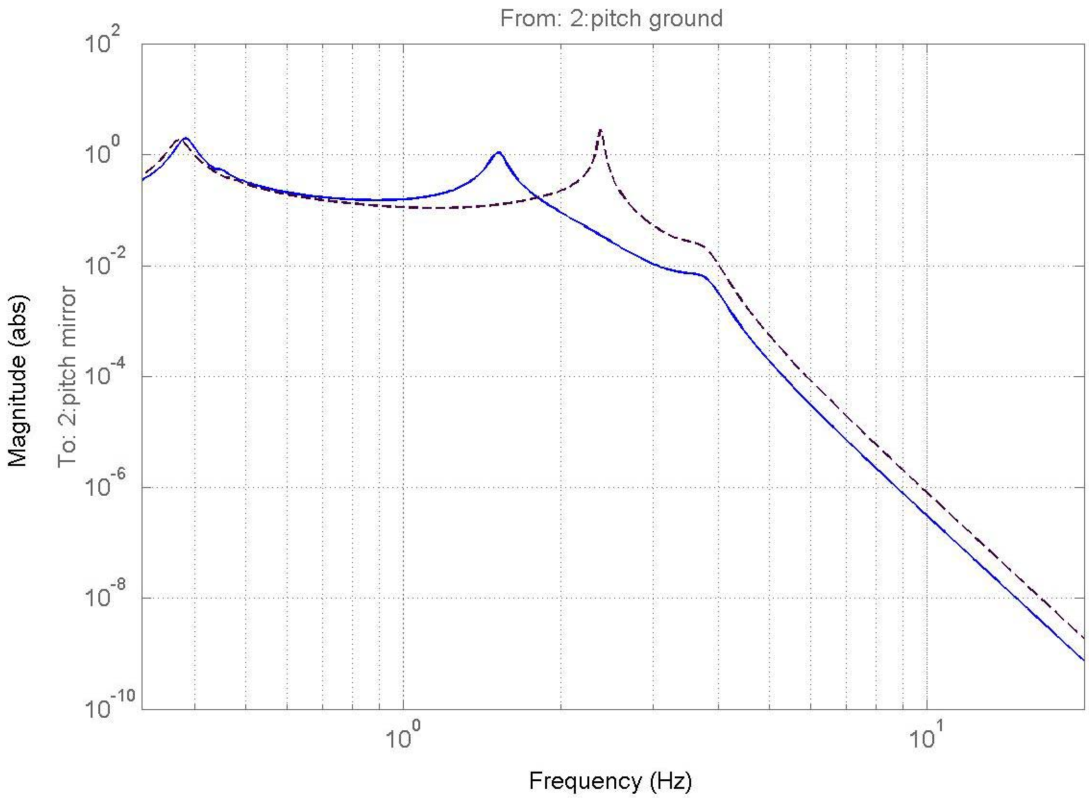


Figure 1. Pitch transfer functions for ETM (blue, solid line) and CP (black, dotted line)

Better coupling of pitch modes in the CP design could be achieved if required by decreasing the spacing (in the longitudinal direction) of the wires in the final stage, set at 0.015 m half-spacing for the curves shown. There is no "in principal" reason why this parameter should be the same for the ETM and CP suspensions.

The only other direction for which the transfer functions show significant differences is for the roll modes, where for the CP one of the modes is noticeably more poorly damped. However again the settling time is acceptable. For other directions the differences between the ETM and CP behaviour are less marked.

We note that the use of wire in this suspension implies that its suspension thermal noise is much more than that of an ETM. However this is not expected to cause any problems since the noise requirements for the CP (TBD) will be considerably relaxed from those of an ETM.

## 4. Conclusion

It has been proposed that the compensator plate (CP) is hung as the ultimate mass in the ITM reaction chain, with the CP being a 22kg silica mass of the same dimensions as a sapphire test mass. The penultimate mass would be made of metal with a mass of 58 kg. We have investigated the behaviour of such a chain in terms of mode frequencies, coupling of modes and damping, and conclude that this is a viable solution. This analysis has been done using a simplified model of penultimate mass. As the design is developed, further checks on the expected behaviour should be carried out. It is noted that the spacing of the wires supporting the CP can be varied if required for optimizing performance.

## 5. Consideration of lighter CP

Since this document was written, it has been proposed by Phil Willems that a lighter mass should be used for the compensator plate – see reference
http://www.ligo.caltech.edu/~willems/Web%20pages/Thermal%20Compensator%20System/Design%20of%20a%20suspended%20compensator%20plate%202.doc
The basic argument is that a thinner plate would be superior in performance, by introducing less thermal lensing into the interferometer to be corrected. It would also be much less expensive. He proposes to use an 11 kg CP (dimensions: thickness 6.5 cm, diameter 31.4 cm) with a ~69 kg penultimate mass to keep the overall sum of these masses the same as previously considered. I have analysed this combination in a quadruple suspension using the MATLAB model, with a penultimate mass of the following dimensions: thickness 13 cm, diameter 31.4 cm, and suitable density to give the appropriate mass ( around 6860 kg/m$^3$), corresponding to steel with holes.

The most significant change in behaviour is that the pitch mode associated with the 11 kg mass on the significantly heavier penultimate mass is rather more decoupled from the other pitch modes than with the previous design. This mode could still be damped by overdamping the other modes. Figure 2 below shows bode magnitude plots for pitch, where blue is 40kg/40kg penultimate and test mass as for an ETM suspension (with silica fibres), green is 58/22 CP suspension and turquoise is 69/11 CP suspension (both with steel wires). Local control gain was increased by factor of 10 from blue to turquoise, (green in between) to get damping times less than 10s in all cases. To get these plots the spacing of wires on test mass was also changed to improve coupling (reduced to 1 cm full separation for the 69/1l model compared to 3 cm for 40 /40 model and 1.6 cm for the 58/22 model). The mode which is less well damped in the 69/11 model can be seen at ~0.8 Hz .
The conclusion from this analysis is that an 11 kg CP is acceptable from a dynamics/control perspective. A RODA on the CP design is currently being revised to reflect this new size (M040005-01).

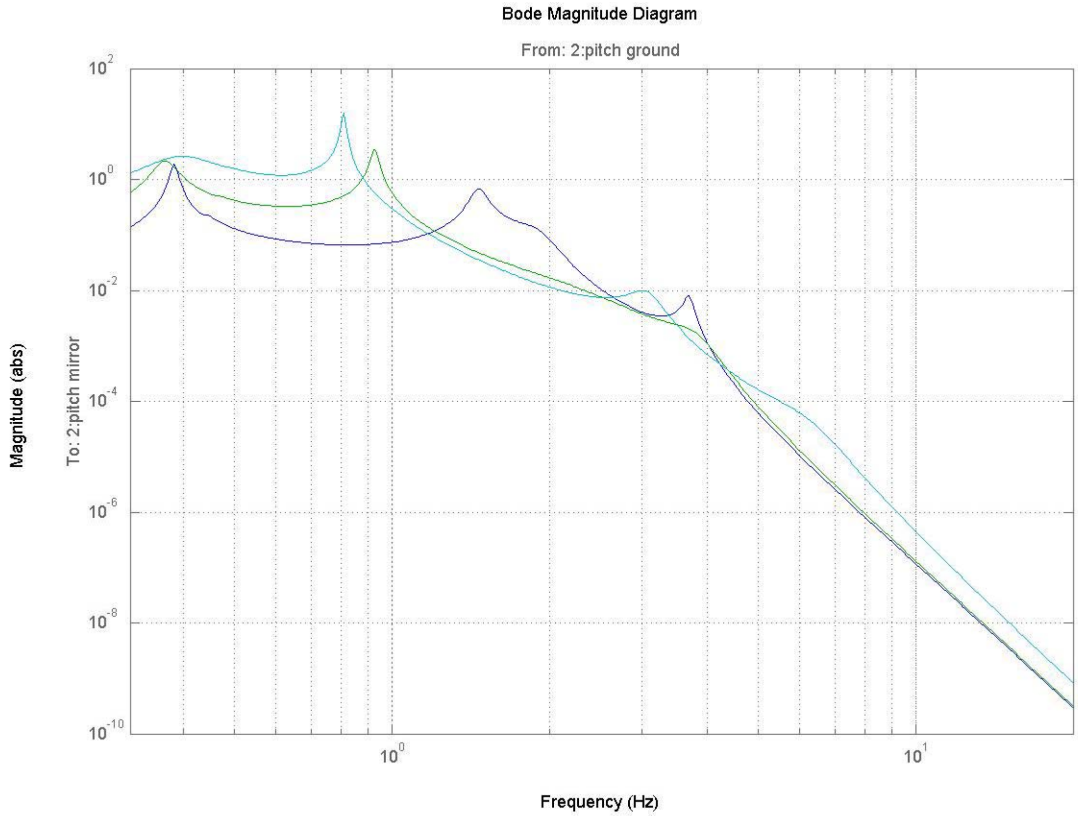


Figure 2. Pitch bode magnitude plots for two sizes of compensator plates compared to ETM suspension. Blue is 40kg/40kg penultimate and test mass (ETM suspension), green is 58/22 and turquoise is 69/11 corresponding to 22 kg and 11 kg compensator plates respectively. Local control gain increased by factor of 10 from blue to turquoise, (green in between) to get damping times less than 10s in all cases.

**Appendix A**

List of parameters for ETM model.

pend =

```
     g: 9.8100
    nx: 0.1300
    ny: 0.5000
    nz: 0.0840
  denn: 4000
    mn: 21.8400
   Inx: 0.4678
   Iny: 0.0436
   Inz: 0.4858
    ux: 0.1300
    uy: 0.5000
    uz: 0.0840
  den1: 4000
    m1: 21.8400
   I1x: 0.4678
   I1y: 0.0436
   I1z: 0.4858
    ix: 0.1300
    ir: 0.1570
  den2: 3980
    m2: 40.0660
   I2x: 0.4938
   I2y: 0.3033
   I2z: 0.3033
    tx: 0.1300
    tr: 0.1570
  den3: 3980
    m3: 40.0660
   I3x: 0.4938
   I3y: 0.3033
   I3z: 0.3033
    ln: 0.4450
    l1: 0.3040
    l2: 0.3420
    l3: 0.6000
   nwn: 2
   nw1: 4
   nw2: 4
   nw3: 4
```

rn: 5.4000e-004
r1: 3.5000e-004
r2: 3.1000e-004
r3: 2.0000e-004
Yn: 2.2000e+011
Y1: 2.2000e+011
Y2: 2.2000e+011
Y3: 7.0000e+010
lnb: 0.4800
anb: 0.0961
hnb: 0.0045
ufcn: 2.3628
stn: 8.9866e+008
intmode_n: 73.5303
l1b: 0.4200
a1b: 0.0583
h1b: 0.0049
ufc1: 2.5555
st1: 8.9994e+008
intmode_1: 104.5764
l2b: 0.3400
a2b: 0.0500
h2b: 0.0045
ufc2: 2.1106
st2: 7.9192e+008
intmode_2: 146.5517
dm: 0.0010
dn: 0.0010
d0: 0.0010
d1: 0.0010
d2: 0.0010
d3: 0.0010
d4: 0.0010
twistlength: 0
d3tr: 0.0010
d4tr: 0.0010
sn: 0
su: 0.0030
si: 0.0030
sl: 0.0150
nn0: 0.2500
nn1: 0.0900
n0: 0.2000
n1: 0.0700
n2: 0.1200
n3: 0.1635

n4: 0.1585
n5: 0.1585
tln: 0.4162
tl1: 0.2768
tl2: 0.3412
tl3: 0.6020
l_suspoint_to_centreofoptic: 1.6363
l_suspoint_to_bottomofoptic: 1.7933

Parameters used in blade optimization program opt.m
alpha = 1.53; %shape factor
ye = 186e9 (Young's modulus of maraging steel)

List of parameters for CP model *where different from above* (this is for the 22 kg CP discussed in sections 1 to 4).

ix: 1.3000e-001
ir: 1.5700e-001
den2: 5760
m2: 5.7985e+001
I2x: 7.1464e-001
I2y: 4.3898e-001
I2z: 4.3898e-001
tx: 1.3000e-001
tr: 1.5700e-001
den3: 2200
m3: 2.2147e+001
I3x: 2.7295e-001
I3y: 1.6767e-001
I3z: 1.6767e-001
r3: 1.6000e-004
Y3: 2.2000e+011

List of parameters for CP model with 11 kg CP (as in section 5)

g: 9.8100
nx: 0.1300
ny: 0.5000
nz: 0.0840
denn: 4000
mn: 21.8400
Inx: 0.4678
Iny: 0.0436
Inz: 0.4858
ux: 0.1300
uy: 0.5000

uz: 0.0840
den1: 4000
m1: 21.8400
I1x: 0.4678
I1y: 0.0436
I1z: 0.4858
ix: 0.1300
ir: 0.1570
den2: 6860
m2: 69.0584
I2x: 0.8511
I2y: 0.5228
I2z: 0.5228
tx: 0.0650
tr: 0.1570
den3: 2200
m3: 11.0735
I3x: 0.1365
I3y: 0.0721
I3z: 0.0721
ln: 0.4450
l1: 0.3040
l2: 0.3420
l3: 0.6000
nwn: 2
nw1: 4
nw2: 4
nw3: 4
rn: 5.4000e-004
r1: 3.5000e-004
r2: 3.1000e-004
r3: 1.1300e-004
Yn: 2.2000e+011
Y1: 2.2000e+011
Y2: 2.2000e+011
Y3: 2.2000e+011
ufcn: 2.3300
ufc1: 2.4800
ufc2: 1.8100
dm: 0.0010
dn: 0.0010
d0: 0.0010
d1: 0.0010
d2: 0.0010
d3: 0.0010
d4: 0.0010

twistlength: 0
d3tr: 0.0010
d4tr: 0.0010
sn: 0
su: 0.0030
si: 0.0030
sl: 0.0050
nn0: 0.2500
nn1: 0.0900
n0: 0.2000
n1: 0.0700
n2: 0.1400
n3: 0.1635
n4: 0.1585
n5: 0.1585
tln: 0.4162
tl1: 0.2768
tl2: 0.3432
tl3: 0.6020
l_suspoint_to_centreofoptic: 1.6382
l_suspoint_to_bottomofoptic: 1.7952

(blade optimisation program not used for uncoupled vertical frequencies of blades ufcn, ucf1 and ufc2 - the values given correspond to those expected for the ETM controls prototype blade design)