Static and simple dynamic analysis of BTF blades
Justin Greenhalgh, RAL, May/July 2004

Aim: To compare FEA, blade equations, and measured results of the BTF blades. To extend this to the controls prototype blades in order to arrive at a suitable bend radius.

This continues work reported in T040115, which explained how the parameters were chosen for the BTF blades.

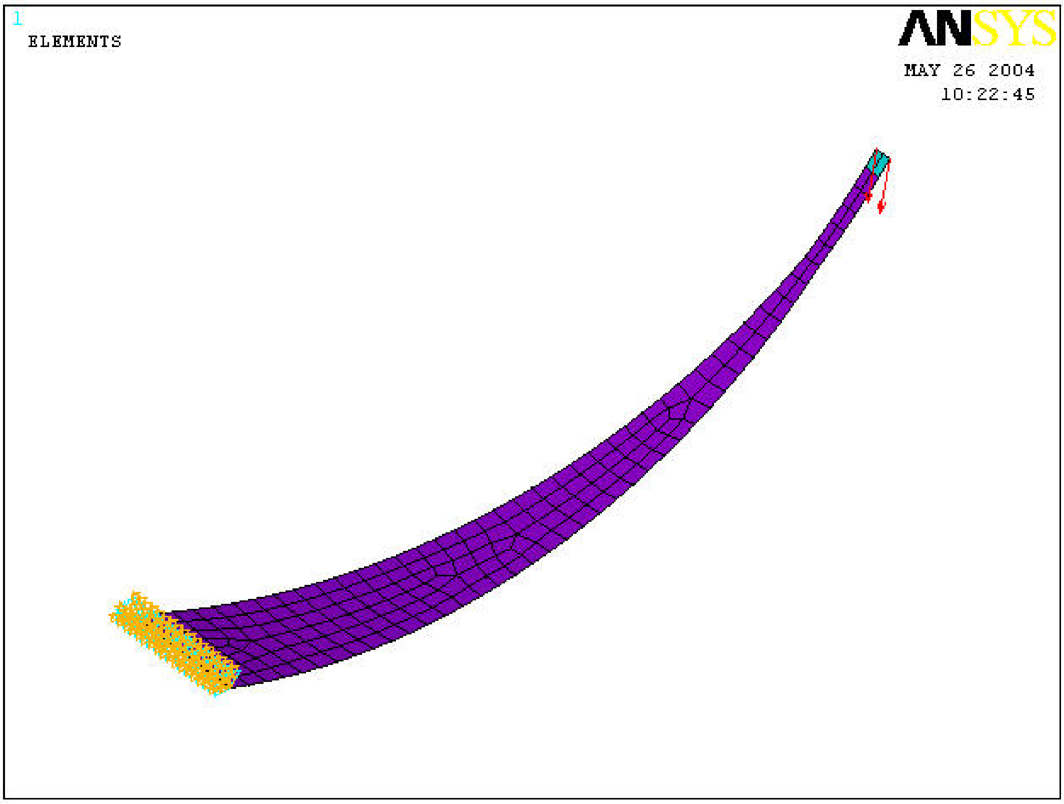
1. Basic model

The dimensions of the model have been taken from the drawings used to produce the BTF blades, reproduced in appendix 2. The blades are near-triangles (not trapezoidal), with a truncated triangular portion and a plain section near the tip of the blade to allow fixing of the wire clamp. The wire break-off point is at the tip of the triangle. Key dimensions are (in metres):

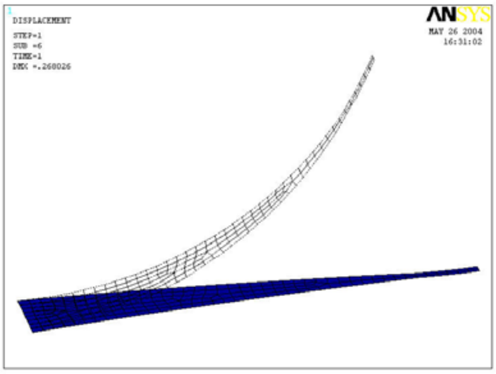
trilength=.48	Length of triangle to wire breakoff point
blength=0.469	Length of blade
rootwidth=0.095	Width at root
tipwidth=0.013	Width of plain portion at tip
bthick=0.0044	Thickness of blade
bendrad=0.4278	Radius of bend at neutral axis of blade (differs from band radius on drawing by half the thickness).
tipload=61.936*9.81	Nominal load is 61.936 kg

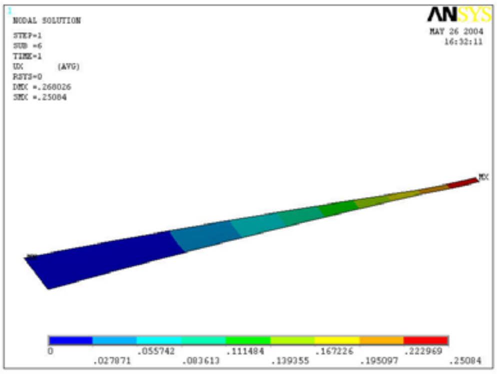
For the reasoning behind this choice of dimensions see T040115.

The basic model uses a cylindrical co-ordinate system to specify the blade shape. It includes a thick portion at the tip, beyond the end of the blade, to simulate the wire clamp and to allow loading at the wire break-off point. This extra piece is shown in blue in the diagram below.

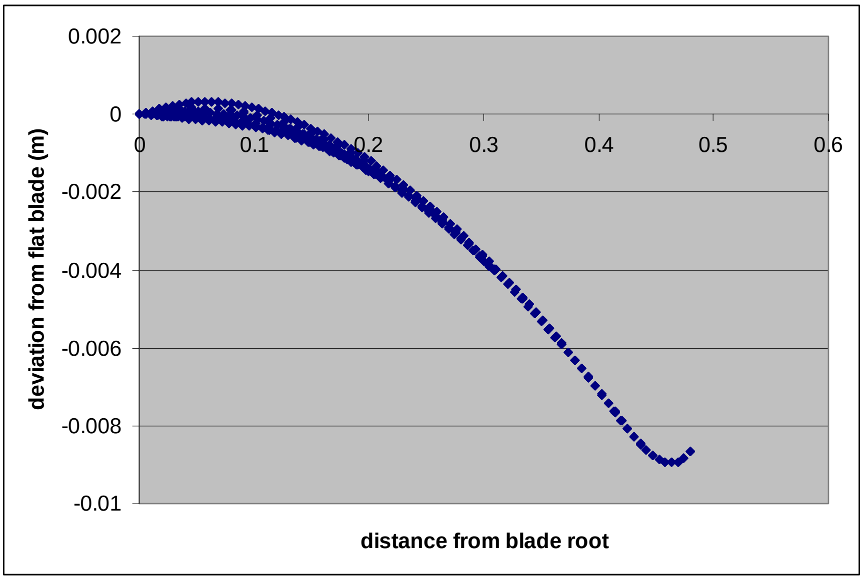


The bend radius is 427.8mm, giving a theoretical undeflected tip height of, from the ANSYS geometry, (428-186)=242mm. ANSYS can be used to calculate the deflection under the nominal load of 61.936 kg. See macro in appendix 1 for a calculation including geometric nonlinearity. The maximum deflection is 250.84mm.





By updating the geometry to reflect the distorted shape and then plotting the co-ordinates of the nodes we can see the distorted shape with respect to a theoretical flat blade:



Note that in the region near the root the blade develops a curvature across its width due to Poisson effects. This is expected in a relatively thin, wide beam. The classic "curl-up" near the tip, due to the fact that the blade is wider than a triangle there, is also clearly to be seen.

2. Comparison with blade equations

This is a pseudo-triangular blade. By that I mean, a blade of a basically triangular shape, and with the load applied at the tip of the triangle, but with the tip cut away and with a rectangular portion near the tip - both changes being made to allow to for the wire clamp. Given this, I would expect the blade equations, if applied with the same young's modulus as the FEA and with alpha=1.5 (see, eg, T030285) should give a deflection slightly higher than the FEA. The FEA will be stiffer because it includes the stiffness of the rectangular portion of the blade near the tip.

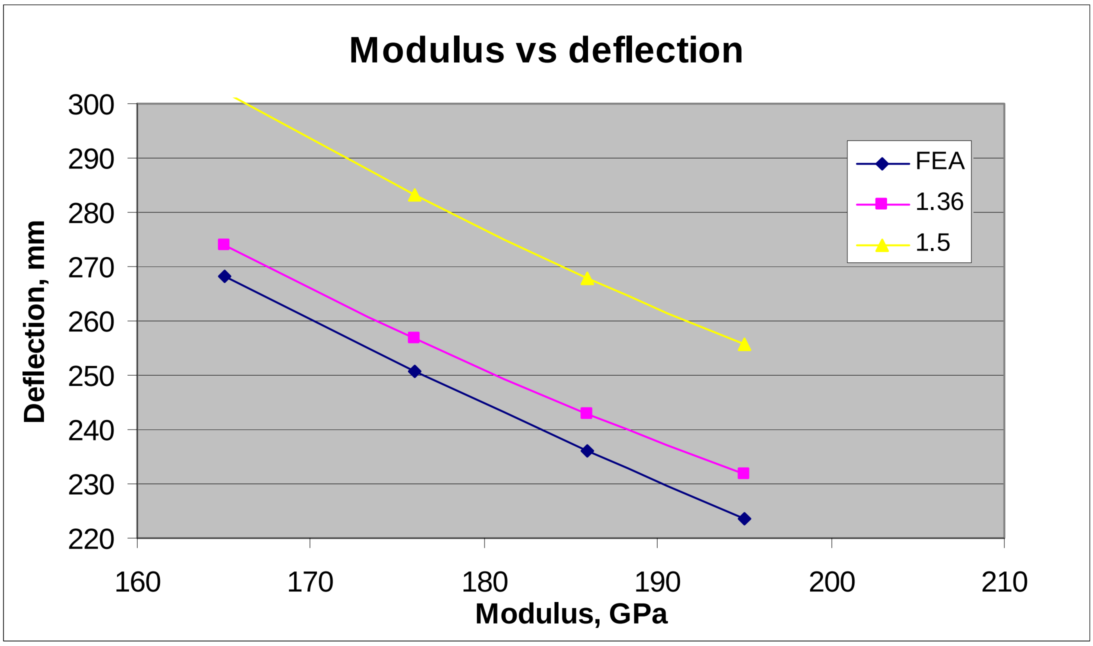
Method	E (GPa)	alpha	Result
Blade eqns	186	1.36	242.9 (design value)
FEA, nonlinear	176	n/a	250.84
Blade eqns	176	1.5	283.1
FEA, nonlinear	186	n/a	236.1
FEA, nonlinear	165	n/a	268.3
FEA, nonlinear	195	n/a	223.7

The exact numbers used in the blade equations fro the first line are shown in this extract from the spreadsheet:

constants	Value	Units
l (length)	0.48	m

a (root width)	0.095	m
h (thickness)	0.0044	m
E (young's modulus)	1.86E+11	Mpa
alpha (shape factor)	1.36	
mt (total mass on spring)	61.936	kg
m (mass of next stage, per spring)	10.95	kg
g (gravitational acceleration)	9.81	m/s^2
elastic limit of Marval 18	1.60E+09	Mpa
calculated values		
I (2nd moment of area)	6.74373E-10	
lambda (tip deflection)	2.429E-01	m
k (spring constant)	2501.911637	n/m
f (uncoupled vertical frequency)	2.40574203	hz
SigmaMAX (max blade stress)	9.51E+08	Mpa
does SigmaMAX exceed elastic limit?	NO	
ratio of elastic limit to SigmaMAX	0.59	
undeflected radius (read from graph)	0.426323713	m

For the blade equations, the result will vary simply as (alpha/E). The FEA result does not quite vary as (1/E). Summarising in a graph:



Interestingly, the "geometrical" value of alpha=1.5 is a long way from the FE results – too much to be explained by the added stiffness of the rectangular portion near the tip. The well-tried value of alpha = 1.36 is much closer.

3. Comparison with experimental results

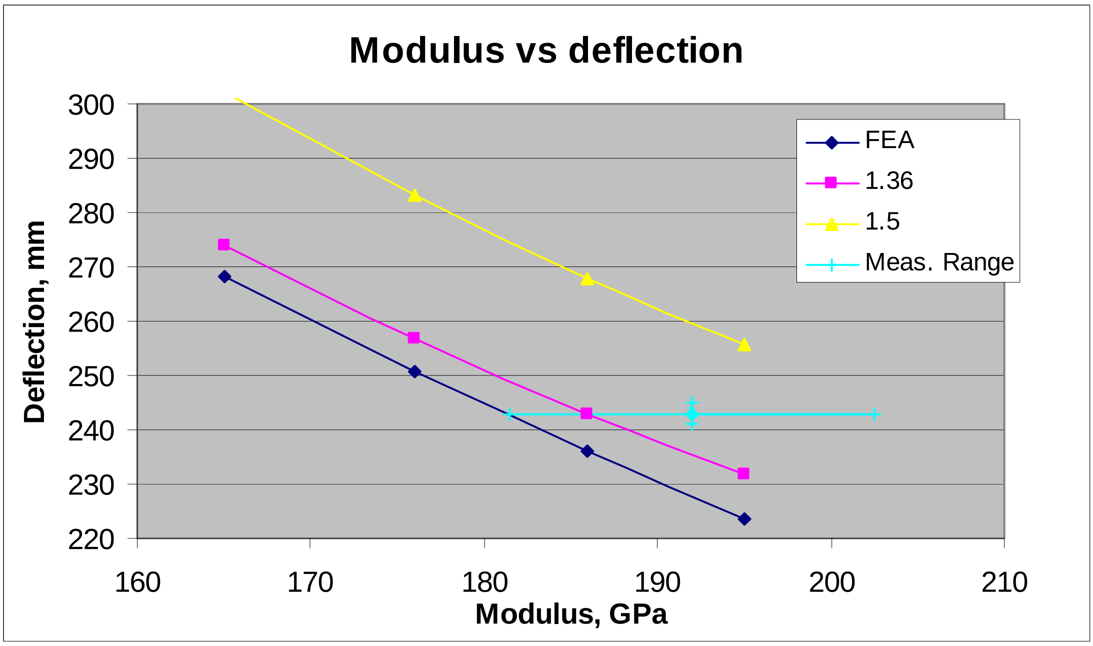
Tests have been made to measure the Young's modulus of samples of material cut from the same plate as the springs.

The results are tabulated below

Samples cut near midplane of sheet			Samples cut near surface of sheet	
1	178		1	187
2	200		2	196
3	192		3	188
4	204		4	197
5	186		5	183
Ave +/- 1SD	192+/-10.5			190.2+/-6.1

We intend to remeasure the moduli of all the samples to check for experimental error, but results taken on a the same machine with a similar method on a cast aluminium alloy were much less spread, suggesting that at most of the spread seen above is a real material variability. If this is the case, it will not be possible to predict the stiffness of a blade with any great accuracy. (The failure stress was between 1766 and 1811 MPa).

A measurement has been made of one blade in the blade test facility, loaded with a mass of 61.478 +/-.035 kg. The deflection was 241+/-2 mm. Scaling this up for the nominal load of 61.936 kg gives a deflection of 243+/-2mm.

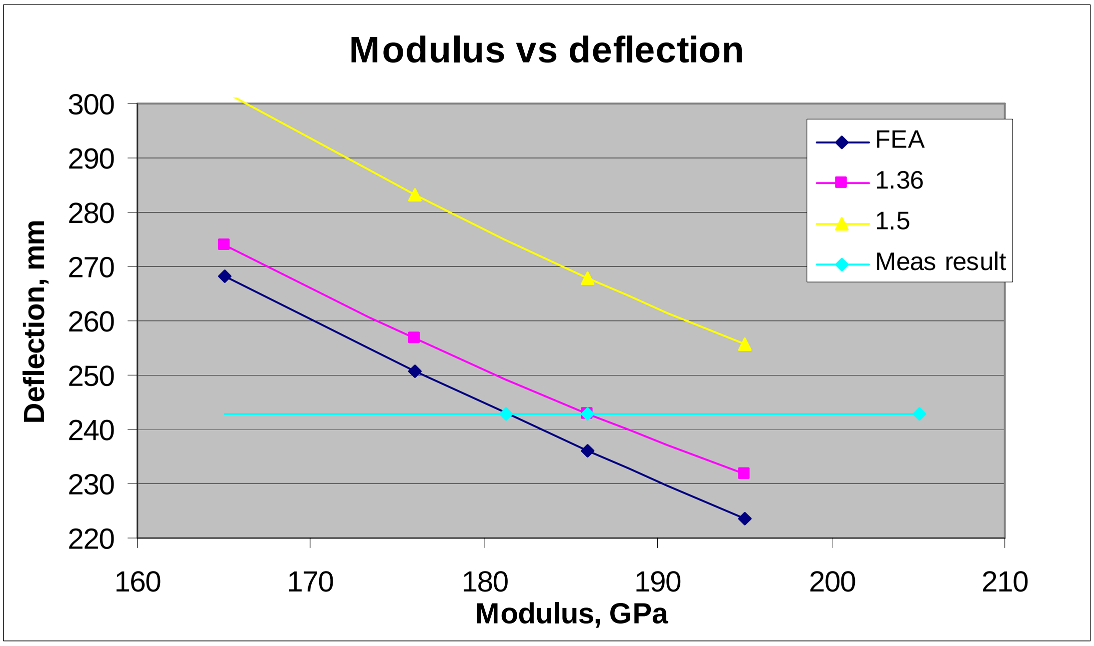


Natural frequency was measured at 100 oscillations in 103 seconds +/-1 sec, giving a frequency of 0.97 +/-0.01 Hz.

4. Commentary and conclusions so far

- The blade equations with alpha=1.5 do not match the nonlinear FEA very well. This could be explained by the fact that there are large deflections involved or that the blade, having width and curling laterally, is not behaving as a perfect beam.
- The blade equations with alpha=1.36 match the nonlinear FEA much better. Alpha=1.36 has been found empirically to be a "good" value.
- The measurements of modulus are variable and the mean value is higher than we were expecting. With the measured modulus value, the measured deflection matches alpha=1.36 more closely than any other method.

We now need to decide how to proceed with the design of the CP blades. Given the results above, we may choose to ignore the odd measured modulus result and instead decide what modulus would have had to be used to give the observed deflection result. We can then apply that method to the CP blades. The results are



Method	Modulus to give observed deflection in BTF blades	Corrected bend radius to give a flat blade under load	Derived "COUPLED" frequency with 61.478kg
Nonlinear FEA	181.31	See below	See below
Blade equations, alpha = 1.36	185.92	426.1mm	1.015 Hz
Blade equations, alpha = 1.5	205.06	426.1mm	1.015 Hz

Blade equations: Both the blade equation methods should give the same result as we are in fact only adjusting the ratio alpha/E to match the measured result. **The blade equations with the tried and trusted formula of E=186, alpha=1.36 give almost exactly the right answer in this case**.

FEA: A modulus of 181.31 gave a deflection of 242.9mm as expected. Using that value I reran the analysis reported in the start of T040114 to find the natural frequency, with these parameters:

```
! values of parameters
trilength=.48
blength=0.469
rootwidth=0.095
hroot=rootwidth/2
tipwidth=0.013
htip=tipwidth/2
inter=trilength*tipwidth/rootwidth
taperl=trilength-inter
tipmass=61.478
bthick=0.0044
maryoung=1.8131e11
marpoiss=0.3
mardens=7800
```

Frequency was 0.679 Hz which is clearly too low by a significant amount. I do not understand this result.

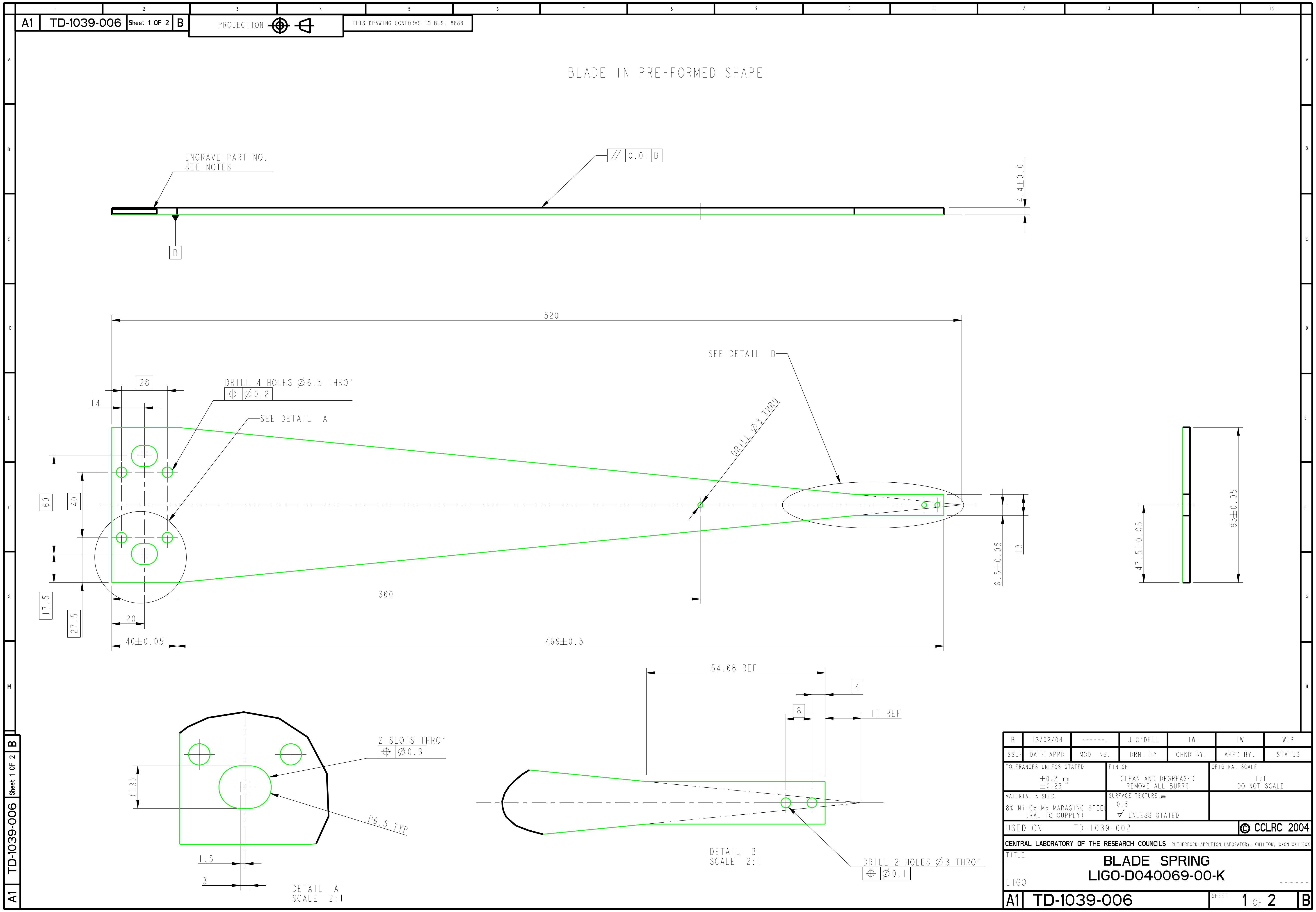
5. Implications for controls prototype blades

Some aspects of the above are puzzling, but it seems to me that the best combination of accuracy and simplicity is to use the blade equations with E=186 and alpha=1.36.

Appendix. Basic macro for nonlinear statics.

```
!Macro for nonlinear
statics on BTF blade

finish
/CLEAR,START
*abbr,doit,doit

/PREP7
!*
! values of parameters
trilength=.48
blength=0.469
rootwidth=0.095
hroot=rootwidth/2
tipwidth=0.013
htip=tipwidth/2
inter=trilength*tipwidt
h/rootwidth
taperl=trilength-inter

bthick=0.0044
maryoung=1.76e11
marpoiss=0.3
mardens=7800
dampratio=5e-5
tipload=61.936*9.81
bendrad=0.4278
!*
raddeg=180/3.1415926
thtip=blength/bendrad*
raddeg
thwaist=taperl/bendrad
*raddeg
thtri=trilength/bendrad
*raddeg

!*
ET,1,SHELL93
R,1,bthick, , , , , ,
R,2,bthick*10
!*
MPTEMP,,,,,,,,
MPTEMP,1,0
MPDATA,EX,1,,mary
oung
MPDATA,PRXY,1,,m
arpoiss
MPTEMP,1,0
MPDATA,DENS,1,,m
ardens

/VIEW,  1, -
0.370848664746    , -
0.543743333662    ,
0.752870808940
/ANG,   1,  -
84.2975399159

csys,1
k,1,bendrad,0,-hroot
,2,bendrad,thwaist,-htip
,3,bendrad,thtip,-htip
,4,bendrad,thtip,htip
,5,bendrad,thwaist,htip
,6,bendrad,0,hroot
,7,bendrad,thtri,-htip
,8,bendrad,thtri,htip

L,1,2
,2,3
,3,4
,4,5
,5,6
,6,1
,3,7
,7,8
,8,4
AL,1,2,3,4,5,6

AL,3,7,8,9

aplot
ESIZE,hroot/4,0

real,1
amesh,1

real,2
amesh,2

eplot
!AATT, MAT, REAL,
TYPE, ESYS, SECN

csys,0

DL,6,,all,0
FK,7,FX,tipload/2
FK,8,FX,tipload/2

FINISH
/SOL
!*
ANTYPE,0
ANTYPE,0
NLGEOM,1
NSUBST,10,0,0
/STATUS,SOLU
SOLVE
FINISH
/POST1
PLDISP,0
PLDISP,1
:end
```

Appendix 2 – drawings of blades


A1 TD-1039-006 Sheet 1 OF 2 B
PROJECTION
THIS DRAWING CONFORMS TO B.S. 8888
BLADE IN PRE-FORMED SHAPE
ENGRAVE PART NO.
SEE NOTES
// 0.01 B
4.4±0.01
B
520
SEE DETAIL B
28
14
DRILL 4 HOLES Ø6.5 THRO'
Ø0.2
SEE DETAIL A
DRILL Ø3 THRU
60
40
17.5
27.5
360
20
40±0.05
469±0.5
6.5±0.05
13
47.5±0.05
95±0.05
54.68 REF
4
8
11 REF
2 SLOTS THRO'
Ø0.3
(13)
R6.5 TYP
1.5
3
DETAIL A
SCALE 2:1
DETAIL B
SCALE 2:1
DRILL 2 HOLES Ø3 THRO'
Ø0.1
B
13/02/04
------.
J O'DELL
IW
IW
WIP
ISSUE
DATE APPD
MOD. No.
DRN. BY
CHKD BY.
APPD BY.
STATUS
TOLERANCES UNLESS STATED
±0.2 mm
±0.25 °
FINISH
CLEAN AND DEGREASED
REMOVE ALL BURRS
ORIGINAL SCALE
1:1
DO NOT SCALE
MATERIAL & SPEC.
8% Ni-Co-Mo MARAGING STEEL
(RAL TO SUPPLY)
SURFACE TEXTURE µm
0.8
UNLESS STATED
USED ON TD-1039-002
© CCLRC 2004
CENTRAL LABORATORY OF THE RESEARCH COUNCILS RUTHERFORD APPLETON LABORATORY, CHILTON, OXON OX11 0QX.
TITLE
BLADE SPRING
LIGO-D040069-00-K
LIGO

A1 TD-1039-006
SHEET 1 OF 2
B
A1 TD-1039-006 Sheet 1 OF 2 B

A1	TD-1039-006	Sheet 2 OF 2	B

PROJECTION

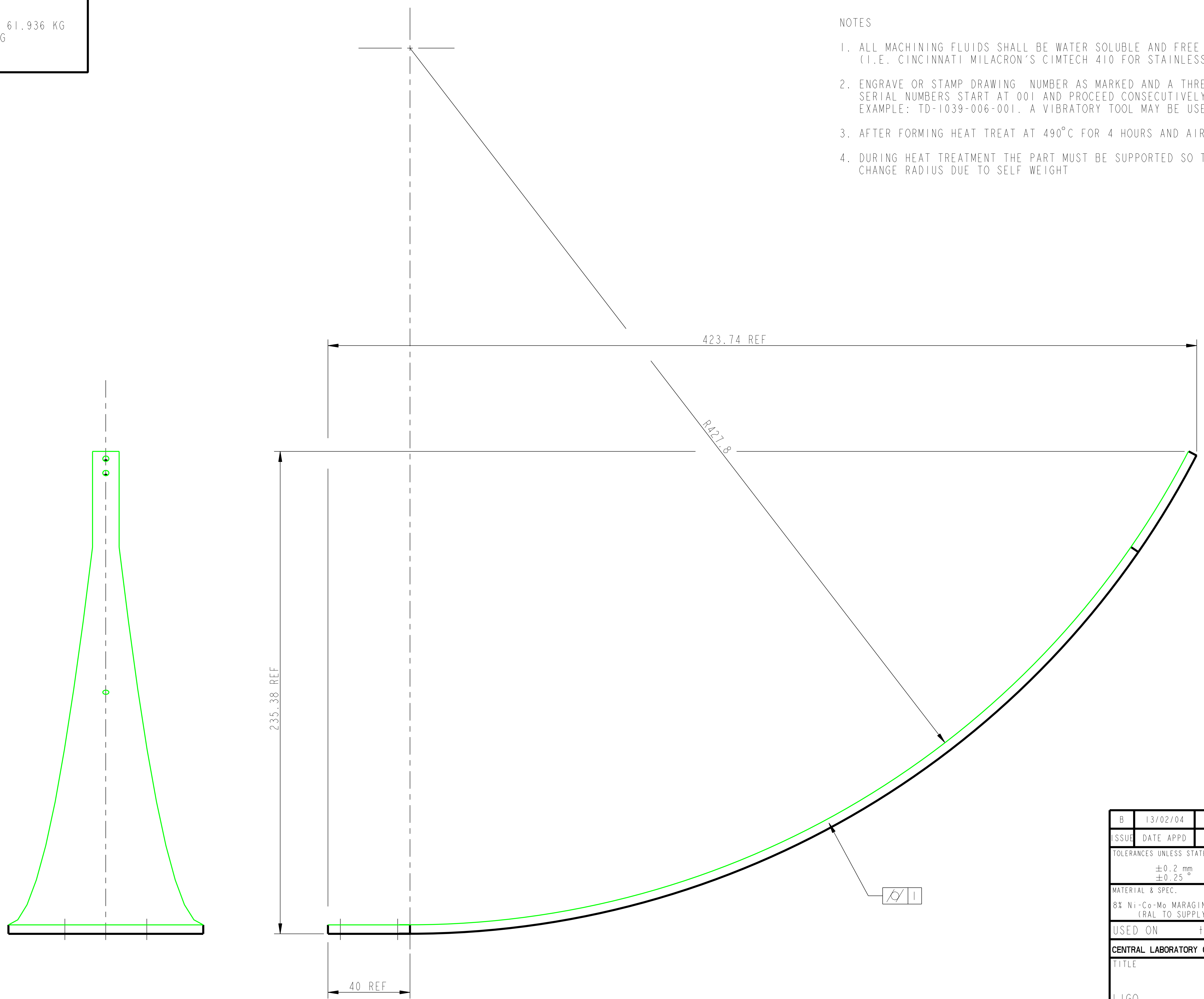
THIS DRAWING CONFORMS TO B.S. 8888

FOR INTERNAL USE ONLY:

E=186Mpa
ALPHA=1.36
TOTAL SUSP MASS = 61.936 KG
US MASS = 10.95 KG

NOTES

1. ALL MACHINING FLUIDS SHALL BE WATER SOLUBLE AND FREE OF SULFUR, CHLORINE AND SILICONE, (I.E. CINCINNATI MILACRON'S CIMTECH 410 FOR STAINLESS STEEL)
2. ENGRAVE OR STAMP DRAWING NUMBER AS MARKED AND A THREE DIGIT SERIAL NUMBER. SERIAL NUMBERS START AT 001 AND PROCEED CONSECUTIVELY. USE 2MM HIGH CHARACTERS. EXAMPLE: TD-1039-006-001. A VIBRATORY TOOL MAY BE USED.
3. AFTER FORMING HEAT TREAT AT 490°C FOR 4 HOURS AND AIR COOL
4. DURING HEAT TREATMENT THE PART MUST BE SUPPORTED SO THAT IT DOES NOT CHANGE RADIUS DUE TO SELF WEIGHT



B	13/02/04	------.	J O'DELL	IW	IW	WIP
ISSUE	DATE APPD	MOD. No.	DRN. BY	CHKD BY.	APPD BY.	STATUS

TOLERANCES UNLESS STATED	FINISH	ORIGINAL SCALE
±0.2 mm ±0.25 °	CLEAN AND DEGREASED REMOVE ALL BURRS	1:1 DO NOT SCALE
MATERIAL & SPEC. 8% Ni-Co-Mo MARAGING STEEL (RAL TO SUPPLY)	SURFACE TEXTURE μm 0.8 UNLESS STATED	

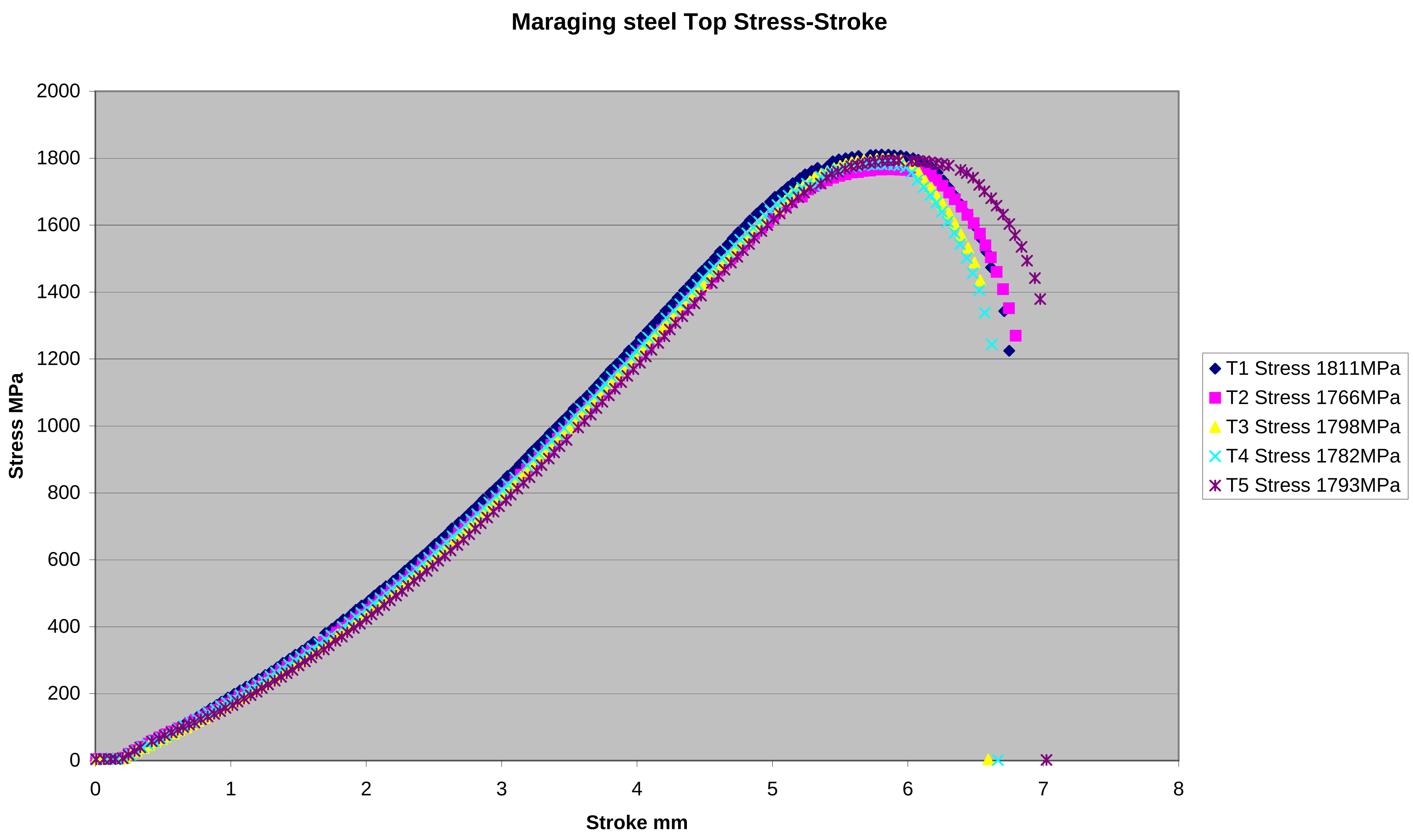
USED ON td-1039-600

CENTRAL LABORATORY OF THE RESEARCH COUNCILS RUTHERFORD APPLETON LABORATORY, CHILTON, OXON OX11 0QX.

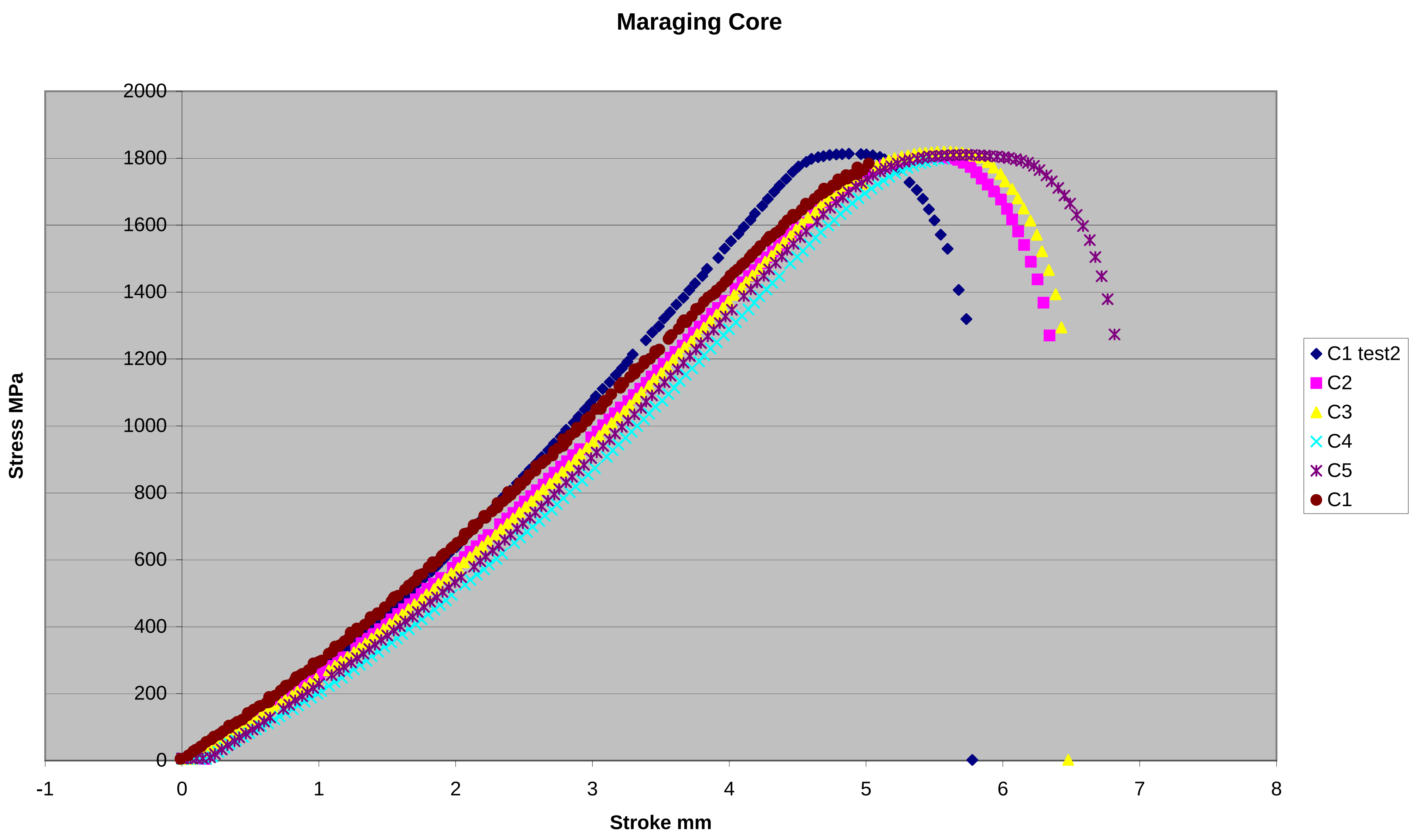
TITLE
BLADE SPRING
LIGO-D040069-00-K

LIGO ------

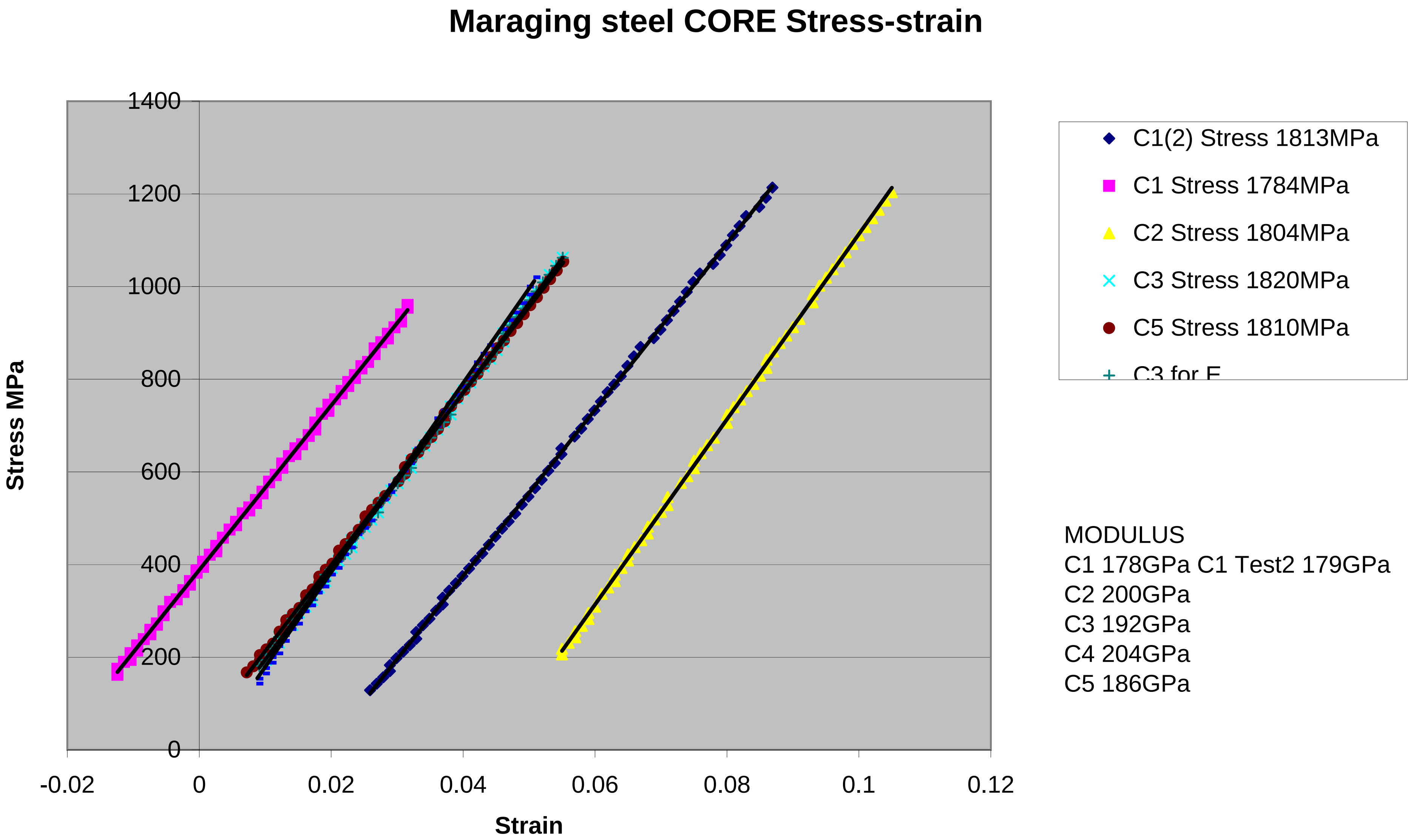
A1	TD-1039-006	SHEET 2 OF 2	B

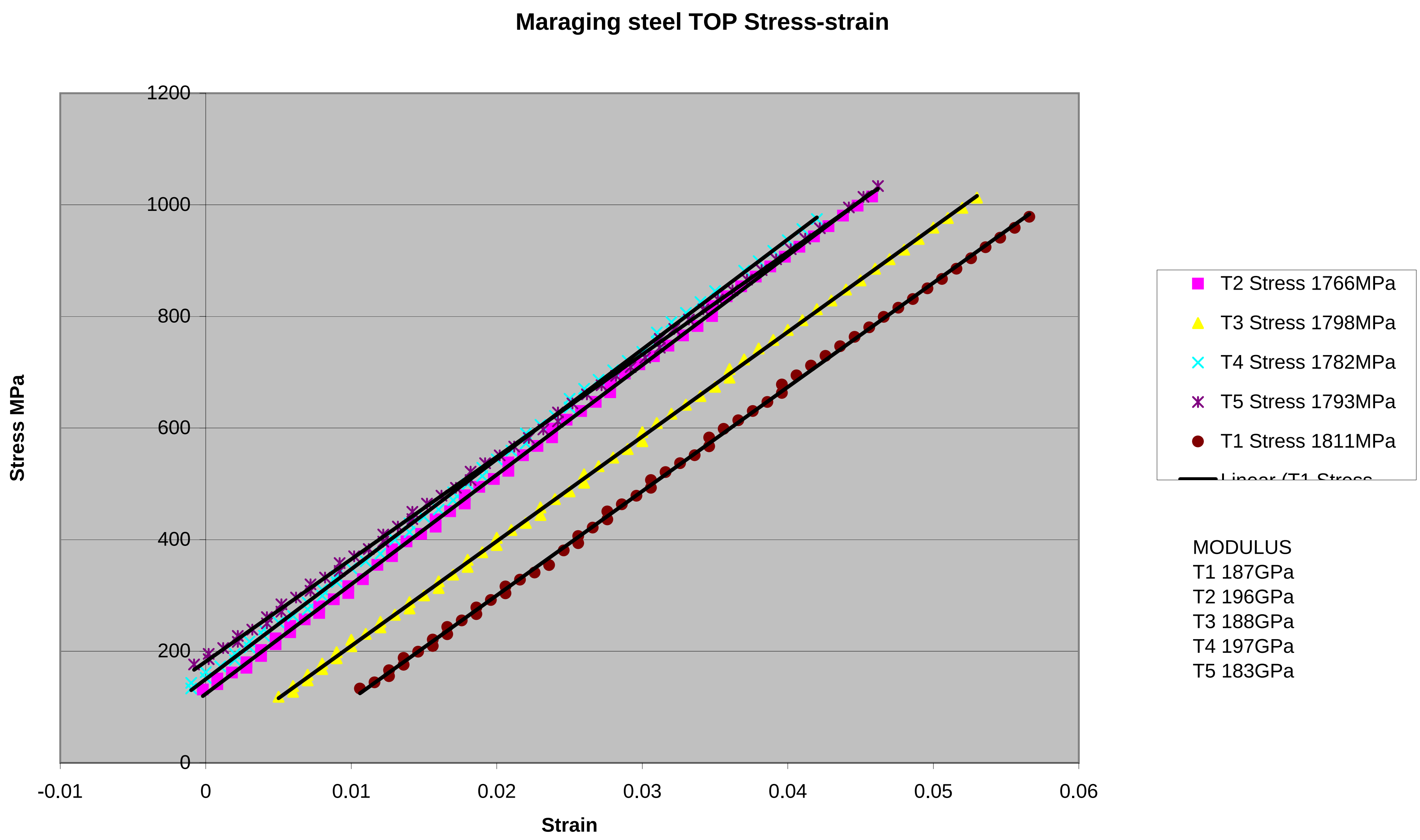


Maraging steel Top Stress-Stroke
Stress MPa
2000
1800
1600
1400
1200
1000
800
600
400
200
0
0
1
2
3
4
5
6
7
8
Stroke mm
T1 Stress 1811MPa
T2 Stress 1766MPa
T3 Stress 1798MPa
T4 Stress 1782MPa
T5 Stress 1793MPa



Maraging Core
Stress MPa
2000
1800
1600
1400
1200
1000
800
600
400
200
0
-1
0
1
2
3
4
5
6
7
8
Stroke mm
C1 test2
C2
C3
C4
C5
C1


Maraging steel CORE Stress-strain
Stress MPa
1400
1200
1000
800
600
400
200
0
-0.02
0
0.02
0.04
0.06
0.08
0.1
0.12
Strain
C1(2) Stress 1813MPa
C1 Stress 1784MPa
C2 Stress 1804MPa
C3 Stress 1820MPa
C5 Stress 1810MPa
C3 for E
MODULUS
C1 178GPa C1 Test2 179GPa
C2 200GPa
C3 192GPa
C4 204GPa
C5 186GPa



Maraging steel TOP Stress-strain
Stress MPa
1200
1000
800
600
400
200
0
-0.01
0
0.01
0.02
0.03
0.04
0.05
0.06
Strain
T2 Stress 1766MPa
T3 Stress 1798MPa
T4 Stress 1782MPa
T5 Stress 1793MPa
T1 Stress 1811MPa
MODULUS
T1 187GPa
T2 196GPa
T3 188GPa
T4 197GPa
T5 183GPa