Parametric instabilities and their control in advanced interferometer GW detectors

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Abstract: A detailed simulation of Advanced LIGO test mass optical cavities shows that parametric instabilities will excite many acoustic modes in the test masses in the frequency range 28-35 kHz and 68-72 kHz. Using nominal Advanced LIGO optical cavity parameters with fused silica test masses, parametric instability excites 32 acoustic modes, with parametric gain R up to 250. For the alternative sapphire test masses only 11 acoutics modes are excited (in the same frequency range) with R up to 17. Fine tuning of the test mass radii of curvature cause the instabilities to sweep through various modes with R as high as ~10^4. The cavity can be tuned away from the highest gain instabilities using simple thermal compensation with negligible degradation of the noise performance. Additional control systems will be required to suppress all the unstable modes. This will be easier and will risk less noise injection in the case of sapphire test masses.

To achieve sufficient sensitivity to detect numerous predicted sources of gravitational waves, the three long baseline laser interferometer gravitational wave detectors [1, 2, 3, 4] need to achieve about one order of magnitude improved sensitivity. This improvement is planned to be achieved using larger lower acoustic loss test masses and substantially higher laser power [5]. It has already been pointed out that this improvement brings with it the risk of parametric instability [6, 7]. The instability arises due to the potential for acoustic normal modes of the test masses to scatter light from the fundamental optical cavity mode into a nearby higher order mode, mediated by the radiation pressure force of the optical modes acting on the acoustic mode. The instability can occur if two conditions are met. Firstly there must be a substantial spatial overlap of the acoustic mode shape with the higher order cavity mode shape. Secondly the optical frequency difference between the cavity fundamental mode and higher order mode must match the acoustic mode frequency.

Parametric instabilities was observed and controlled in the niobium bar gravitational wave detector NIOBE [8]. If not controlled, instabilities cause acoustic modes to ring at very large amplitudes, sufficient to disrupt operation of a sensitive detector.

We show here that for the proposed Advanced LIGO (AdvLIGO) parameters, the conditions for instability are indeed met for a substantial number of acoustic modes, specifically in the frequency range 28-35kHz and 68-72 kHz. Because the acoustic mode density in this frequency range is much greater for fused silica test masses than for sapphire test masses, the number of parametrically unstable modes is much greater for fused silica, and they generally have much higher parametric gain. After demonstrating the magnitude of the instabilities, we will present a method by which the parametric instabilities may be detuned. Again this is more effective for sapphire than for fused silica because the mode spacing in the relevant frequency range is about 6 times greater for sapphire than fused silica. We will also show that it is unlikely to be possible to predesign against the parametric instabilities unless (a) the error of calculating normal mode frequencies in standard Finite Element Modeling (FEM) software can be improved to less than the cavity bandwidth (~30 Hz), (b) the test mass density inhomogeneity is known, (c) the mirror radius of curvature can be specified to better than one percent.

Under some circumstances the coupling between optical modes and acoustic modes can be symmetric [9]. This means that the energy coupled from a Stokes mode [6] to an acoustic mode converts back to the anti-

Stokes mode [6], thus preventing the instability. In the case of km-scale GW detectors this condition is not satisfied, as we discussed below. The bandwidth of the arm cavity of the km-scale detector is very low. The acoustic modes are far outside this bandwidth, so there is negligible coupling of the acoustic modes to the cavity fundamental mode (TEM_{00}). Instead they couple to the high order transverse modes (TEM_{mn}). The frequency differences between the TEM_{00} mode and TEM_{mn} modes are,

$$\Delta_{-} = \omega_0 - \omega_1 = \frac{\pi c}{L}\left(k_1 - \frac{m+2n}{\pi}\arccos\left(\sqrt{(1-\frac{L}{R_1})(1-\frac{L}{R_2})}\right)\right)$$

$$\Delta_{+} = \omega_{1a} - \omega_0 = \frac{\pi c}{L}\left(k_{1a} - \frac{m+2n}{\pi}\arccos\left(\sqrt{(1-\frac{L}{R_1})(1-\frac{L}{R_2})}\right)\right) \quad (1)$$

Here ω_0 is the fundamental mode frequency, ω_1 is the Stokes mode frequency and ω_{1a} is the anti-Stokes mode frequency, L is the cavity length, R_1 and R_2 are the mirror radii, k_1 and k_{1a} are longitudinal mode indices, and m and n are transverse mode indices.

By inspection of equation 1, Δ_- is not necessary equal to Δ_+. The TEM_{mn} modes are not always symmetric to the TEM_{00} mode. Thus the Stokes mode and anti-Stokes mode are not normally resonant in the cavity simultaneously as shown in figure 1. Even in the case that the TEM_{mn} mode frequencies are symmetric to the fundamental mode, the overlapping between the optical field and the acoustic mode field will be different. In addition, the mechanical impedance matching of the radiation pressure force applied to the acoustic modes is not necessarily equal, and depends on the optical field distribution. Thus the coupling between an acoustic mode and the various optical modes will normally be unequal. Symmetric coupling is therefore very rare.

Braginsky [7] has shown that the effective parametric gain R is given by* :

$$R = \frac{2PQ_m}{mcL\omega_m^2}\left(\frac{Q_1\Lambda_1}{1+\Delta\omega_1^2/\delta_1^2} - \frac{Q_{1a}\Lambda_{1a}}{1+\Delta\omega_{1a}^2/\delta_{1a}^2}\frac{\omega_{1a}}{\omega_1}\right) \quad (2)$$

When R>1 parametric effect will excite the acoustic mode. Here P is the total power inside the cavity, Q_1 *and* Q_{1a} are the quality factors of the Stokes and anti-Stokes modes, Q_m is the quality factor of acoustic mode, $\delta_{1(a)}=\omega_{1(a)}/2Q_{1(a)}$, m is the test mass's mass, L is the cavity length, $\Delta\omega_{1(a)}=\omega_0-\omega_{1(a)}-\omega_m$ is the possible detuning from the ideal resonance case, and Λ_1 and Λ_{1a} are the overlap factors between optical and acoustic modes. The overlap factor is defined as [6],

$$\Lambda_{1(a)} = \frac{V(\int f_0(\vec{r}_\perp) f_{1(a)}(\vec{r}_\perp) u_z d\vec{r}_\perp)^2}{\int |f_0|^2 d\vec{r}_\perp \int |f_{1(a)}|^2 d\vec{r}_\perp \int |\vec{u}|^2 dV} \quad (3)$$

Here f_0 and $f_{1(a)}$ describe the optical field distribution over the mirror surface for the fundamental and Stokes (anti-Stokes) modes respectively, $\vec{u}$ is the spatial displacement vector for the mechanical mode, u_z is the component of $\vec{u}$ normal to the mirror surface. The integrals $\int d\vec{r}_\perp$ and $\int dV$ correspond to integration over the mirror surface and the mirror volume V respectively.

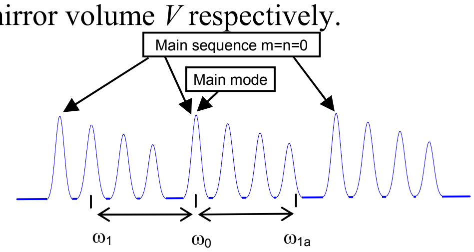
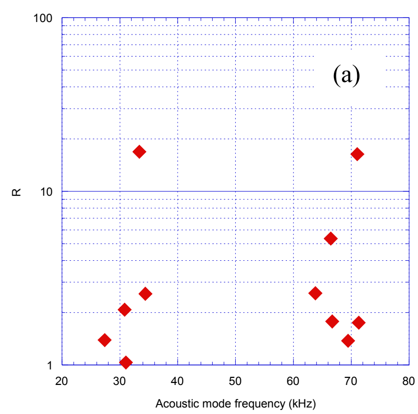


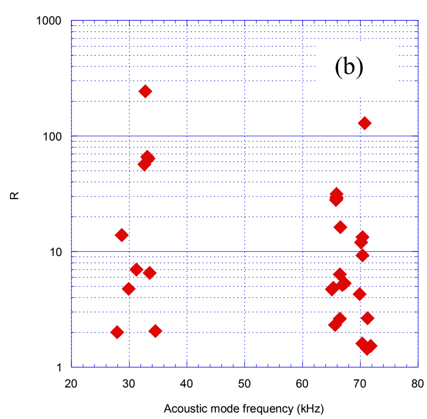
FIG. 1. High order transverse modes of the cavity and Stokes and anti-Stokes frequencies scattered by the acoustic mode of frequency ω_m. Here $\omega_1=\omega_0-\omega_m$ and $\omega_{1a}=\omega_0+\omega_m$.

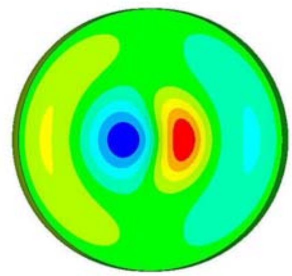
Using FEM (ANSYS®), we show in Figure 2 (a) and 2 (b) the parametric gain R of the AdvLIGO test masses [10] (Sapphire and Fused Silica) acoustic modes close to the first and the second order transverse modes. The simulation for the acoustic modes close to the transverse

* In the unusual case that the Stokes mode is within about 0.2 Hz of the optical cavity mode for the proposed AdvLIGO parameters, the R factor for the acoustic mode is also affected by the power recycling conditions.

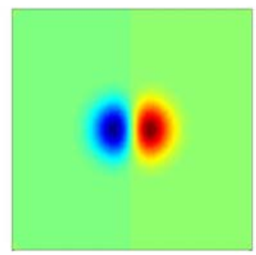
modes higher than the second order is not included in this letter. Higher order modes generally have lower parametric gain due to their diffraction losses. Here only those modes with R>1 are displayed. Figure 2 (c) shows a particular acoustic mode structure with frequency close to the frequency difference between TEM_{00} and TEM_{10} modes. Figure 2 (d) shows the TEM_{10} mode optical field distribution. The similarity of these mode structures is apparent.







(c)



(d)

Fig. 2. (a) and (b), all the acoustic modes with R>1 for sapphire and fused silica respectively, assuming AdvLIGO parameters [10], (c) a typical test mass acoustic mode structure, and (d) the field distribution of the cavity TEM_{10} mode showing a high overlap of the acoustic and optical mode structures.

Because of the large acoustic mode density around 30 and 70 kHz there are significant numbers of acoustic modes which have the potential (R>1) to be unstable. Fused silica test masses have ~3 times more potentially unstable acoustic modes. For the nominal AdvLIGO parameters, the maximum parametric gain R for sapphire test masses is ~17, compared with ~250 for fused silica.

It is possible that the parametric gain could be much larger than the values mentioned above for several reasons. (a) Standard FEM methods for calculating the acoustic mode frequencies have errors much larger than the cavity bandwidth [7]; (b) The suspension system may change acoustic mode frequencies; (c) An error of one km (~2%) in the mirror radius of curvature results in a cavity mode spacing error of ~20 Hz which is comparable to the cavity bandwidth (~30 Hz). Finally thermal lensing causes the radii of curvature of test masses to vary from their nominal values. Thus the worst case, where the acoustic mode frequency is very close to the frequency difference between the fundamental mode and a high order transverse mode can not be definitely avoided. For instance, in a sapphire test mass, if the acoustic mode at frequency of 33.388 kHz (the highest dot in figure 2 (a)) has a 511 Hz error; the parametric gain R could be as large as $\sim 10^4$.

The fact that the cavity mode spacing changes with the mirror radius of curvature (see Eq.1) also provides an opportunity to tune out the most unstable acoustic modes. Lawrence et al [11] and Degallaix, et al [12] have demonstrated that by using a heating ring near the front of the test mass you can adjust the test mass radius of curvature to effectively compensate for thermal lensing. The GEO project [13] has used this method to compensate the mismatch of radii of curvature of two interferometer mirrors. Here we propose a similar method, with the heating ring at the back of the test mass to tune the cavity mode frequencies. Figure 3 shows a typical temperature distribution in a test mass. In the large radius of curvature mirrors of AdvLIGO substantial changes in radius of curvature can be achieved. Figure 4 shows the AdvLIGO end test mass radius of curvature and the relative R as a function of the maximum temperature difference across the test mass when heated by a heating ring with variable heating power. The radius of curvature changed from ~54 km to ~ 48 km, corresponding to the maximum temperature difference across the test mass from 0 °K to ~0.15 °K for sapphire (average mirror temperature changed from 300 °K to ~303°K) and from and 0 °K to ~1.5 °K for fused silica (average mirror temperature changed from 300 °K to ~301°K). If one consider only a single acoustic mode this tuning is sufficient to reduce R to 1% of its original value. Unfortunately, there are many potential acoustic modes around. When tuning the cavity modes away from a particular acoustic mode we generally increase the coupling to nearby acoustic modes. In sapphire test masses, the frequency gap is ~1 kHz. Tuning the cavity mode to a point between two acoustic modes minimizes the parametric gain of both. The acoustic mode gap of ~ 200 Hz for fused silica test masses makes such tuning much less effective. Thus we see that in fused silica (figure 5 (b)) it is impossible to tune to parametric gain R to less than 100, an order of magnitude higher than that for sapphire (figure 5 (a)). Figure 6 shows the total numbers of acoustic modes whose parametric gain R are greater than 1 and 10 as a function of mirror radius curvature for sapphire and fused silica respectively. The optimum tuning of fused silica leads to 28 modes with R ranging from 1 to 113 when the radius curvature reduced to 32 km. In sapphire there are 11 modes with R ranging from 1 to 9 at the optimum tuning point corresponding to about 45 km radius of curvature.

Tuning the arm cavity radius of curvature also changes the TEM_{00} mode waist size and may mismatch the arm cavity with the recycling cavities. Over modest tuning ranges, this effect is small. For example, when the radius of curvature of the AdvLIGO end test mass changes from 54 km to 48 km, the arm cavity beam waist changes from 5.9 cm to 5.8 cm. The introduced loss due to the mode mismatching is 300ppm which is acceptable in relation to the recycling mirror transmission (6% for the power recycling mirror and 7% for the signal recycling mirror). For the reason already given the parametric gain compensation will be different for each optical cavity. The inevitable mismatch between the two arm cavities may create extra power at the interferometer dark port. It can be prevented from reaching the photodetector by an output mode cleaner.

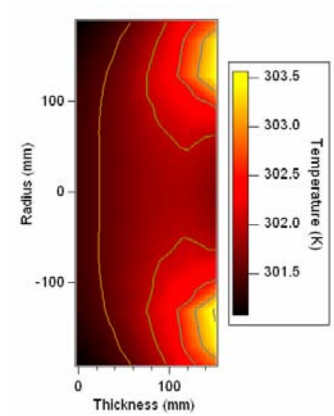


Figure 3: FEM model of the temperature distribution of the mirror with 5W heating power.

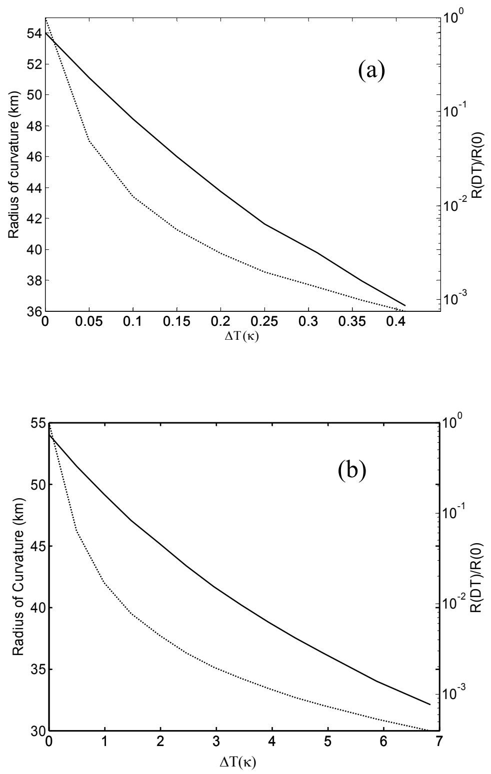


Figure 4: The dependence of the parametric gain R (dotted line) and the mirror radius of curvature (solid line) on the maximum temperature difference across the test mass, (a) for sapphire and (b) for fused silica

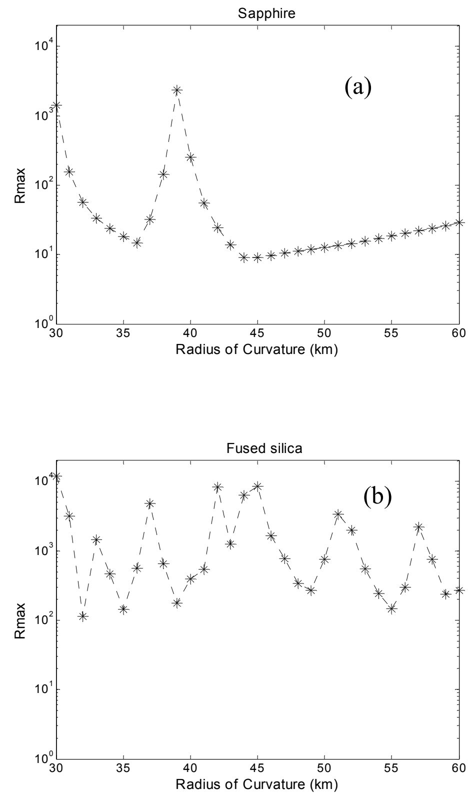
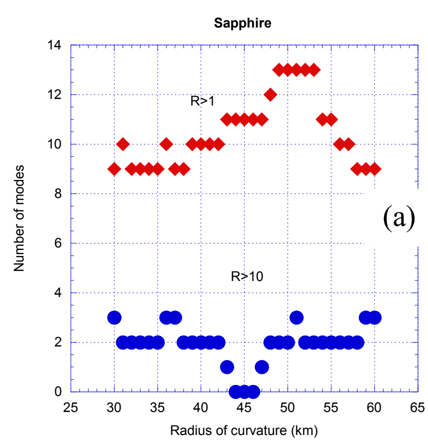


Fig. 5. The dependence of the maximum parametric gain R of all acoustic modes on mirror radius of curvature, (a) for sapphire and (b) for fused silica.



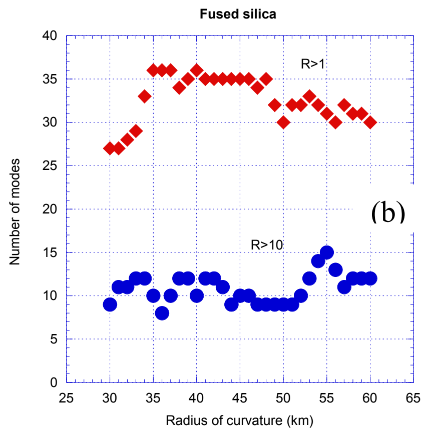


Fig. 6. The numbers of acoustic modes with R>1 and R>10 when tuning the radius of curvature for (a) sapphire, and (b) fused silica.

In summary, it is inevitable that parametric instabilities will appear in AdvLIGO. By thermally tuning the arm cavity mirror radius of curvature we can minimise the instability gain. The thermal tuning is feasible and need not introduce extra noise. While the data has been applied to AdvLIGO parameters, it is also directly relevant to VIRGO interferometer.

We note that the instabilities discussed here refer to only one test mass. Each test mass will experience instability and the optimum tuning for pairs test masses will require further study.

Braginsky et al [14] has proposed the use of small but high finesse detuned cavities as a means of low noise "tranquilizing" of parametric instabilities. The extra cavities needed in the scheme create extra complexity into an already complex system. Feedback schemes similar to the demonstrated cold damping of thermal noise [15] could be another solution, again adding complexity.

The similar analysis for the Gingin high optical power facility will be presented in a separate article. The results show that the Gingin facility is ideally suited for experimentally testing parametric instabilities and their control.

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