Cross-coupling in quadruple pendulum suspension.

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1. Introduction

Using his Mathematica model of the quadruple pendulum suspension design for Advanced LIGO, Mark Barton has carried out a series of cross-coupling investigations, putting various asymmetries into the suspension and looking at the 36 transfer functions from the 6 degrees of freedom (DOFs) of the support to the 6 DOFs of the optic. A summary from Mark, with links to the detailed results, can be found at

http://www.ligo.caltech.edu/~mbarton/SUSmodels/results/justin/index.html

The full text at this link is included as an appendix. In this summary, Mark has detailed the 6 different types of asymmetries which he has investigated. Four of these (cases 1 to 4) are concerned with the behaviour when one blade of a pair has a spring constant less than the other, and were proposed for analysis by Justin Greenhalgh. This behaviour has been investigated since we know that matching blades to a high degree of accuracy is not easy, and could in principal involve getting many more blades manufactured than ultimately used. Thus an idea of tolerances on allowed mismatch would be useful. The other two asymmetries investigated (cases 5 and 6) are concerned with changing the length of one pair of wires with respect to another (where a pair is lying to the front or back looking face-on) at a stage in the suspension where there are 4 wires. The cases were done using the conceptual design quad values of 14 Nov. 2003 as a baseline.

We have taken a look at these results, to draw some conclusions regarding the performance of the suspension when such asymmetries are present. Possible issues include any significant increase in residual seismic noise at the test mass, and any potential detrimental effects on global control. For the first of these, the cross-coupling value at 10 Hz and above is the important parameter. For the latter issue, the amount of cross-coupling around the 0.1 to 1 Hz region leading to possible significant enhancement of rms noise needs to be considered.

2. Cases investigated.

#1 The upper left blade spring has been made 0.7836% laxer so that the tip sits 2 mm low, and the masses have been leveled by making the upper left wire 2 mm/cos(upper wire angle) = 2.14364 mm shorter.

#2 The upper left blade spring has been made laxer by 1.5 %, and (conceptually at least) the unloaded angle of the blade has been adjusted to keep the tip of the blade at its nominal position.

#3 The upper left blade spring has been made laxer by 1.5 %, and the tip of the blade has been allowed to sag so that the lower masses hang at an angle.

#4 The upper left blade spring has been made laxer by 1.5%, and the orientation of the lower masses has been restored by cutting out a counterweight of mass 0.2 kg from the top mass and moving it 0.346923 m in y from the centre.

#5 The front two intermediate wires have been made longer by 0.1%.

#6 The front two intermediate wires have been made longer by 0.1%, and the pitch of the optic has been corrected by a 0.2 kg counterweight 0.05 m off-centre on the top mass.

Three more cases, as in 1,2, and 3 above but with perturbations 1/5 the size, were also run.

3. Some general observations

Mark has already produced a summary in his write-up in which some general points are made, and these are not reproduced in full here. Some points:

a) The strength of the cross-coupling effects appears to be linear for the magnitudes investigated.
b) In all cases the new cross-couplings begin to roll off steeply above the highest participating mode.

4. Some detailed observations

In general for seismic isolation considerations the most important cross-coupling types are those which couple directly into the longitudinal direction. For other types, there is then a further cross-coupling factor into the longitudinal (e.g. pitch or yaw will cross-couple into longitudinal at the mirror due to an offset of the beam from the centre of the optic face). This introduces a further reduction in magnitude and such cases are therefore unlikely to be so significant. Thus we have concentrated to start with on the couplings into the 'x' direction.

4.1 Cross-couplings into the x (longitudinal) direction

Using Mark's summary table, we identified 3 types to look at: z to x, yaw to x and roll to x. A fourth (y to x) was not considered since isolation in y is typically of same order as in x and thus this route is not going to be as significant as (say) z to x, where the isolation in z is poorer, given the magnitudes quoted in Mark's table.

It should be noted that none of these cross-couplings appeared in the first 4 investigations (those associated with blade mismatch). They only appear with front to back wire length mismatch.

4.1.1 z to x.

The typical transfer function from z of support to z of optic is around 4 x 10^{-4} at 10 Hz for a symmetric model, and we further have assumed a cross-coupling factor of 10^{-3} into the x direction – giving an overall TF of 4 x 10^{-7}. Thus anything of that order would be significant. The values for cases 5 and 6 are found to be 10^{-9} and 10^{-8} respectively at 10 Hz and falling off rapidly above – thus these would produce a negligible increase in noise. As noted above, 5 and 6 correspond to wire lengths differing by 0.1% (0.3 mm in 30 cm).

4.1.2 yaw to x and roll to x.

These transfer functions have typical values at 10 Hz of 10^{-12} to 10^{-14} in m/rad. Input yaw and roll should be of order a few times 10^{-13} rad /√Hz at 10 Hz. Thus the resulting motion in x from these cross-couplings is insignificant compared to the noise requirement of 10^{-19} m / √Hz at 10 Hz.

In summary none of the above will compromise the seismic isolation at 10 Hz and above as estimated from the symmetric model.

4.2 Cross-coupling into pitch

4.2.1 x to pitch.

One of the findings with the GEO suspensions was that due to a significant tapering of the ends of the silica fibres, rather than a simple cylindrical cross-section as modelled, there was increased longitudinal to pitch coupling in the suspensions over that predicted by our standard GEO model. (ref "The Status of GEO", K A Strain et al , SPIE presentation, June 2004), This had an adverse effect on the lock acquisition, due to excitation of the suspensions around the microseismic frequency band (~0.15 to 0. 3 Hz). This led to excessive pitching of the mirrors, and corrective steps had to be taken to reduce this effect (see above ref.). We will take steps in the LIGO suspensions to carry out careful checking of the profiles of the ribbons or fibres so as not to repeat this problem. However it is also worth checking that the cross-coupling introduced by asymmetries does not enhance the longitudinal (x) to pitch coupling which is inherent in the symmetric model.

Note that x to pitch was not one of the cross-couplings which Mark identified in his table of results as having significant cross-couplings appearing relative to the symmetrical pendulum suspension, and so we do not expect to see an appreciable difference. Indeed this is the case. An example of the transfer function from case 6 is shown in Figure 1, where it can be compared to a symmetric MATLAB model, also shown. Thus we can conclude that in the cases studied the asymmetries are not introducing further noise through this coupling,

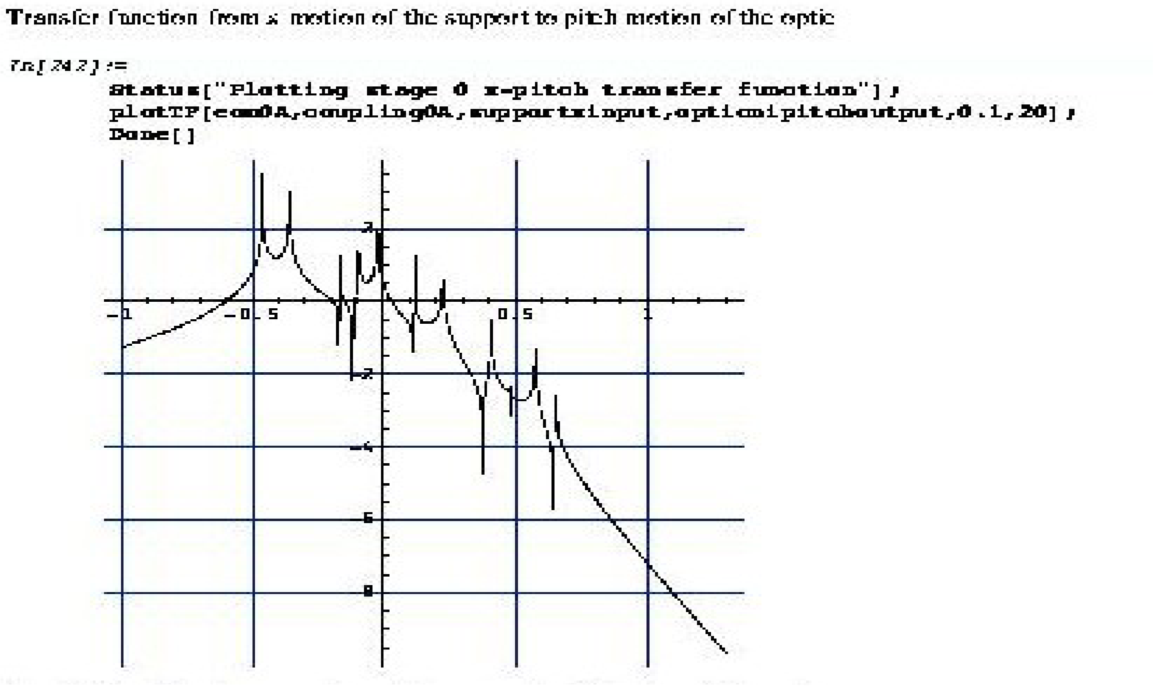
4.2.2 z to pitch

One more type of cross-coupling which we should consider is z to pitch. With symmetry there is no such cross coupling. However it is introduced in cases 5 and 6. This is highlighted in Mark's table. The transfer function for case 6 is shown in Figure 2. We can look at two features. Firstly there is significant coupling just below 1 Hz. It is of the same magnitude as the peak coupling seen in the x to pitch transfer functions, in a slightly higher frequency band. However since the peak is not around the microseismic frequency region, and in magnitude is no worse than the coupling seen in x to pitch from a symmetric model, we conclude that it should not make lock acquisition significantly harder to do.

We can also look at the magnitude at 10 Hz to check that the seismic isolation is not decreased. The TF at 10 Hz is around 10^{-6}. The input noise level is expected to be $2\text{x}10^{-13}$ m/√ Hz. Thus the pitch noise at the mirror would be $2\text{x}10^{-19}$ rad/√ Hz. Assuming a 1mm offset, this translates to a longitudinal motion of $2\text{x}10^{-22}$ m/√ Hz, well below the target noise level of 10^{-19} m/√ Hz.

5 Conclusions.

In each of the examples highlighted above, we have concluded that no significant detrimental effects are seen with the levels of coupling introduced in the cases studied.



Transfer function from x motion of the support to pitch motion of the optic

```
In[242]:=
        Status["Plotting stage 0 x-pitch transfer function"];
        plotTF[eom0A,coupling0A,supportxinput,opticmipitchoutput,0.1,20];
        Done[]
```

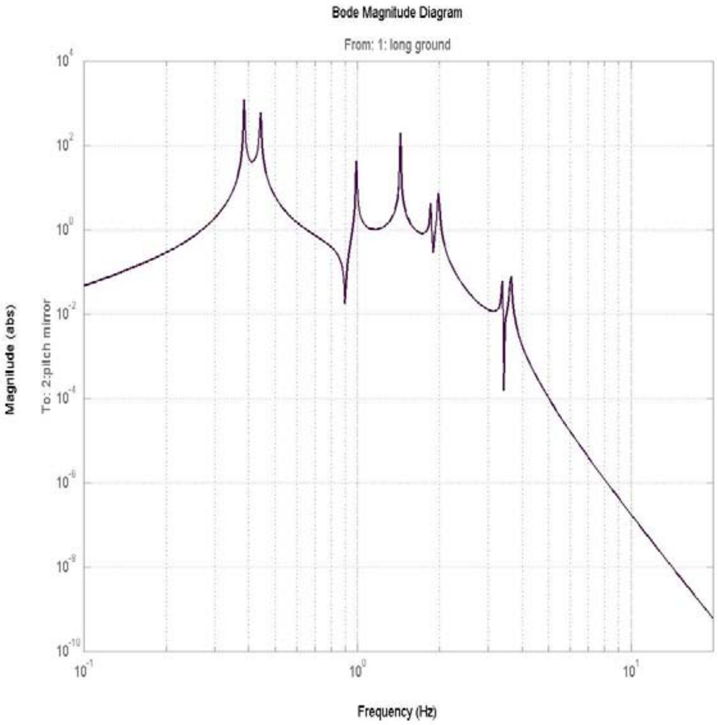
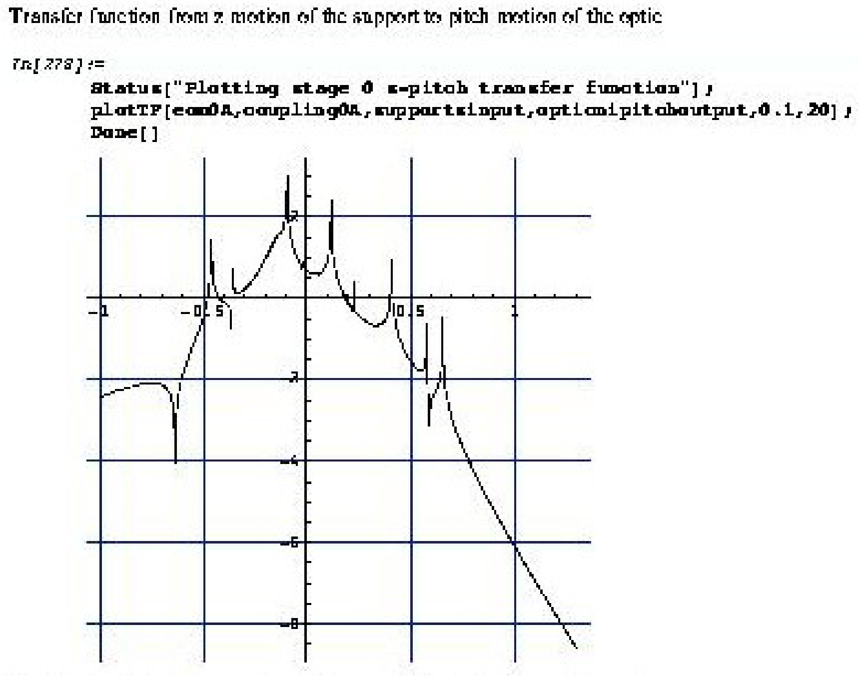


Figure 1 Longitudinal (x) to Pitch Transfer Functions.

Upper curve is from Mark’s analysis of case 6, lower curve from the symmetric MATLAB model. Note that overall the curves are very similar, with the addition of more modes in the asymmetric case. Small differences in the positions of the common peaks can be accounted for by slight differences in the parameters used in the models analysed.



Transfer function from z motion of the support to pitch motion of the optic

```
In[278]:=
        Status["Plotting stage 0 z-pitch transfer function"];
        plotTF[eom0A,coupling0A,supportzinput,opticmipitchoutput,0.1,20];
        Done[]
```

Figure 2. z to pitch transfer function for case 6. (y-axis: rad/m)

Appendix. (M Barton)

Content of link
http://www.ligo.caltech.edu/~mbarton/SUSmodels/results/justin/index.html

Justin's queries from email of 10/29/03, plus further asymmetrical cases of interest

Scenarios and Results Archives

All the PDF transfer function summaries in a single archive: justinsresults.zip.

#0 (quad20031114.zip): Norna's conceptual design quad of 11/14/03 as a baseline. Archive contains a case directory with case calculation notebook, precomputed results directory and a PDF summary of the 36 transfer functions from the 6 DOFs of the support to the 6 DOFs of the optic. To do anything beyond viewing the existing results, the case directory needs to be installed in the model directory of the v3.1 quad model.

#1 (quad20031114J1.zip): Perturbations as per Justin's query #1 of email of 10/29/03. The upper left blade spring has been made 0.7836% laxer so that the tip sits 2 mm low, and the masses have been leveled by making the upper left wire 2 mm/cos(upper wire angle) = 2.14364 mm shorter.

#2 (quad20031114J2.zip): Perturbations as per Justin's query #2 of email of 10/29/03. The upper left blade spring has been made laxer by 1.5 %, and (conceptually at least) the unloaded angle of the blade has been adjusted to keep the tip of the blade at its nominal position.

#3 (quad20031114J3.zip): Perturbations as per Justin's query #3 of email of 10/29/03. The upper left blade spring has been made laxer by 1.5 %, and the tip of the blade has been allowed to sag so that the lower masses hang at an angle.

#1b (quad20031114J1b.zip): As for #1 above, but 1/5 the perturbation as a linearity test.

#2b (quad20031114J2b.zip): As for #2 above, but 1/5 the perturbation as a linearity test.

#3b (quad20031114J3b.zip): As for #3 above, but 1/5 the perturbation as a linearity test.

#4 (quad20031114J4.zip): Variation on #2/#3. The upper left blade spring has been made laxer by 1.5%, and the orientation of the lower masses has been restored by cutting out a counterweight of mass 0.2 kg from the top mass and moving it 0.346923 m in y from the centre.

#5 (quad20031114J5.zip): The front two intermediate wires have been made longer by 0.1%.

#6 (quad20031114J6.zip): The front two intermediate wires have been made longer by 0.1%, and the pitch of the optic has been corrected by a 0.2 kg counterweight 0.05 m off-centre on the top mass.

Quick Summary

The following table shows which new cross-couplings appeared relative to the symmetrical pendulum. See "Results Archives" section above for the detailed descriptions of each case. Each entry is the power of 10 of a subjective estimate of the maximum level of cross-coupling maintained across a significant frequency range (e.g., a factor of two). No other input/output combinations (except of course for those which were already large in the symmetrical pendulum) were significantly different from rounding error: 10^-15 or so. In all cases the new cross-couplings begin to roll off steeply above the highest participating mode - in nearly all cases by 10 Hz.

Transfer Function	#1	#2	#3	#4	#5	#6	unit
x->y					-7	-6	1
y->x					-5	-5	1
x->z					-2	-1	1
z->x					-2	-1	1
x->yaw	-2	-4	-2	+1!	-5	-5	m/rad
yaw->x	-3	-6	-3	-1	-6	-7	rad/m
x->roll					-6	-6	rad/m
roll->x					-7	-7	m/rad
y->z	-2	-2	-2	-1	-5	-6	1
z->y	-2	-2	-2	-1	-6	-6	1
y->yaw					-1	0	rad/m
yaw->y					-3	-2	m/rad
y->pitch					-5	-4	rad/m
pitch->y					-11	-7	m/rad
z->yaw					-5	-5	rad/m
yaw->z					-7	-8	m/rad
z->pitch					1!	1!	rad/m
pitch->z					-6	-5	m/rad
z->roll	-1	-1	-1	0	-5	-5	m/rad
roll->z	-3	-3	-3	-1	-7	-7	rad/m
yaw->pitch	-3	-5	-1	-1	-5	-6	1
pitch->yaw	-3	-6	-2	-2	-6	-7	1
yaw->roll					-2	-2	1
roll->yaw					-1	-1	1
pitch->roll					-8	-7	1
roll->pitch					-5	-5	1

It is apparent that geometrical asymmetry (as in #1 and #3) produces more strong cross-couplings than just spring asymmetry. The strong cross-couplings in #1, #2 and #3 were down about a factor of 5 in #1b, #2b and #3b (as opposed to 25), indicating the effects are linear (as opposed to quadratic). Scenario #2 is never the worst case and is 2-3 orders of magnitude better in several cases.

Counterweighted configurations (#4, #6) seem to be significantly worse than uncorrected configurations (#3, #5) or ones corrected by other means (#1, #2).

Note on interpretation of angles

The Mathematica code uses a yaw/pitch/roll convention for angles. For an active transformation (i.e., one which physically rotates an object) the component rotations are applied in the order roll, pitch then yaw, about the global x, y and z axes respectively. Because finite rotations don't commute, there is some subtlety about the interpretation of small angular displacements when the equilibrium position has non-zero values for the orientation angles of the objects, as in several of the present cases.

The issue is that infinitesimal yaw, pitch, and roll displacements specified by the normal modes are increments to the equilibrium angles rather than independent rotations. Because roll is applied first, an increment to roll is modulated by any pitch and yaw rotations which come after it. If these are non-infinitesimal, then an increment to roll is no longer a pure x-axis rotation. All the transfer functions reported here have had a correction applied, which effectively moves all the infinitesimal dynamic rotations after all the finite static rotations. Without this correction there are a lot more apparently non-zero cross-couplings.