LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY

LIGO Laboratory / LIGO Scientific Collaboration

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Some Power Scaling Options for the Lightwave 10 Watt Lasers

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1 Introduction

With the installation of thermal compensation in Initial LIGO, the point design nature in terms of operating power of initial LIGO has been somewhat removed. Hence the interferometers may be able to handle increased input power. This paper addresses some potential scaling ideas for the 10-Watt Lightwave Electronics MOPA Lasers. To do this I have put together a numerical model that solves the one dimensional laser gain saturation problem. I compare this model to data taken from the LASTI PSL laser. I also make some predictions about the likely performance if the power of the master laser was significantly increased. I also consider the likely power increase if a commercially available amplifier was used to amplify the output of the laser.

2 Theory

2.1 Basic Theory

The saturable gain coefficient of a laser-amplifying medium is given by

$$g = \frac{g_0}{1 + I / I_s} \qquad (1)$$

Where g_0 is the laser small signal gain coefficient, I is the intensity of the light incident on the gain medium and I_s is the gain medium saturation intensity, which for a four level laser system is

$$I_s = \frac{h\nu}{\sigma_{21}\tau_f}$$

Where h is Planck's constant, ν is the frequency of the light, σ_{21} is the stimulated emission cross section and τ_f is the upper state lifetime of the gain medium.

The amplification of the light is given by

$$\frac{dI(x)}{dx} = \frac{g_0 I(x)}{1 + I(x)/I_s} - \alpha(x)I(x) \tag{2}$$

Where α is the loss coefficient of the material

In the small signal limit ie when no saturation is occurring Equation 2 is solvable analytically and results in

$$\frac{Iout}{Iin} = Exp[(g_0 - \alpha)l] = G_0 \tag{3}$$

Unfortunately in the case where saturation occurs Equation 3 cannot be solved analytically and numerical methods must be use

2.2 Numerical Model

To determine the power gain of a saturated rod we need to solve the equation of single passed laser gain medium we need to perform the integration

$$I(l) = \int_0^l [\frac{g_0 I(x)}{1 + I(x)/I_s} - \alpha(x)I(x)]dx \tag{4}$$

This is relatively simple to solve numerically by dividing the gain medium up into N segments. The segments need to be small enough such that intensity in segment j can be accurately described by

$$I(x_j, t_n) = I(x_{j-1}, t_{n-1}) + \frac{g_0 \Delta x I(x_{j-1}, t_{n-1})}{1 + I(x_{j-1}, t_{n-1})/I_s} - \alpha I(x_{j-1}, t_{n-1}) \tag{5}$$

Where Δx is the spacing between the grid points. Also t_n-t_{n-1}=$\Delta x/(nc)$, where n is the refractive index of the laser material and c is the speed of light in a vacuum. With a single pass amplifier this is quite trivial to do and the beam is simply traced through rod. With a double passed rod the situation becomes a little more complex as it is necessary to take into account the intensity of the beam traveling in the opposite direction. Fortunately the gain in a Nd:YAG laser system is homogeneous so the saturating intensity is simply the sum of the two fields. This ignores the effects of standing waves in gain medium. At present I do not know how to take this into account.

The two directional model considers a wave traveling though the gain medium but takes into account the gain saturating effects of the counter propagating wave that occupies the same region of space, so equation 5 becomes

$$I_{forward}(x_j,t_n)=I_{forward}(x_{j-1},t_{n-1})+\frac{g_0\Delta x I_{forward}(x_{j-1},t_{n-1})}{1+(I_{forward}(x_{j-1},t_{n-1})+I_{backward}(x_{j+1},t_{n-1})/I_s}-\alpha I_{forward}(x_{j-1},t_{n-1})$$

$$I_{backward}(x_j,t_n)=I_{backward}(x_{j+1},t_{n-1})+\frac{g_0\Delta x I_{backward}(x_{j+1},t_{n-1})}{1+(I_{forward}(x_{j-1},t_{n-1})+I_{backward}(x_{j+1},t_{n-1})/I_s}-\alpha I_{backward}(x_{j+1},t_{n-1})$$

Naturally with this algorithm it is necessary to run the code for longer than it would take for the wave to double pass the rod. This is because the first wave to hit the laser rod does not see the gain saturated by itself at a later time. In practice it has been found necessary to allow the code to run for 4 roundtrip times.

A model was written in Matlab to numerically solve these equations. Figure 2.1 and Figure 2.2 show some of the results of these calculations

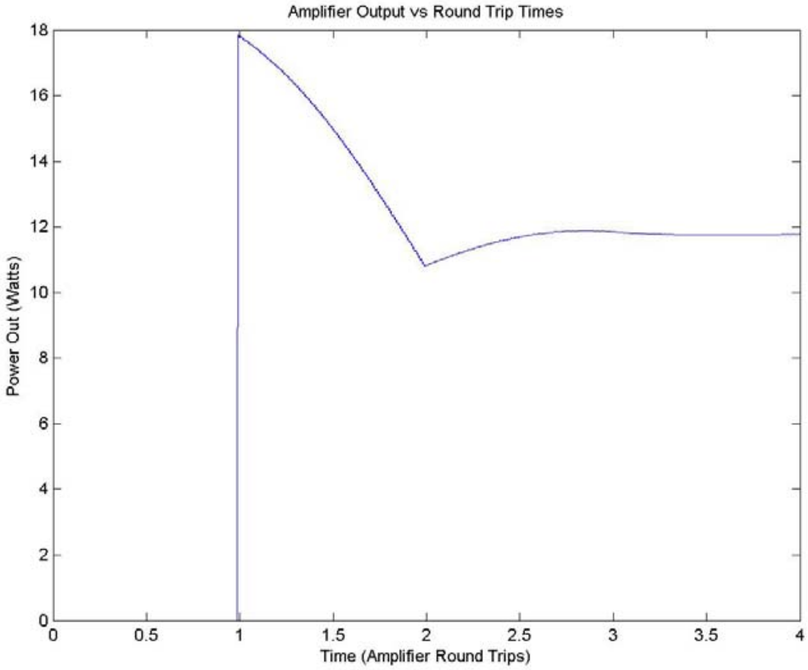


Figure 2-1 Predicted time response of the amplifier during turn-on of the master field. This assumes an abrupt turn on of the master field

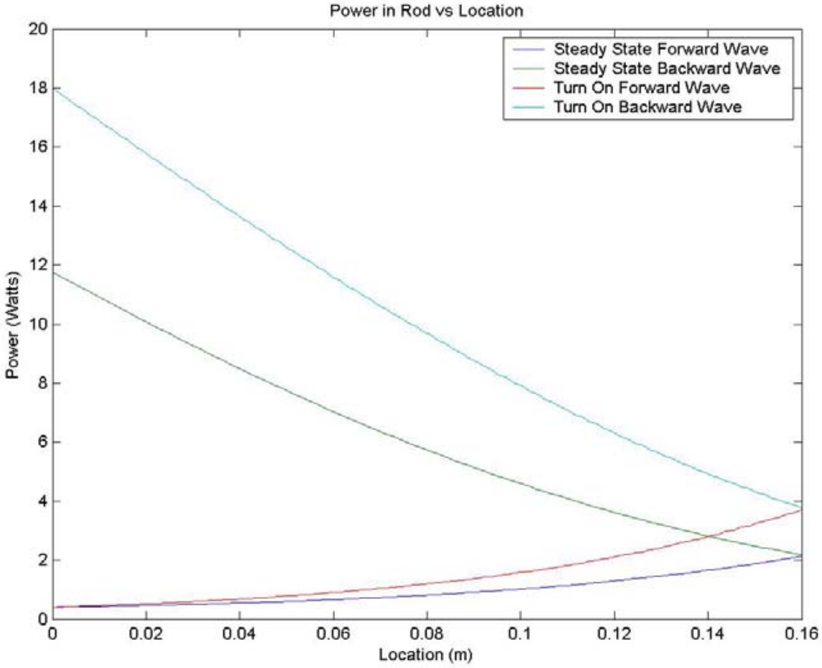


Figure 2-2 This figure illustrates the difference in the power profile throughout the rod between one round trip time after master field turn on and the steady state condition

3 Results from the LASTI Lightwave 10 Watt MOPA

To test the model we measured the saturation properties of the LASTI 10 Watt laser. The power transmission from the NPRO to the input of the amplifier was measured and found to 80%. The transmission through the Faraday isolator was found to be 90%. To investigate the power saturation properties of the amplifier we placed neutral density filters immediately in front of the NPRO which. Using the filters, the master power incident on the amplifier was varied from 2.5 mW to 400 mW. The results of this are listed below with the results from the theory. The parameters that were varied to achieve these fits were small signal gain (G_0=1250) and spot size in amplifier (w=0.41 mm)

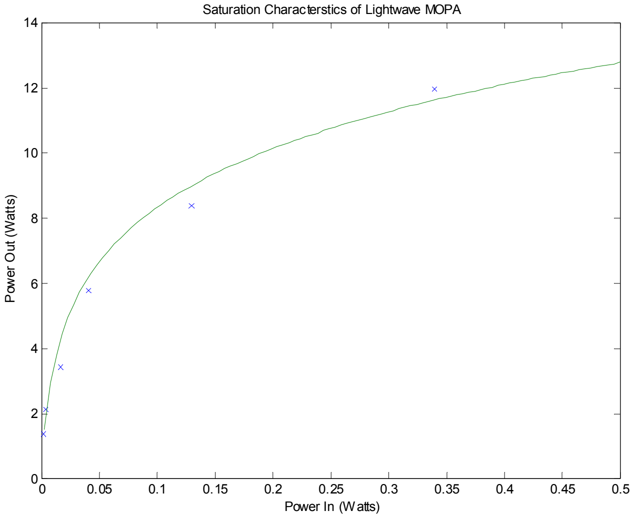


Figure 3-1 Power out versus the power in with linear/linear axis

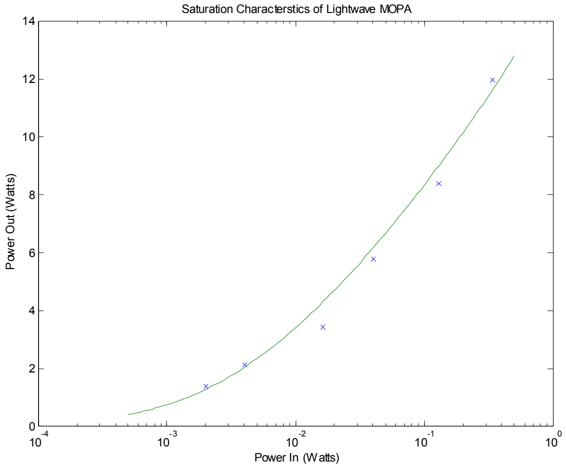


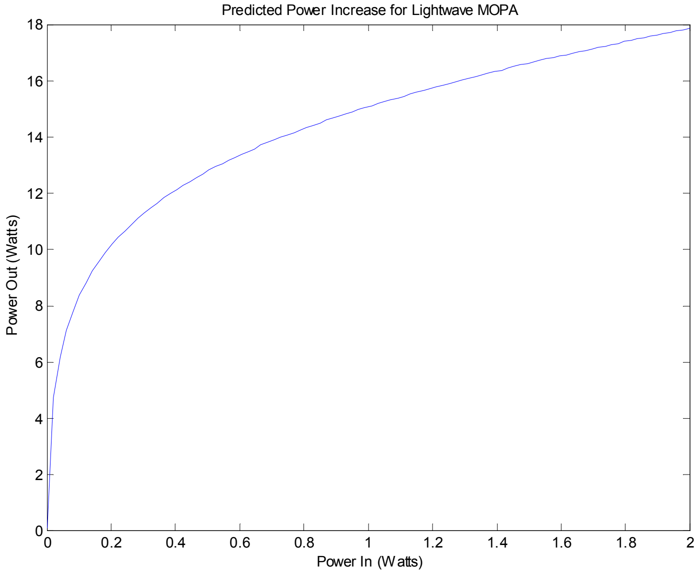
Figure 3-2 Power out versus power with log/linear axes

4 Power Upgrade Options for the Lightwave 10 Watt Laser

In this section I will use the theory to predict the output of the laser system when the master laser is upgraded from 400 mWatts to 2 Watts. This can be accomplished by swapping the current Lightwave master laser to a 2-Watt Innolight Mephisto. The second power upgrade that is considered is placing an off the shelf amplifier from CEO lasers. The third power upgrade option considered is to term the Lightwave 10 Watt MOPA into an injection-locked oscillator

4.1 Master Laser Power Upgrade

The theory allows us to predict the effect of increasing the master laser power. This is shown in the following figure, which shows the predicted power immediately outside the laser amplifier. Using a 2 Watt laser and assuming that 1.6 Watts reaches the amplifier, the theory predicts an output of 16.9 Watts. Assuming that 90% of this power makes it through the Faraday, the output of the MOPA in this configuration will be 15.2 Watt.



4.2 Adding Another Power Amplifier

The case studied here is the likely power increase if we used a commercially available pump module to amplify the output of the Lightwave 10W Lasers. The module I chose to model are available from CEO laser corporation and have the following characteristics

Crystal Length	67 mm
Crystal Geometry	Rod with flat/flat faces
Crystal Diameter	2 mm
Output Power	35 W / 50 W
Test cavity output coupler reflectivity	80 %

Figure 4-1 Properties of the Cutting Edge Optronics RB series pump modules

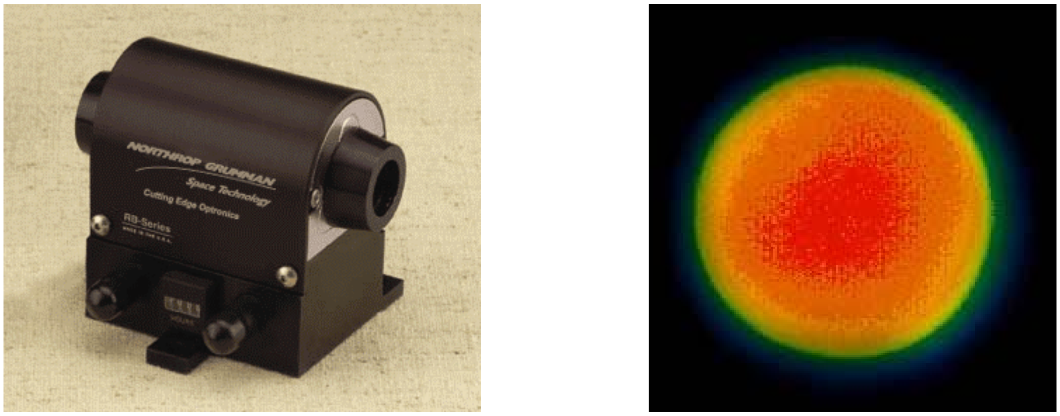


Figure 4-2 Pump module from Cutting Edge Optronics (Left) with fluorescence profile (Right)

The output power of the amplifier was obtained by installing the laser head in a multimode laser cavity whose output coupler was 80 %. The power extraction measurement was done by Cutting Edge Optronics as is listed in the specifications. To evaluate the likely output of these modules when they are used in an amplifier it is necessary to reverse engineer out the small signal gain.

This was done by realizing that the power levels in various parts of the laser resonator are connected as shown in Figure 4-3. The small signal gain of the amplifier was then obtained by iteratively entering various values of the small signal gain into the gain saturation model until the correct output power from the rod was achieved for power incident in the rod. For finding the small signal gain coefficient the mode area was set to the same size as the rod, which for a multimode resonator is a reasonable approximation.

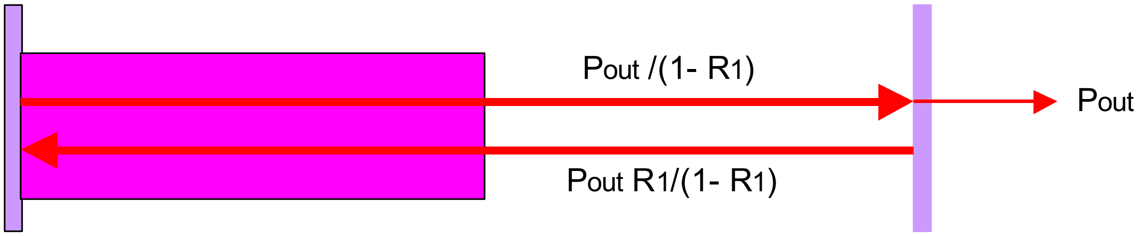


Figure 4-3 Modeling the laser resonator as an amplifier

The above method was chosen because the using the uniform saturation formula was found to over predict the maximum power extractable from the resonator at the 10 % level. By this method I mean simply applying the formula

$$g_0 l = \tfrac{1}{2}\ln\left(\frac{1}{R_1 R_2}\right)\left(\frac{2P_{out}}{AI_s(1-R_1)}+1\right)$$

Where R_1 is the reflectivity of the output coupler and R_2 can be used to represent losses in the cavity. It was realized that this formula was the cause of the inaccuracy because when the losses were set to zero ie $R_2 = 1$ and the R_1 was set to 0.99 and P_{in} was set to be very high lead to the correct output. These values reflect the case where the gain is very highly saturated and the field though the cavity is very close to uniform

The result of the amplifier output are shown as Figure 4-4

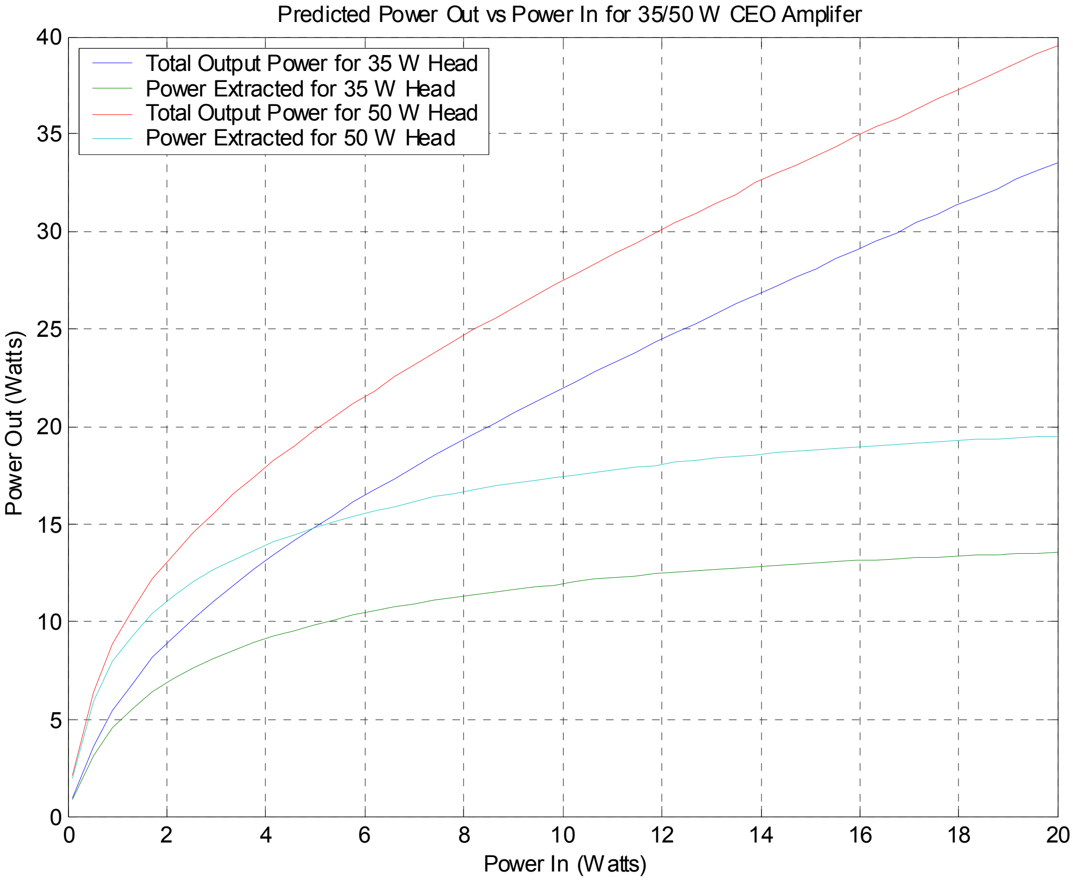


Figure 4-5 Predicted Ideal Power Out of Amplifier

It should be noted in this case that such an amplifier is known to introduce significant depolarization due to thermally induced birefringence. This should be able to be fixed using a 45 degree single pass Faraday rotator in the retro-reflection path when the amplifier is double pass. This is likely to reduce the power gain by 10%.

4.3 Converting the Lightwave 10 Watt Amplifier to an Injection-Locked Oscillator

To model the amplifier as an injection-locked amplifier a single pass code was used. This code was used to numerically find the self-consistent circulating power in a resonator. Virtually the same approach was used as in Section 4.2. The results are shown below:

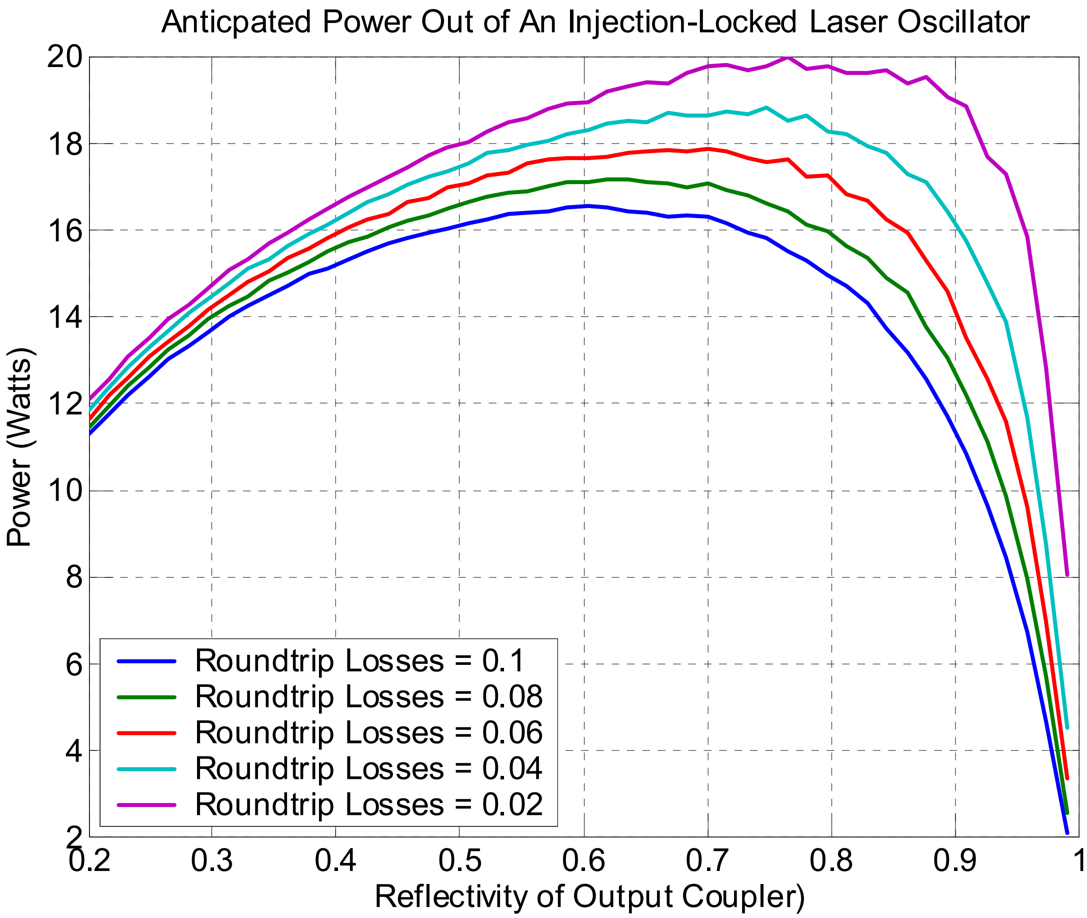


Figure 4-6 The predicted output power from the a Lightwave 10 Watt amplifer with a small signal double pass gain of 1250 and a mode radii of 0.41 mm. (Same as the LASTI Lightwave 10 W laser)

The locking range of such an oscillator is given by:

$$f_{locking} = \frac{T.FSR}{\pi}\sqrt{\frac{P_{master}}{P_{slave}}}$$

Where T is the transmission of the output coupler, FSR is the free spectral range of the cavity, P_{master}, P_{slave} is the power of the master laser and the slave laser respectively. For an output power of 20 Watt, master laser power of 0.4 W, roundtrip length of cavity = 0.5 m and an output coupler transmission of 20% gives a locking range of 5.5 MHz.

Appendix 1. Matlab function that calculates the power from a saturable double passed optical amplifier

```
D:\PSL\saturated_laser_gain.m                                              Page 1
April 3, 2004                                                         6:33:06 PM
function res=saturated_laser_gain(power_in,w,Go,alpha,l)

% saturated_laser_gain(power_in,w,Go,alpha,l)
% power_in is the power incident on the gain medium
% w is the spot size (radii) of the incident laser mode
% alpha is the loss coefficent where total round trip loss
% ie loss =exp(-2 l alpha)
% l is the length of the gain medium

% This equation evaluates the power output from an optical amplifier given
% the mode area and the small signal gain. The optical parameters of the gain medium are ↙
set for Nd:YAFG
% Scattering losses can also be added. It solves the laser gain equation as described in ↙
Koechner P 87.
% It does this by breaking the crystal up into N sections and solving the
% equation numerically in each section. It solves these equations for
% double passing the laser medium. It does this by proogating a field
% through the gain medium



% Laser Gain Medium Parameters and Constants

c=3.0e8;            % Speed of light in a vacuum
h=6.626176e-34;     % Planck's Constant
sigma21=6.5e-23;    % Cross section for the lasing transition
tf=230e-6;          % Fluencence lifetime of the upper lasing state of Nd:YAG
lambda=1.064e-6;    % Wavelength of the laser

A=pi*w^2;                        % The area of the laser mode
Iin=power_in/A;                  % The intensity of the laser mode
Is=h*(c/lambda)/(sigma21*tf);    % Saturation Intensity for Nd:YAG
go=log(Go)/(2*l); %              % Single pass small signal gain coefficient

% Calculation Paramaters
n=50;               % Number of divisions
delta_x=l/n;        % Length of individual segments



Iinbound=zeros(n);
Ioutbound=zeros(n);
Iinbound(1,1)=Iin;

for ii=1:8*n        % This is calculates the field after 8 round trip times
hold_Iinbound=Iinbound+go*delta_x*Iinbound./(1+(Iinbound+Ioutbound)/Is)-alpha*delta_x*Iin ↙
bound;
hold_Ioutbound=Ioutbound+go*delta_x*Ioutbound./(1+(Iinbound+Ioutbound)/Is)-alpha*delta_x* ↙
Ioutbound;

if (abs(hold_Iinbound-Iinbound)<100*eps) or (abs(hold_Ioutbound-Ioutbound)<100*eps)
    error('Significant Numerical Errors Exist')
end

hold_end=hold_Iinbound(n);
Iinbound(2:n)=hold_Iinbound(1:n-1);
Ioutbound(1:n-1)=hold_Ioutbound(2:n);
```

```
D:\PSL\saturated_laser_gain.m                                Page 2
April 3, 2004                                           6:33:06 PM
```

```
Ioutbound(n)=hold_end;
Inbound(1,1)=Iin;
clear hold_Iinbound hold_Ioutbound hold_end

end

res=Ioutbound(1)*A;
```

Appendix 2. The program used to calculate the small signal gain given the cavity parameters, output power and the rod diameter

```
D:\PSL\Go_find_spatial.m                                      Page 1
April 3, 2004                                           6:40:25 PM

function res=Go_find_spatial(pout,R1,R2,w,l)
% gol_fine(pout,R1,R2,w)
% A function to calculate the small signal gain of a medium given we know
% the power extracted from the gain medium when it is placed in a multimode
% cavity of known parameters. It is basically written to allow the reverse
% engineering of a CEO laser head
% This takes into account the spatial gain inhomogeneties in calculating
% gol
power_error=0.01; % This is the allowable error in power
A=pi*w^2;
c=3.0e8;
h=6.626176e-34;
sigma21=6.5e-23;    % Cross section for the lasing transition
tf=230e-6;          % Fluencence lifetime of the upper lasing state
lambda=1.064e-6;    % Wavelength of the laser


Pcirc_out=pout/(1-R1);
Pcirc_in=R1*pout/(1-R1);
alpha=-log(R2)/(2*l);

Is=h*(c/lambda)/(sigma21*tf);

go_initial=(2*pout/(A*Is*(1-R1))+1)*log(1/(R1*R2))/2;

Go_initial=exp(go_initial*2)
Go_lower=Go_initial*0.66;
Go_upper=Go_initial*1.5;
Go=Go_initial
power_out=saturated_laser_gain(Pcirc_in,w,Go_initial,alpha,l);

delta_power=power_out-Pcirc_in-pout;
nn=0;

while abs(delta_power)/pout > power_error

    if delta_power > 0
        Go=(Go+Go_lower)/2;
    else
        Go=(Go+Go_upper)/2;
    end
    power_out=saturated_laser_gain(Pcirc_in,w,Go,alpha,l);
    delta_power=power_out-Pcirc_in-pout;
    nn=nn+1;
end
disp(nn)
res=Go;
```

Appendix 3. Single pass saturable amplifier power gain Matlab code

```
D:\PSL\saturated_laser_gain_sp.m                                      Page 1
April 7, 2004                                                     8:36:58 PM
function res=saturated_laser_gain_sp(power_in,w,Go,alpha,l)
close all
% saturated_laser_gain(power_in,w,Go,alpha,l)
% power_in is the power incident on the gain medium
% w is the spot size (radii) of the incident laser mode
% alpha is the loss coefficent where total round trip loss
% ie loss =exp(-2 l alpha)
% l is the length of the gain medium

% This equation evaluates the power output from an optical amplifier given
% the mode area and the small signal gain. The optical parameters of the gain medium are ↙
set for Nd:YAFG
% Scattering losses can also be added. It solves the laser gain equation as described in ↙
Koechner P 87.
% It does this by breaking the crystal up into N sections and solving the
% equation numerically in each section. It solves these equations for
% double passing the laser medium. It does this by proogating a field
% through the gain medium



% Laser Gain Medium Parameters and Constants

c=3.0e8;            % Speed of light in a vacuum
h=6.626176e-34;     % Planck's Constant
sigma21=6.5e-23;    % Cross section for the lasing transition
tf=230e-6;          % Fluencence lifetime of the upper lasing state of Nd:YAG
lambda=1.064e-6;    % Wavelength of the laser

A=pi*w^2;                       % The area of the laser mode
Iin=power_in/A;                 % The intensity of the laser mode
Is=h*(c/lambda)/(sigma21*tf);   % Saturation Intensity for Nd:YAG
go=log(Go)/l; %             % Single pass small signal gain coefficient

% Calculation Paramaters
n=50;               % Number of divisions
delta_x=l/n;        % Length of individual segments



Iinbound=zeros(n);
Iinbound(1)=Iin;
power_out=zeros(n);

for ii=1:n        % This is calculates the field after a single pass
hold_Iinbound=Iinbound+go*delta_x*Iinbound./(1+Iinbound/Is)-alpha*delta_x*Iinbound;

if (abs(hold_Iinbound(1)-Iinbound(1))<100*eps)
error('Significant Numerical Errors Exist')
end

Iinbound(2:n)=hold_Iinbound(1:n-1);
Inbound(1,1)=Iin;
clear hold_Iinbound
%power_out(ii)=Iinbound(n)*A;

end
```