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Thermal noise in Sapphire mirrors in a high-sensitivity interferometer

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Thermal noise in Sapphire mirrors in a high-sensitivity interferometer

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We report direct broadband measurements of thermal fluctuations in Sapphire mirrors in a high-sensitivity Fabry-Perot interferometer. Our results agree well with the thermoelastic-damping noise predictions of Braginsky, et al. and Cerdonio, et al. We use this measurement, along with a previously-reported measurement of coating thermal noise, to predict the noise floor of an advanced gravitational wave detector with both Sapphire and fused-silica mirrors.

PACS numbers:

MIRROR MATERIALS FOR ADVANCED INTERFEROMETRIC GRAVITATIONAL WAVE DETECTORS

There is currently a large, multi-national effort to construct interferometric gravitational-wave detectors, including LIGO [1], GEO [2], VIRGO [3], TAMA [4], and ACIGA [5], and to establish the field of gravitational-wave astronomy. As the community builds, operates, and learns from these first-generation detectors, advanced second-generation designs are also being explored that will utilize new sensing techniques and mirror materials to substantially reduce the fundamental noise floor and hence the astrophysical reach [6]. One candidate material for advanced mirror substrates is Sapphire, which is expected to show significantly lower levels of thermal noise than the fused-silica mirrors of first-generation detectors [7]. However, while fused-silica is relatively well-established as a mirror substrate material, until now no high-sensitivity interferometer using Sapphire mirrors had ever been constructed. In this paper we report on the first high-sensitivity interferometer with Sapphire mirrors, and we report an observation of the fundamental thermal noise floor in this interferometer, set by the Sapphire mirror substrates.

THEORY

It has been known for some time that thermoelastic damping loss can contribute to thermal noise in mechanical systems [8]. However, that this mechanism can contribute significantly to the noise floor of an interferometer with Sapphire mirrors was first pointed out by Braginsky and collegues in 1999 [9]. This noise source can be thought of either as thermoelastic-damping mediated thermal noise, as described by the fluctuation-dissipation theorem, or as a coupling between intrinsic temperature fluctuations in the bulk of the mirror and the mirror's thermal expansion coefficient. Either picture gives the same result: This "Braginsky noise" will set the ultimate sensitivity limit of an advanced gravitational wave detector that uses Sapphire mirrors.

Braginsky made a quantitative prediction of this noise source valid in the limit of high frequencies or large spot sizes. Because our spot size is necessarily smaller than that of LIGO's, we must use an expression valid at all frequencies. Cerdonio, et al. have derived this result, and it is [10]

$$S_B(\omega) = \frac{8}{\sqrt{\pi}} \frac{\alpha^2(1+\sigma)^2}{\kappa} k_B T^2 w_0 J\left[\Omega\right], \tag{1}$$

where the dimensionless integral $J\left[\Omega\right]$ is given by

$$J\left[\Omega\right] = \frac{\sqrt{2}}{\pi^{3/2}} \int_0^\infty du \int_{-\infty}^{+\infty} dv \frac{u^3 e^{-u^2/2}}{(u^+ v^2)[(u^2+v^2)^2+\Omega^2]} \tag{2}$$

and

$$\Omega = \frac{\omega}{(2\kappa/\rho C w_0^2)}.$$

In these expressions, α is the thermal expansion coefficient of the mirror substrate, σ is its Poisson's ratio, κ is the thermal conductivity, ρ is its mass density, C is its specific heat, and ω is the (angular) measurement frequency. The laser spot radius w_0 we use in this paper is the usual one, where the electric field falls off to $1/e$ of its maximum value.

In their original paper, Cerdonio, et al. made a mathematical error that resulted in an incorrect expression for the integral $J\left[\Omega\right]$. For these formulas, we have corrected that error. Also, they, and Braginsky, used a different definition of the laser spot size than is customary. Their spot size is the radius at which the intensity of the beam, rather than the field, falls off to $1/e$ of its maximum value. We adopt the convention that this quantity is designated r_0, and it is related to the usual spot size w_0 by $r_0 = w_0/\sqrt{2}$.

THE INSTRUMENT

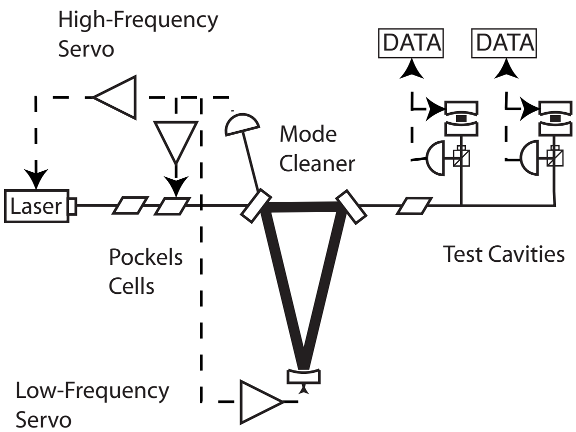

FIG. 1: Schematic of the interferometer. Each of the test mass mirrors and the mode cleaner mirrors are suspended under vacuum. The laser is locked to the mode cleaner via a three-path servo for frequency stabilization, and the test cavities are independently locked to the stabilized beam. The test cavity lengths (1 cm each) are short, to optimize the system for measuring displacement noise, and their radii of curvature of the test-mass mirrors are long (1 meter) to increase the spot size as much as feasible. This is done to reduce the displacement noise as much as possible and thus make the measurement relevant to gravitational wave detection.

Figure 1 shows a diagram of our instrument. The interferometer has two short, independent arm cavities, made up of four identical mirrors. In order to reduce the thermal noise as much as possible, the radii of curvature of these mirrors (1 meter) was chosen to be much larger than the cavity length (1 cm), which gives the largest practical spot size ($160\mu m$) for an interferometer of this size.

The arm cavity lengths were chosen small to reduce the relative contribution of laser frequency noise, which was also a factor in the choice of mode-cleaner length (1 meter).

All of the test-cavity mirrors and the mode cleaner mirrors were suspended under vacuum, with magnet-and-coil damping and actuation systems. The laser is locked to the mode cleaner to stabilize its frequency, and the arm cavities are locked independently to the resulting stabilized and filtered beam. Data is collected from the error signals of the arm cavities, which are differenced to remove any remaining laser frequency noise. Calibration is described in detail in a previous paper [11].

We performed two experiments with this instrument. First, we measured displacement noise using fused-silica test masses. In this first configuration we characterized a variety of noise sources and verified that what appeared to be displacement noise really did originate inside the test cavities. Second, we replaced the fused-silica test masses with Sapphire mirrors of the same geometry, replacing the suspension wires at the same time with thicker ones to keep the violin mode frequencies the same. Any difference, then, between the first and second noise spectra should be due only to the difference between fused-silica and Sapphire substrates. The coatings on both sets of mirrors were identical, both being SiO_2/Ta_2O_5 coatings of the same thickness, made by the same manufacturer [12].

RESULTS

Figure 2 shows a plot of the displacement noise in the interferometer, along with theoretical plots of the Braginsky noise, coating thermal noise (expected), shot noise (measured), and electronic noise (measured). The Braginsky noise prediction agrees well with the observed noise floor of this instrument over approximately one decade in frequency, from 400 Hz to 5 kHz. No parameters were adjusted in the theory to fit the data. Table I gives the parameters used to calculate both the Braginsky noise and the coating thermal noise for our instrument.

Table I: Parameters used to calculate the noise floor of the instrument

α	5.0×10^{-6} [13]
κ	$33W/mK$ [13]
ρ	$3.983 \times 10^3 kg/m^3$ [13]
C	$770J/kgK$ [13]
σ	0.23 [13]
σ_c	0.2 [14]
w_0	$160\mu m$
E	$40 \times 10^{10} N/m^2$ [15]
E_c	$11.0 \times 10^{10} N/m^2$ [15]
d	$4.26\mu m$ [12]
ϕ_c	2.7×10^{-4} [11]

We evaluated the coating thermal noise for these Sapphire mirrors using the method described in Reference [15]. We demonstrated in a previous paper that this method is valid for nominally identical coatings on fused-silica substrates [11]. We measured the shot noise in our photodetectors by shining a heat lamp on them.

One aspect of using Sapphire mirrors became clear when we performed this measurement: Mechanical resonances in the mirrors were much less problematic than with fused-silica mirrors. With fused-silica mirrors in our interferometer, we needed eight notch filters in the servo to keep the lowest-frequency mirror resonances from ringing up. The same resonances in Sapphire mirrors occur at nearly twice the frequencies of those in fused-silica mirrors. Moreover, even though the intrinsic Q's of Sapphire mirrors are expected to be higher than those of fused-silica optics, the *in-situ* Q's are largely determined by the suspensions and are not much higher than those of the fused-silica optics. These two effects combined to

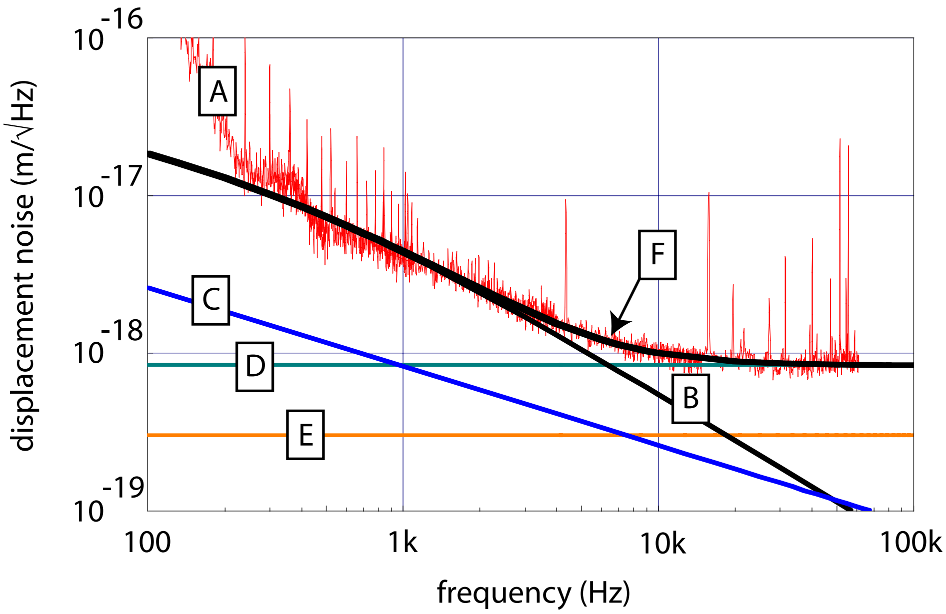

FIG. 2: Displacement noise in the interferometer with Sapphire mirrors. Curve "A" is the total measured noise; "B" is the calculated Braginsky noise; "C" is the expected coating thermal noise; "D" is the measured shot noise; and "E" is the electronic noise in the measurement, principally due to the photodetectors. Curve "F" is the sum, in quadrature, of the Braginsky noise and the shot noise. There are no undetermined parameters in these curves. No fits to the data were performed.

remove the mirrors modes far enough away, in frequency, from the unity-gain frequency of the servo that no notch filters were required. Operating this interferometer with Sapphire mirrors was much easier than with fused-silica.

CONCLUSIONS

We have constructed a high-sensitivity, Fabry-Perot based interferometer to measure the intrinsic noise in Sapphire mirrors. The theoretical prediction of Cerdonio [10] agrees well with our observed noise floor, provided we include the necessary correction to the integral $J\left[\Omega\right]$.

Based on these results, along with our previously-reported coating thermal noise measurement [11], we may predict the relative thermal noise levels in an advanced interferometric gravitational wave detector, using either fused-silica or Sapphire mirrors. Using the expected laser spot size, power, and other parameters [13], we have calculated this expected noise floor. The results, along with the expected quantum-noise floor [13, 16], are shown in Figure 3. We expect Sapphire mirrors to produce a substantially lower noise floor than fused-silica above $100Hz$, even though the Sapphire mirrors' thermal noise is dominated by coating noise above about $200Hz$. Note that we assume silica-tantala for the coating material, which is the current choice of material for mirror coatings in existing gravitational wave detectors. It has been shown that most of the mechanical loss in such a coating is intrinsic to the tantala layers [17], and if a lower-loss coating material could be developed, the noise floor of an interferometer with Sapphire mirrors could be further reduced.

ACKNOWLEDGMENTS

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[1] B. C. Barish and R. Weiss, Physics Today **52**, 44 (1999).

[2] H. Lück and the GEO600 team, Classical and Quantum Gravity **14**, 1471 (1997).

[3] B. Caron, A. Dominjo, C. Drezen, R. Flaminio, X. Grave, and M. more, Classical and Quantum Gravity **14**, 1461 (1997).

[4] K. Kawabe and the TAMA collaboration, Classical and Quantum Gravity **14**, 1477 (1997).

[5] D. G. Blair, General Relativity and Gravitation **32**, 371 (2000).

[6] E. Gustafson, D. Shoemaker, K. A. Strain, and R. Weiss, Internal Working Note LIGO-T990080-00-D, LIGO Project (1999).

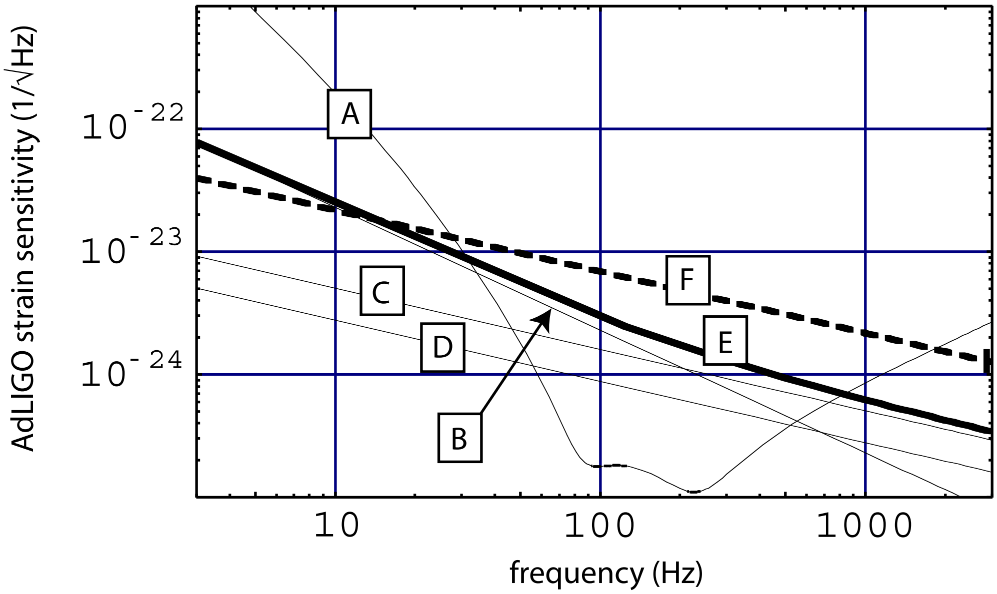

FIG. 3: Expected noise curve of an advanced-LIGO interferometer with Sapphire or fused-silica mirrors. Curve "A" shows the expected quantum noise (radiation-pressure and shot noise); "B" gives the Braginsky noise in Sapphire mirrors; "C" and "D" give the expected coating thermal noise and structural-damping-mediated substrate thermal noise, respectively, of Sapphire; and "E" is the total thermal noise for Sapphire mirrors. Curve "F" shows the expected thermal noise for fused-silica mirrors, including both coating noise and structural-damping-mediated substrate noise. For fused-silica mirrors, substrate thermal noise is greater than coating noise. Sapphire is the better choice, in spite of Braginsky noise and the relatively high level of coating thermal noise if we assume silica-tantala coatings.

[7] S. Rowan, G. Cagnoli, P. Sneddon, J. Hough, R. Route, E. K. Gustafson, M. M. Fejer, and V. Mitrofanov, Phys. Lett. A **265**, 5 (2000).

[8] P. R. Saulson, Phys. Rev. D **42**, 2437 (1990).

[9] V. B. Braginsky, M. L. Gorodetsky, and S. P. Vyatchanin, Phys. Lett. A **264**, 1 (1999).

[10] M. Cerdonio, L. Conti, A. Heidmann, and M. Pinard, Phys. Rev. D **63**, 082003 (2001).

[11] E. D. Black, A. Villar, K. Barbary, A. Bushmaker, J. Heefner, S. Kawamura, F. Kawazoe, L. Matone, S. Meidt, S. R. Rao, et al., Phys. Lett. A **328**, 1 (2004).

[12] *Research electro-optics*, http://www.reoinc.com.

[13] L. S. Finn, *BENCH: Science-grounded figures of merit for comparing interferometric gravitational wave detector designs*, Open-source interferometer characterization program, v. 2.0; http://ligo.mit.edu/∼pf/Bench/.

[14] G. Harry, Tech. Rep. T040029-00-R, LIGO (2004).

[15] G. M. Harry, A. M. Gretarsson, P. R. Saulson, S. E. Kittelberger, S. D. Penn, W. J. Startin, S. Rowan, M. M. Fejer, D. R. M. Crooks, G. Cagnoli, et al., Class. Quantum Grav. **19**, 897 (2002).

[16] A. Buonanno and Y. Chen, Class. Quantum Grav. **18**, L95 (2001).

[17] S. D. Penn, P. H. Sneddon, H. Armandula, J. C. Betzweiser, G. Cagnoli, J. Camp, D. R. M. Crooks, M. M. Fejer, G. A. M., G. M. Harry, et al., Class. Quantum Grav. **20**, 2917 (2003).