LIGO Laboratory / LIGO Scientific Collaboration

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TRIPLE PENDULUM AT LASTI
INSTALLATION AND CHARACTERIZATION

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Advanced LIGO Suspensions/Isolation Working Group

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1 INTRODUCTION

In order to test and facilitate the future implementation of the triple pendulum suspension for advanced LIGO. LASTI has been carrying out some tests on a full scale controls prototype of the triple pendulum suspension for a modecleaner mirror. This triple pendulum, assembled in Caltech has been installed in the X HAM of LASTI in order to :

1. Test and improve the installation procedure
2. Characterize the prototype and compare it with the numerical model
3. Identify new problems (if any) occurring in real conditions
4. Test the control loop and check they meet the damping requirements

2 INSTALLATION

The pendulum has been installed on the optical table of the X HAM in LASTI. A short time before, the HEPI actuators and piers have been installed on this HAM to be able to use it as a shaking table. The difference with the advanced LIGO configuration is the lack of the internal seismic isolation.

The procedure of installation has been checked and modified to facilitate future implantation of the triple pendulum, the document is E040277

The electronics has been modified to enable us to do some extensive scientific tests on the suspension/HEPI system, the usual suspension/dSPACE electronics have been merged with the HEPI electronics to be able to record and monitor in real time :

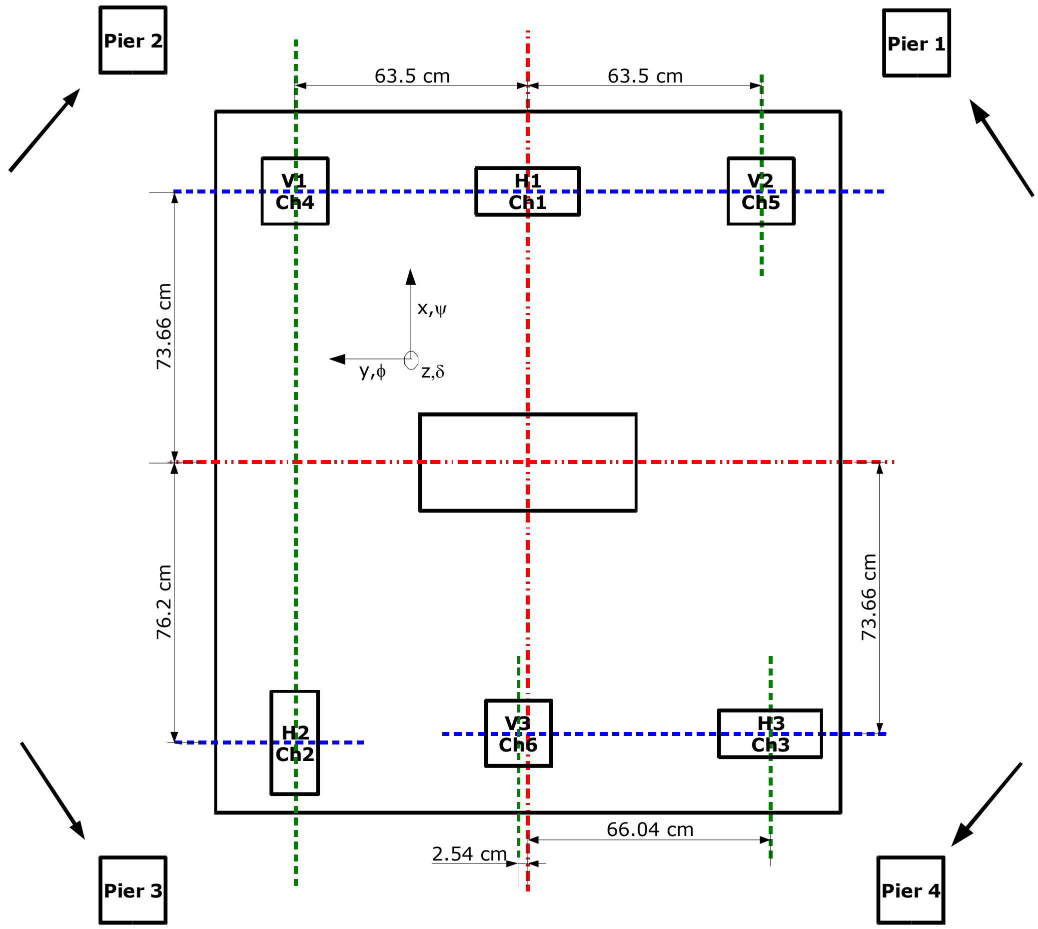
1. The external position of the HEPI platform with position sensors located in the piers
2. The velocity of the optical table with 6 geophones (see below)
3. The position in the 6 dofs of the first mass of the pendulum
4. The position in 3 dofs of the 2 other masses of the pendulum

This configuration also enables us to drive different parts of the system to characterize its dynamic behavior. We can drive :

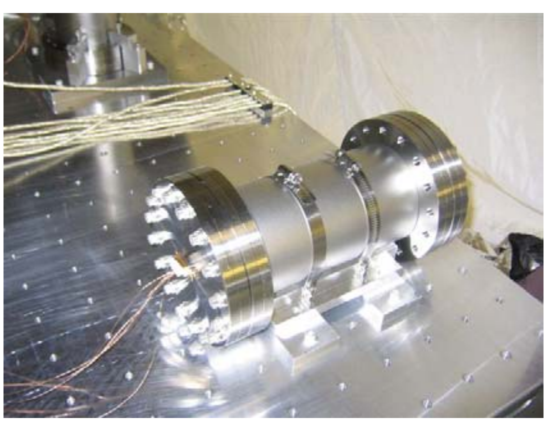
1. The 8 HEPI actuators
2. The 6 dof of the pendulum's first mass
3. The 3 dof of the pendulum's 2 other masses (not used)

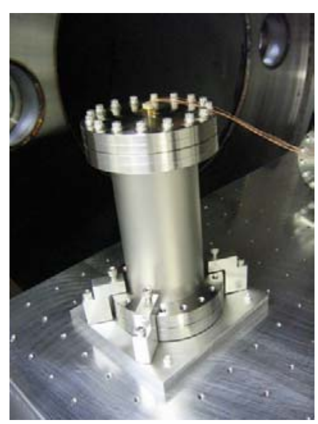
2.1 GEOPHONES AND OPTICAL TABLE

In order to characterize the pendulum dynamic, 6 geophones (L4C 1Hz) have been installed around it on the optical table, they enable us to measure the velocity of the table in the 6 dof. The configuration of the optical table with the pendulum and the 6 geophones is the following one :



Installation of the 6 geophones and the pendulum on the optical table
H1,H3 in Y direction
H2 in X direction
V1,V2,V3 in Z direction

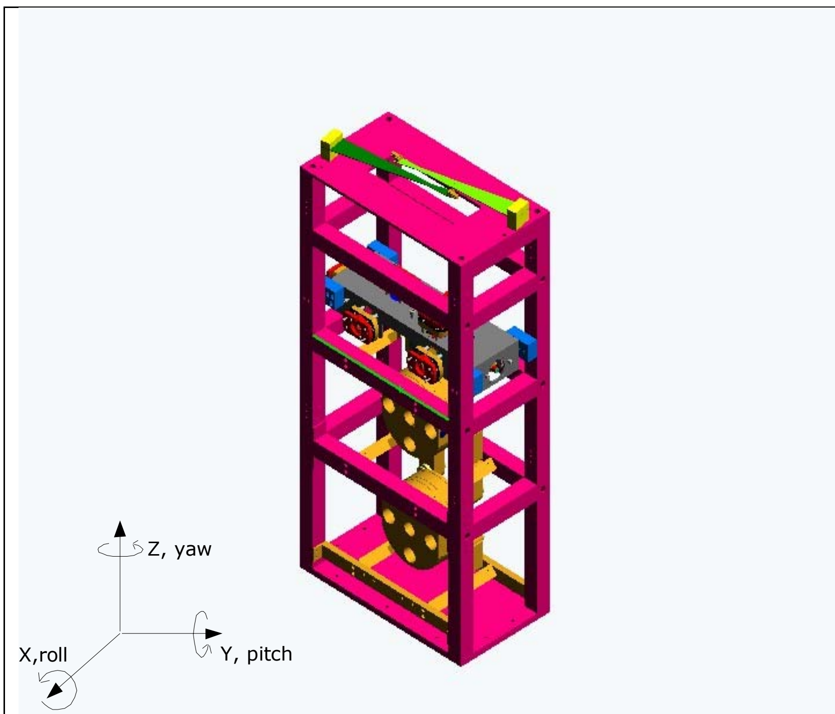




Ham optical table with pendulum and geophones installed

See appendix 1 for details about the coordinate system used for the geophones

2.2 Vocabulary

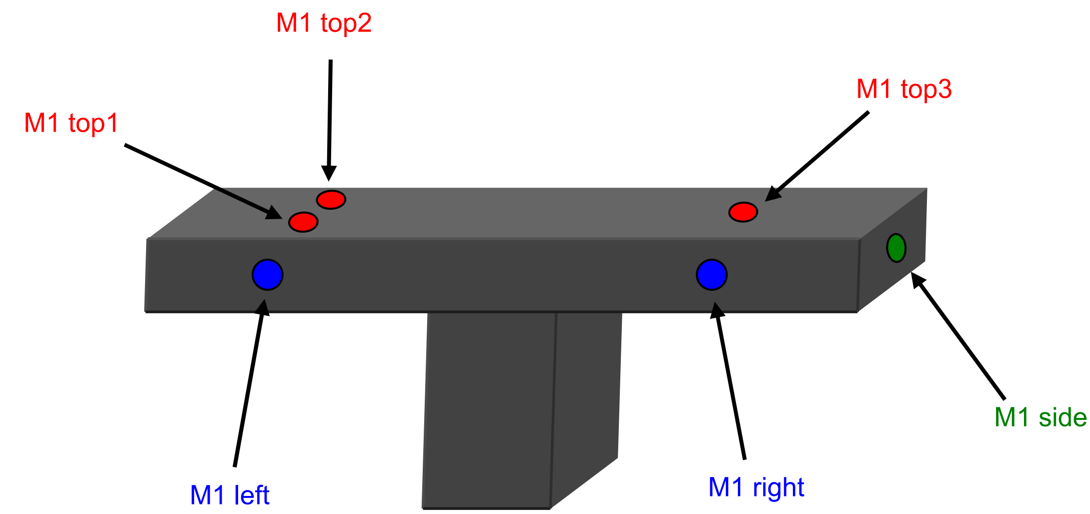


14 sensors and 14 actuators

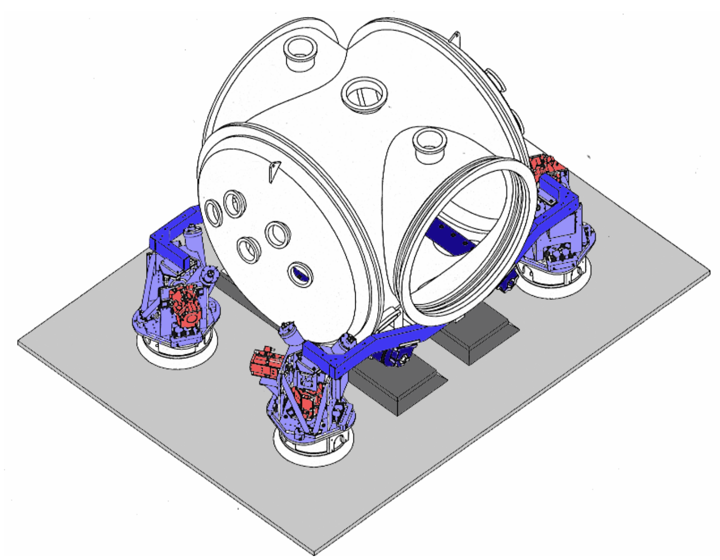
Mass1
1. Top1
2. Top2
3. Top3
4. Left
5. Right
6. Side

Mass2
1. UL (upper left)
2. UR (upper right)
3. LL (lower left)
4. LR (lower right)

Mass3
1. UL
2. UR
3. LL
4. LR



Position of the OSEMS around mass 1



6 sensors on optical table

- 6 geophones (L4C 1Hz)
- OP X, OP Y1, OP Y2
- OP Z1, OP Z2, OP Z3

8 sensors in HEPI piers

- 8 position sensors
- H1, H2, H3, H4
- V1, V2, V3, V4

8 actuators

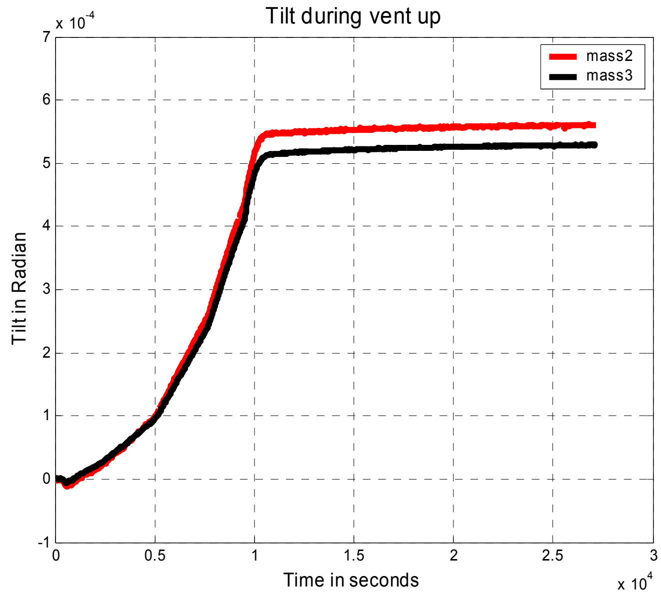
- HEPI actuators in piers
- H1, H2, H3, H4
- V1, V2, V3, V4

2.3 Alignment issues

2.3.1 Vacuum tilt

During the installation, we noticed a problem which was disturbing in LASTI. Although we did the alignment as well as possible, we noticed that pumping the air out of the chamber significantly tilted the floor due to the asymmetry of the chamber (one side of the chamber is connected to the tube, which generates a difference of pressure between the 2 sides of the chamber, thus producing a force that tilts the floor), this tilt produced a large misalignment for the pendulum, the osems of the bottom mass simply went out of range (~1mm motion)).

HEPI is supposed to compensate these problems and make the alignment much easier, unfortunately, in our case, the tilt was so large that we needed to push the hydraulic actuator really far to come back to a correct position and thus we lost the range we had with HEPI and made the driving more difficult.



Tilt measured with the osems on mass 2 and 3 during the vent up of the vacuum chamber
The floor is tilting due to the asymmetry of the chamber

For LASTI the choice has been made to modify the position of the platform with the 8 external springs in order to have the pendulum aligned and the osems in range when the chamber is in vacuum.

For advanced LIGO and the sites, we think that the floor is a lot stiffer and that the tilt due the vacuum should be less, thus we shouldn’t have this problem.

2.3.2 Other alignment issues

During the several months we worked on the pendulum, we had to move the pendulum and noticed how difficult the alignments of the osems were, Many times, after we installed the pendulum on a new table or after a long period locked, we had to realign the osems, even though we made our best attempt to level the table on which the pendulum was sitting. However, this problem never happened in the tank once we solved and understood the “vacuum tilt”.

3 CHARACTERIZATION

After the installation was completed and the procedure checked, the first step was to characterize the pendulum dynamic to compare it with the numerical model. The actuators (osems and HEPI) are used to drive the pendulum while the sensors measure its motion to create the transfer functions.

3.1 ACQUISITION METHOD

The drive can be done with 2 different methods :

- Each actuator one by one : it has been used to calibrate the actuators
- Modally : the actuators are combined to drive a specific mode, this is the method we use to save time and improve accuracy

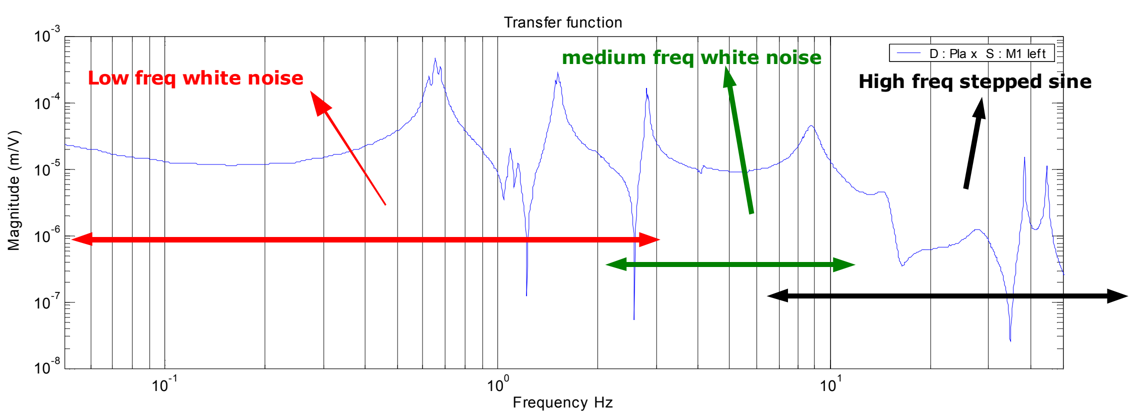
The 28 sensors are all acquired simultaneously by constantly transferring the data acquired by dSPACE to the PC running the software….while it saves time and improve the analysis to acquire every sensors at the same time, it also limits the time sampling we can run the experiment, as a matter of fact, we are limited by 3 factors :

1. The small dSPACE memory
2. The transfer rate between dSPACE and the PC
3. The Matlab calculation ability with large quantity of data

A stepped sine drive at the frequencies we are interested in would be the simplest way but it would also take days to get results, Moreover, since we know that the system is perfectly linear for all small displacements, we can use white noise drive for low frequencies, it has the advantage of being relatively short in time for a very good accuracy (1 hour for one drive). In order to amplify the drive as much as possible, we filter the white noise with a band pass filter to keep only the frequencies we want to study and increase the drive's power (for example a band pass between 0 Hz to 3Hz for very low frequencies and 2Hz to 12Hz for medium ones).

For high frequencies (>~8Hz) however, we can use stepped sine, it is more accurate and takes a very short time in high frequency. In order to prevent the pendulum from moving too much, we filter apply a frequency dependent gain to this stepped sine to reduce the drive for the resonances of the pendulum.

We also calculate and record the coherence between the drive and the sensors and we then merge the data of the different parts by taking the more coherent one



Data merging after acquisition
Low frequencies and medium frequencies are acquired with white noise drive (different parameters for each part)
High frequencies are acquired with stepped sine drive

3.2 DATA CORRECTIONS

The raw data need to be corrected to remove the effect of the whitening boards, sensor's responses and the Analog Digital Conversion. Several corrections are applied:

On the geophone measurements:
- The geophone response is corrected
- The velocity is integrated to position to compare it with other sensors

On the position sensors
- The whitening applied by the HEPI electronics is removed

On the Osems
- The whitening applied by the SUS electronics is removed

On every sensor
- The decimation phase shift is corrected
- The coordinate system is changed to get the transfer functions in the chosen coordinate system $[TF_{new}] = [P][TF_{old}]$

3.3 CALCULATING THE TRANSFER FUNCTION AND GETTING RID OF NOISE

After the first acquisition, we realized that the data were noisy in the range 7-12 Hz, especially for the weak transfer functions such as the cross coupling. Using stepped sine or white noise didn't change many things. We realized that this noise was due to a large amplification of the ground noise by the external structure for these frequencies:

- X resonance at 7Hz
- Y resonance at 9Hz
- Z resonance at 12Hz

You can find other documents describing this issue on the LASTI Ilog

We decided to use a special method to calculate the transfer functions with the white noise drive, this method enabled us to calculate the transfer function while reducing the noise at the same time

3.4 WHITE NOISE TRANSFER FUNCTION CALCULATION

The first thing to do when driving a white noise is to try to increase the power of the drive as much as possible, in order to do that, it is easy to apply a band pass filter (as we saw above) to keep only the frequencies you want to study, which enable you to increase the drive's gain. Another easy change is to add a filter to "bump" the frequencies that are hard to measure because the transmission is weak or because the noise is too large. After we had done that, we realized that the noise for those 3 frequencies was too big to study the cross coupling transfer functions and several other transfer functions which were too weak and dominated by noise. We then decided to use a method to reduce the noise coming from these HEPI resonances :

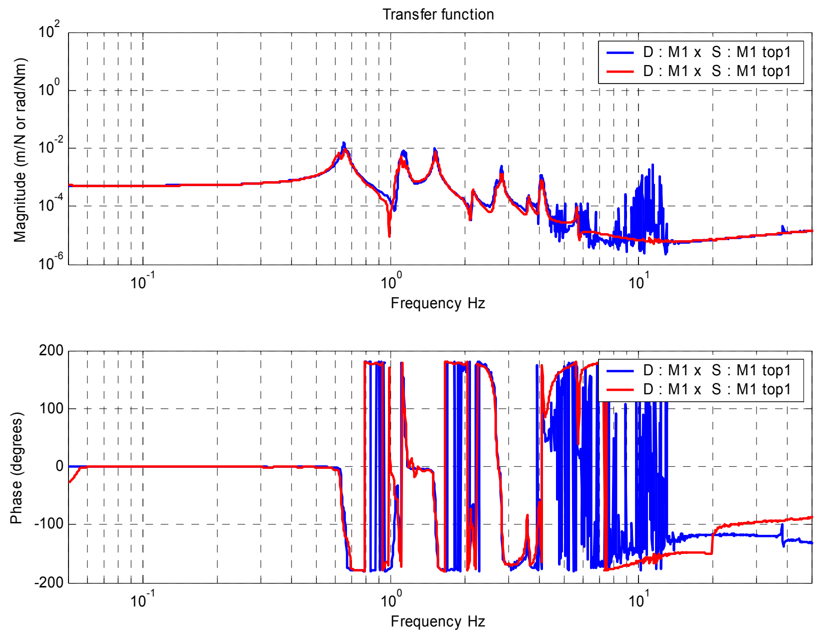
The idea was to put sensors on the floor to sense the ground motion…and to remove directly the coherent part between the sensor measurements and the ground measurements while calculating the transfer functions:

3.4.1 Mathematics

See appendix 2 for details about the noise reduction method in white noise drive acquisition

3.4.2 Results

The method gives excellent results, as you can see on the example below, the noise due to the resonances has been reduced:



Improvement of the transfer function quality thanks to the noise reduction method with white noise drive
Before noise reduction method (blue)
After noise reduction method (red)

3.5 Stepped sine transfer function calculations

We use a stepped sine excitation only for high frequencies, since we have no problem with noise in this range, we don't need to apply any special method to get good and accurate results. A sine is driven at a specific frequency for about 200 periods or more. The calculation is then very simple, we apply a Fourier transform at one point (the frequency of the drive) to get the value of the magnitude and phase of the transfer function.

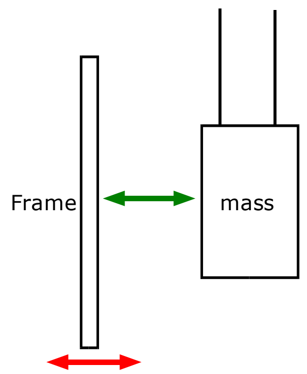
3.6 Getting the transfer function from frame to pendulum

The transfer functions from the optical table motion to the pendulum motion are a bit harder to get than the other.

Those transfer functions show the dynamic of the pendulum when the frame (or optical table) is moving, of course it is impossible to force a pure motion for the optical table, the only thing we can do is to use the HEPI actuators to shake the table and by measuring the table's motion and the pendulum's motion at the same time, we can calculate the transfer function we are looking for.

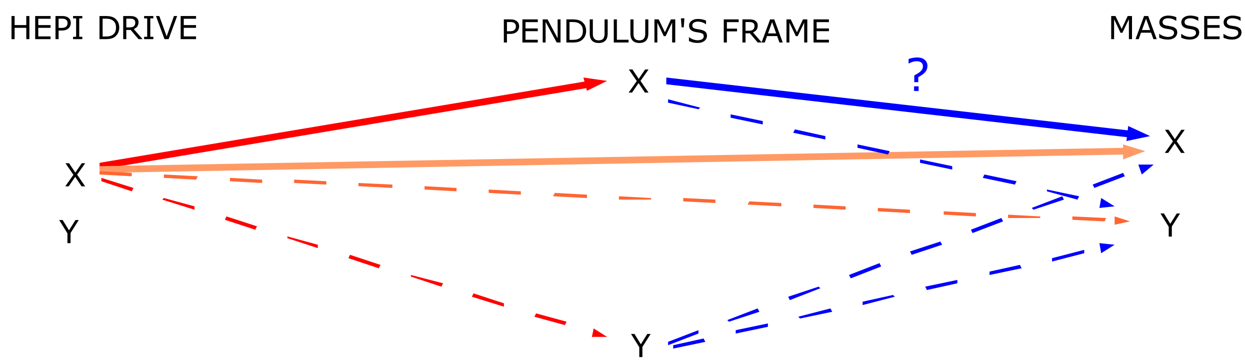
Unfortunately this is not that easy for 2 main reasons :

The osems are not measuring an inertial motion but the relative displacement between the frame and the pendulum's masses



Osems are measuring a relative displacement between the frame and the pendulum's mass
REAL POSITION = OSEM - FRAME

The Drive in one dof is not perfect and actually moves the optical table (and frame) in the 5 other dofs

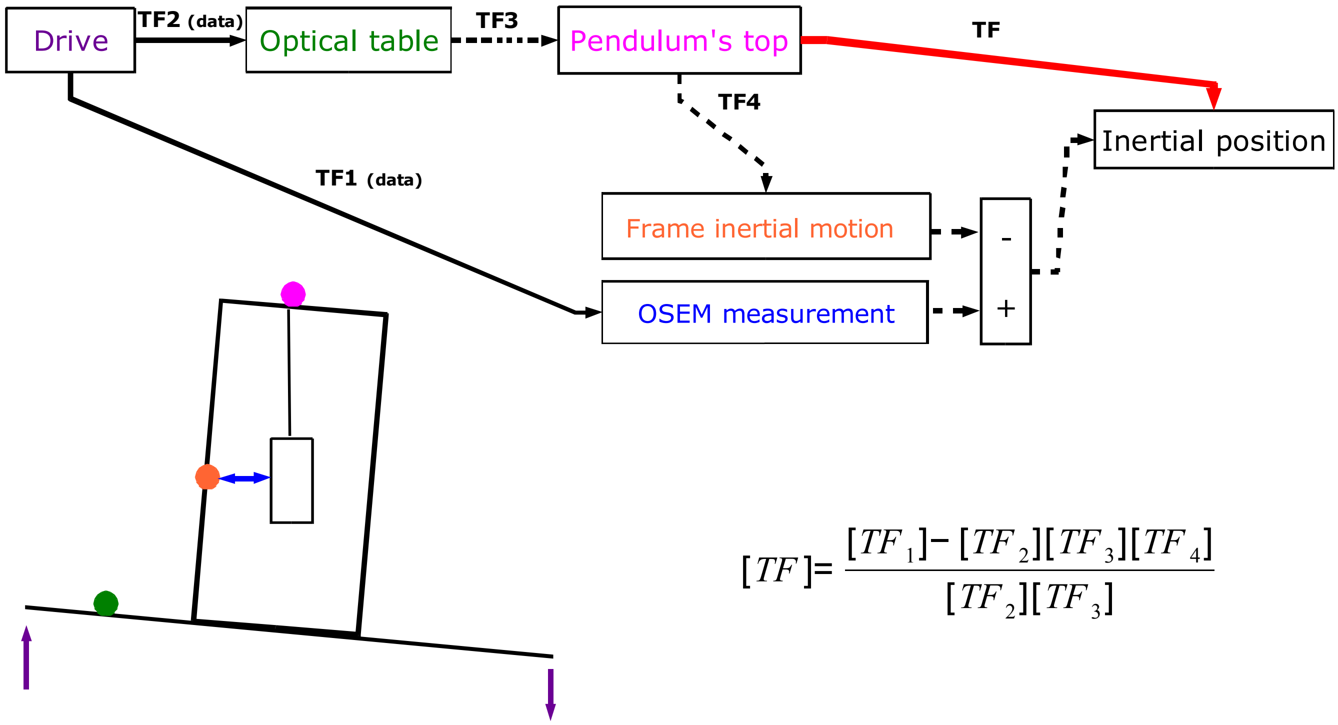


The drive with HEPI is not pure
Additional paths of the motion in dashed lines
We want to calculate the blue transfer functions

The solution to the second problem is simple to solve : we need to take into account all the transfer functions for every drive and to use the matrix form to get the good transfer function :

See appendix 2 for the mathematics

Then, to solve the first problem and to take into account the many different coordinate systems, we can draw a schematic of the system :



$$[TF] = \frac{[TF_1] - [TF_2][TF_3][TF_4]}{[TF_2][TF_3]}$$

How to calculate the real transfer function from frame to mass by taking into account different coordinate systems and relative measurements :

- *Optical table, Pendulum's top and frame motion are all given by geophones (inertial measurement), they correspond to different coordinate systems but are all inertial motion. TF3 and TF4 are simple matrix used to change the coordinate system.*
- *OSEM measurement is the relative displacement between the frame and the pendulum*

The OSEM measurement gives the relative displacement between the suspension frame (sitting on the table) and the top mass of the suspension. The inertial motion of the table is measured by geophones. We consider the frame motion and the table motion to be the same assuming we are at frequencies below the first resonant frequency of the frame. Thus subtracting the OSEM measurement and the geophone measurement we obtain the inertial motion of the mass.

This enables us to calculate the real transfer function by getting rid of the problem of non-pure drive and the problem of relative sensors. As we will see below, the results unfortunately depends a lot on the calibration of the sensors especially when the masses are not moving (anti resonances and high frequencies), in that case, have to subtract 2 very small numbers from each other and that creates a lot of inaccuracy.

3.7 NUMERICAL MODEL

Once the pendulum's dynamic has been characterized, we want to compare it with the numerical model. 3 models have been made for the triple pendulum :

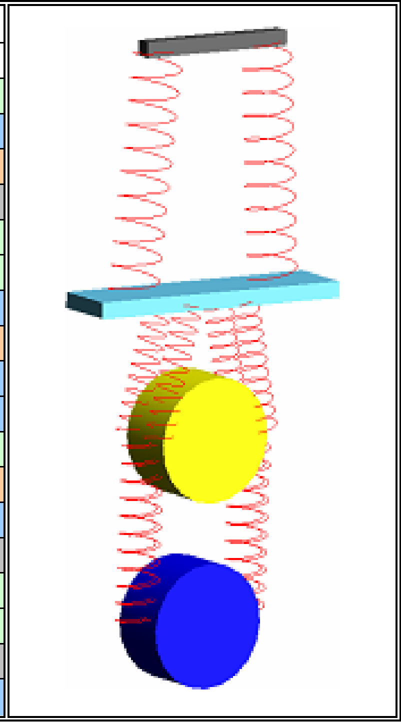
- A Matlab model
- A Mathematica model
- An Adams model (Adams is a dynamic solver)

The Matlab model has been used to compare the modeling with the real measurements, the parameters of the model have been modified to match the pendulum we built for LASTI and 2 problems have been identified with the model:

1. The transfer functions from the roll of the frame (called roll0) to the roll for the masses (roll1, roll2, roll3) were wrong in the Matlab model. It has been corrected by using the Mathematica model and we checked the results with the ADAMS model, the problem was a wrong term in the B matrix of the model.

2. The parameter “bottom wire diameter” was wrong; we thought we used a wire having a 0.15mm diameter for the assembly. By comparing the model and the real measurement, we noticed some important differences in pitch transfer functions (2 last frequencies off by 10-15%). We finally realized the wire we used was actually a 0.2mm diameter, thus correcting the problem we had.

Mode	Measurement	Adams	Matlab	Adams error %	Matlab error %
1 (x&pitch)	0.65	0.67	0.67	-2.57	-2.69
2 (y&roll)	0.66	0.67	0.68	-2.21	-2.33
3 (yaw)	1.09	1.09	1.09	0.40	0.23
4 (z)	1.10	1.19	1.19	-7.75	-7.86
5 (x&pitch)	1.15	1.11	1.11	3.42	3.37
6 (x&pitch)	1.51	1.53	1.53	-1.04	-1.26
7 (y&roll)	1.52	1.53	1.53	-0.43	-0.64
8 (yaw)	1.97	1.96	1.96	0.70	0.56
9 (y&roll)	2.16	2.14	2.14	1.00	0.84
10 (y&roll)	2.67	2.85	2.84	-6.71	-6.45
11 (x&pitch)	2.82	2.83	2.82	-0.26	-0.01
12 (yaw)	3.55	3.52	3.51	0.95	1.16
13 (y&roll)	3.61	3.77	3.77	-4.48	-4.56
14 (z)	4.06	4.25	4.26	-4.74	-4.87
15 (x&pitch)	4.10	4.40	4.40	-7.37	-7.43
16 (x&pitch)	5.65	5.70	5.70	-0.83	-0.88
17 (z)		46.04	46.05		
18 (y&roll)		65.47	65.49		



Although the Matlab model was accurate enough for the triple pendulum, especially with the Mathematica model to validate it; we worked on a third model with a dynamic equations solver software called ADAMS which is available at MIT. This software enables us to build the pendulum in 3D and by describing the different parts dynamically (you need to design and describe the parts with their mass, inertia…. and the links between those parts), the software then solve the dynamic of the system by solving the Lagrange equations. You can then visualize your modes, export your TF and frequencies, export the entire state space model…. This was not necessary for the triple pendulum but we thought it would be a good test to try the software and also a good start for a future possible SEI+SUS model.

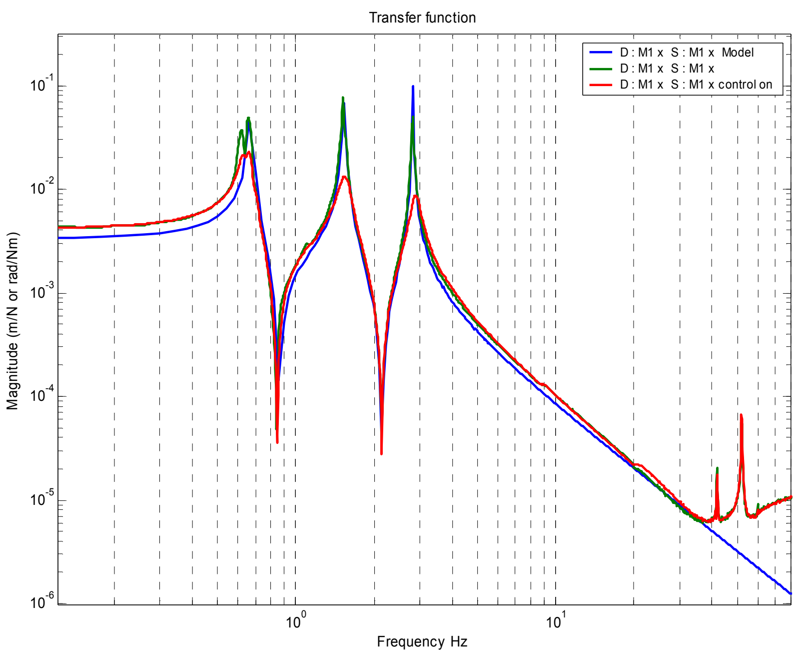
3.8 CONTROL LOOP

The system identification has been with and without local active damping. We can then compare the model, the pendulum and the damped pendulum. We are also recording a step response to study the behavior of the pendulum in real time with control off or on. The control used is the one that has been sent by Caltech and consists in a simple filter on each degrees of freedom with a gain designed to match the control requirements.

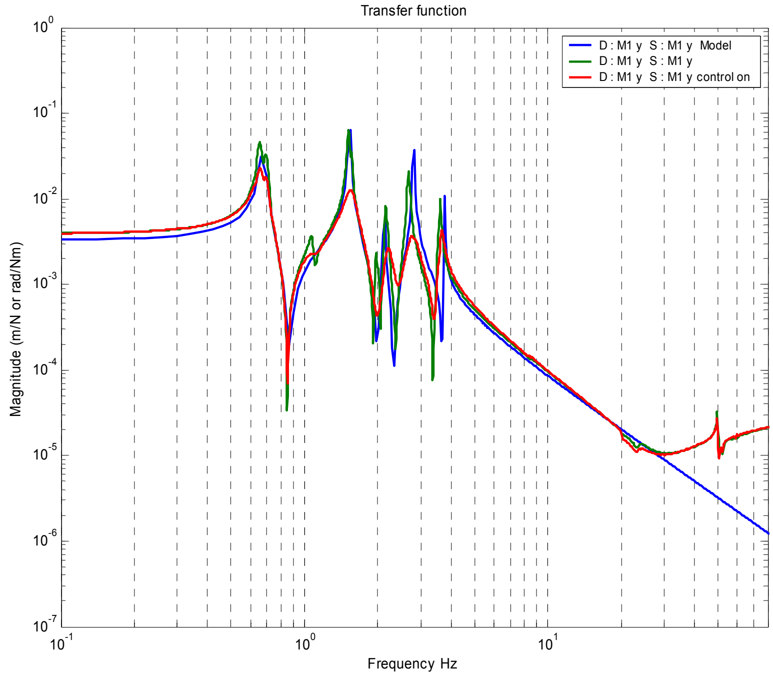
3.9 RESULTS, COMPARISON WITH MODEL

Below are the results of the system identification, we plot the main transfer function (we of course can't show the 312 transfer functions the software calculates so we show only the ones in the main directions :

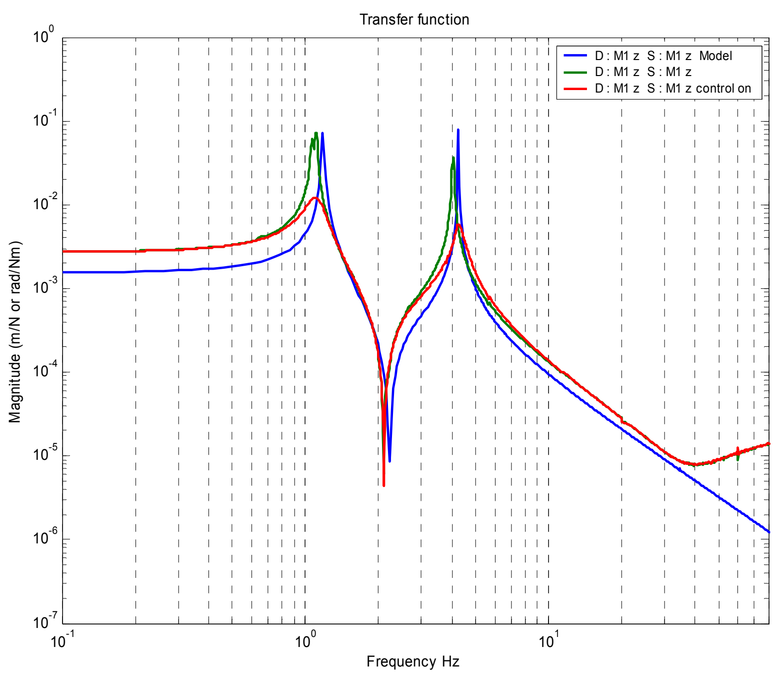
3.9.1 Osem drive :



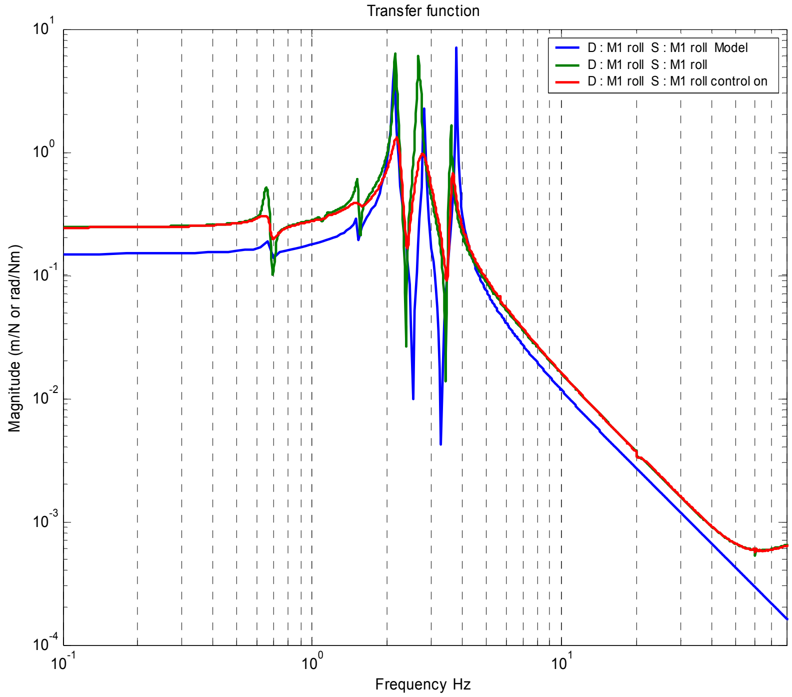
Transfer function from mass1 X drive to mass1 X
Matlab model (blue)
Measurement, control off (green)
Measurement, control on (red)
See 3.9.5 for the 2 high frequencies resonances (41 52 Hz)



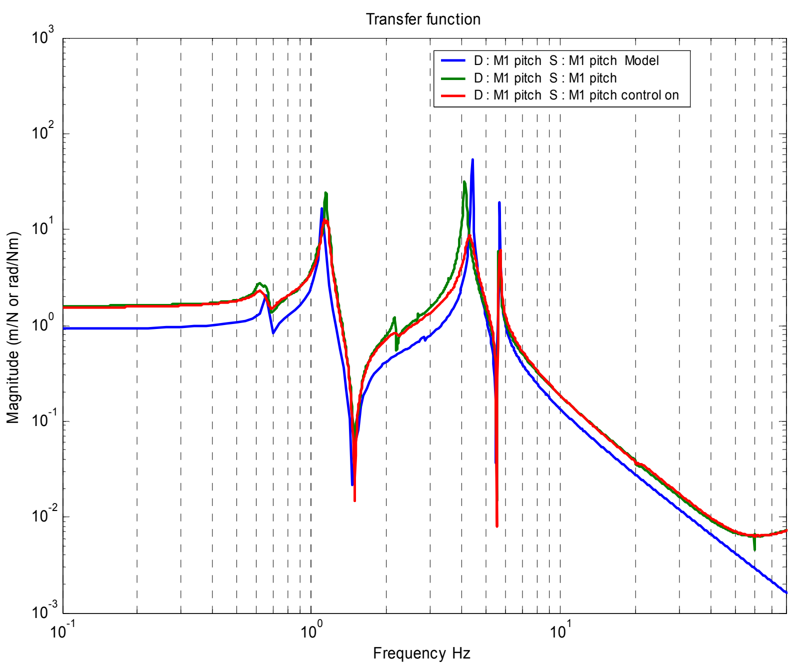
Transfer function from mass1 Y drive to mass1 Y
Matlab model (blue)
Measurement, control off (green)
Measurement, control on (red)



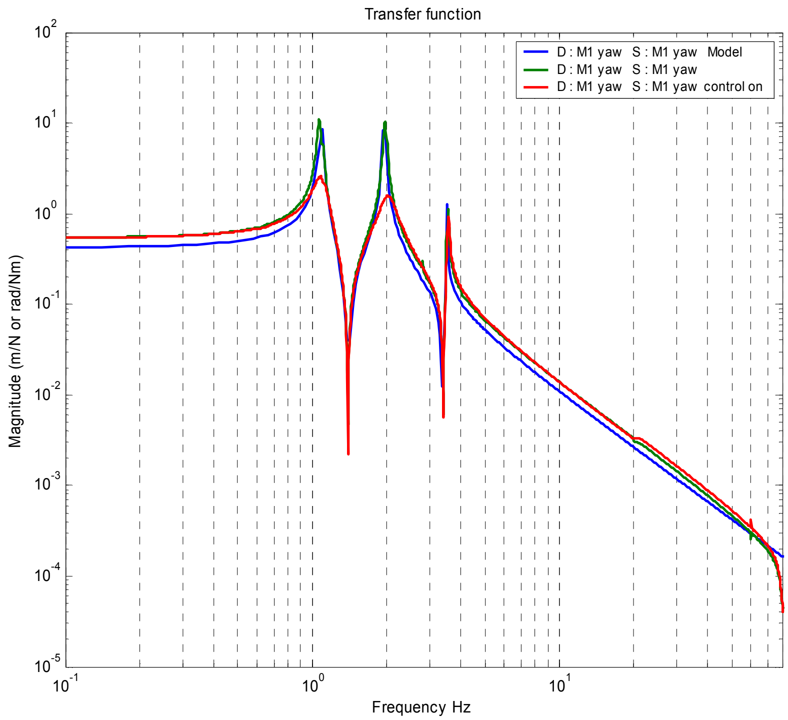
Transfer function from mass1 Z drive to mass1 Z
Matlab model (blue)
Measurement, control off (green)
Measurement, control on (red)



Transfer function from mass1 Roll drive to mass1 Roll
Matlab model (blue)
Measurement, control off (green)
Measurement, control on (red)



Transfer function from mass1 Pitch drive to mass1 Pitch
Matlab model (blue)
Measurement, control off (green)
Measurement, control on (red)



Transfer function from Mass1 yaw drive to mass1 yaw
Matlab model (blue)
Measurement, control off (green)
Measurement, control on (red)

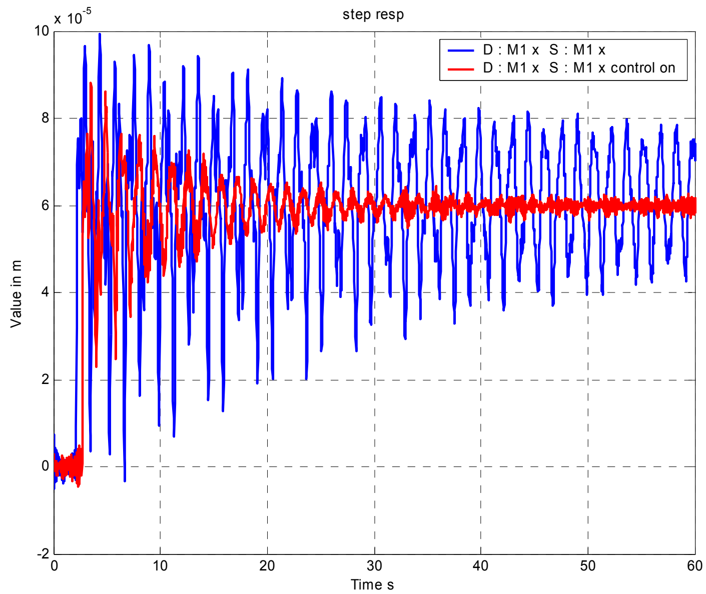
As we can see, the model and the real pendulum match pretty well. The blade stiffness in the model is probably a little bit too high, which cause very small differences in the Z and Pitch transfer functions. The slope in high frequency is good.

What is surprising, even if probably not important, is the difference of the DC value, the magnitude of the transfer function at 0Hz is always bigger on the model than in the measurements, sometimes 3 times bigger. It is probably due to a bad calibration of the sensors or of the actuators.

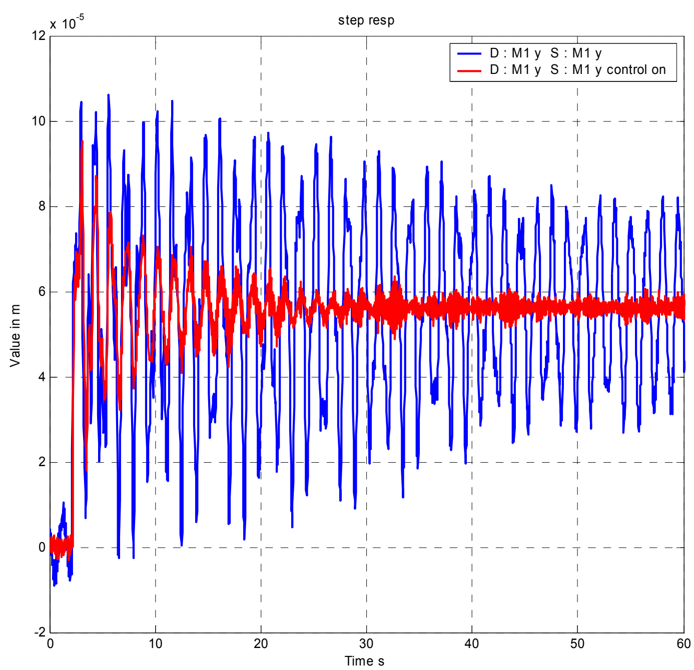
We can also check very well the influence of the control on the transfer functions. Everything works as intended.

Another better way to check the control loop is to plot the step response:

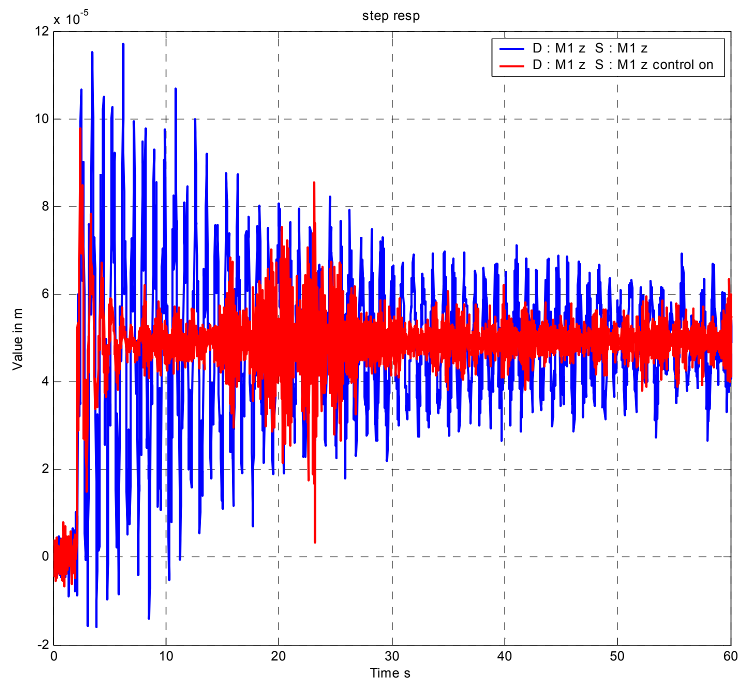
3.9.2 Step response



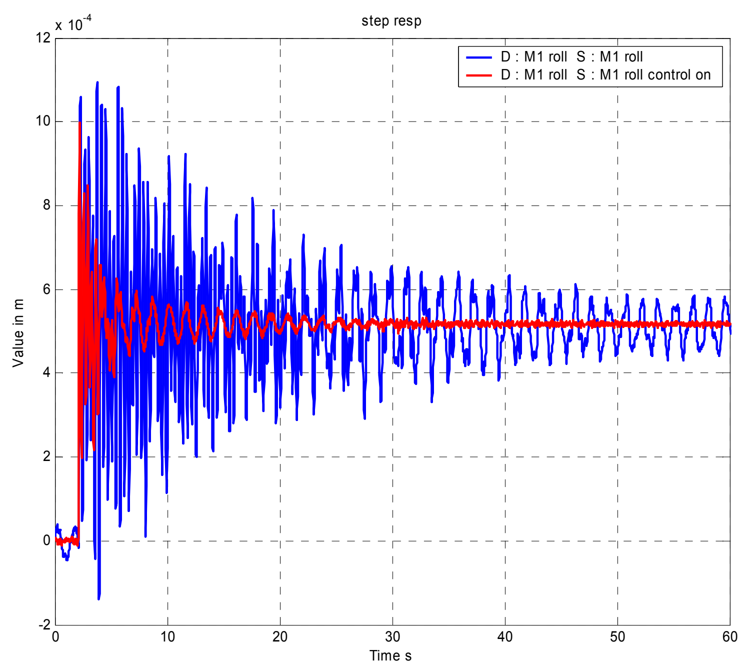
Step response mass1 X to mass1 X
Control off (blue)
Control on (red)



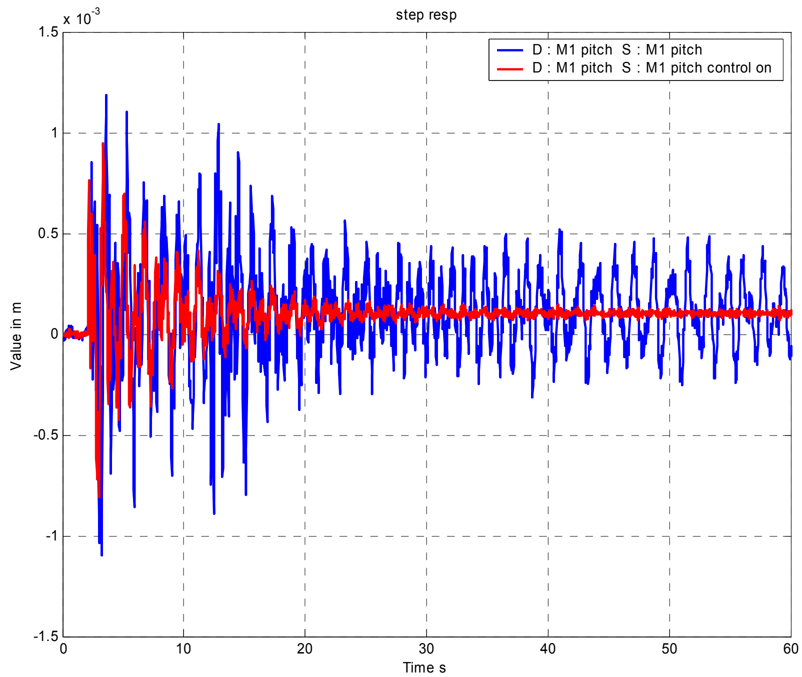
Step response mass1 Y to mass1 Y
Control off (blue)
Control on (red)



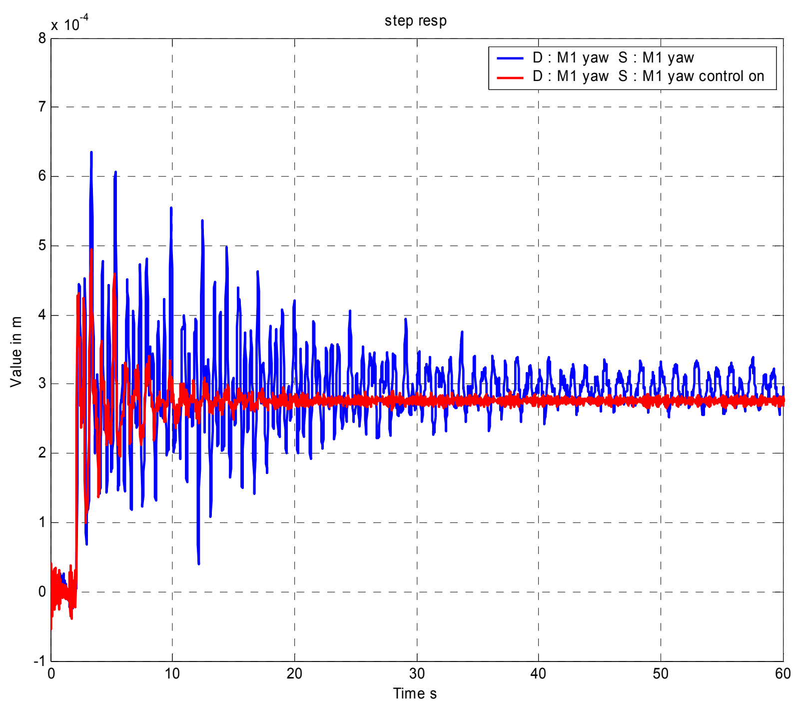
Step response mass1 Z to mass1 Z
Control off (blue)
Control on (red)
Extra noise after damping is due to HEPI structure vertical resonance at 12Hz



Step response mass1 Roll to mass1 Roll
Control off (blue)
Control on (red)



Step response mass1 Pitch to mass1 Pitch
Control off (blue)
Control on (red)



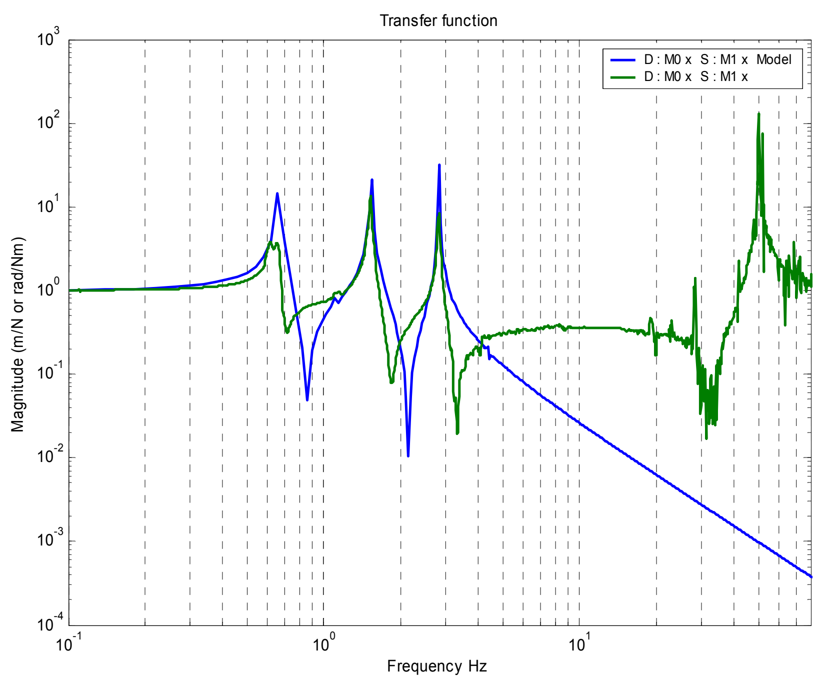
Step response mass1 yaw to mass1 yaw
Control off (blue)
Control on (red)

The active control is working as intended since the damping is achieved in less than 10s with control on. On some plots, we can see extra noise which is due to the motion of the frame because of the resonances of the external platform, it is very easy to check by watching the frequency of this noise, it matches really well with the resonance frequencies measured on the external platform.

The last results concern the drive with the HEPI actuators and the transfer functions between the pendulum frame and the masses.

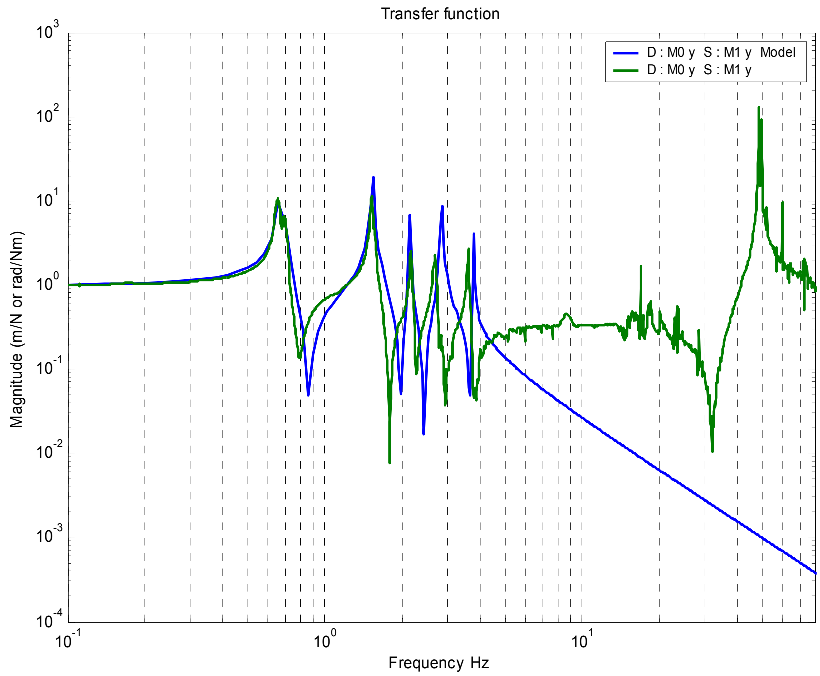
3.9.3 Transfer functions from frame to masses

As we have seen above, the calculation of these transfer functions are a bit more complicated and are very sensitive to sensor calibration (we have to subtract the osems measurement to the geophone's ones and divide it by the geophone's measurements), which cause a very bad accuracy when the subtraction is close to zero (for anti-resonances and high frequencies). It doesn't mean that the measurement are bad, but the 2 kind of sensors are not perfectly calibrated or the frame is not infinitely stiff as assumed and so small perturbation become large ones because of the calculation.

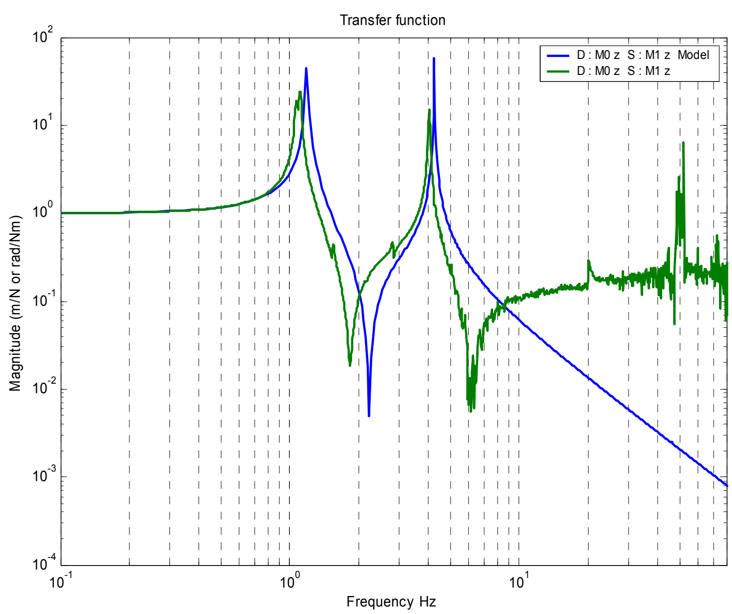


Transfer function from M0 X (frame motion in X) to mass1 X
Matlab model (blue)
Measurement (green)
The TF don't match very well for low amplitudes because of instrument "calibration"

:



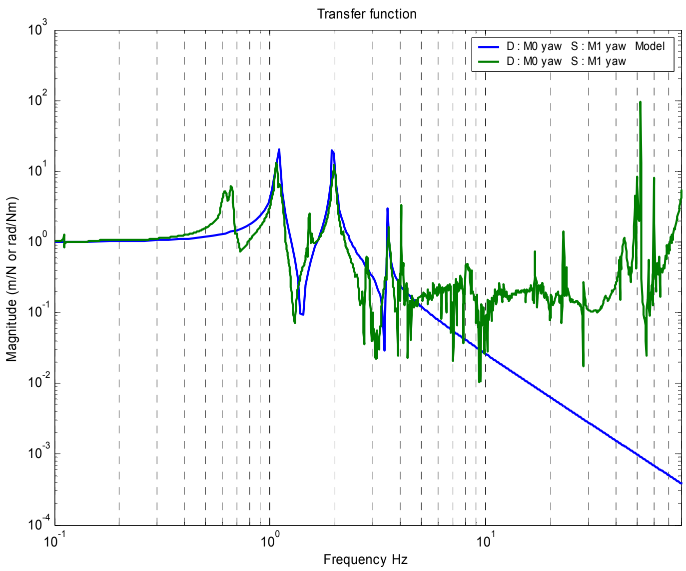
Transfer function from M0 Y (frame motion in Y) to mass1 Y
Matlab model (blue)
Measurement (green)
The TF don't match very well for low amplitudes because of instruments "calibration"



Transfer function from M0 Z (frame motion in Z) to mass1 Z
Matlab model (blue)
Measurement (green)
The TF don't match very well for low amplitudes because of instruments "calibration"

The rotations are even worse, in fact when we tilt the HEPI platform, the arm length produces a lot more horizontal displacement than pure rotation and so we are not exciting the pendulum in rotation at all, which makes the calculation of the transfer function impossible to calculate (the ratio rotation/translation is almost 0).

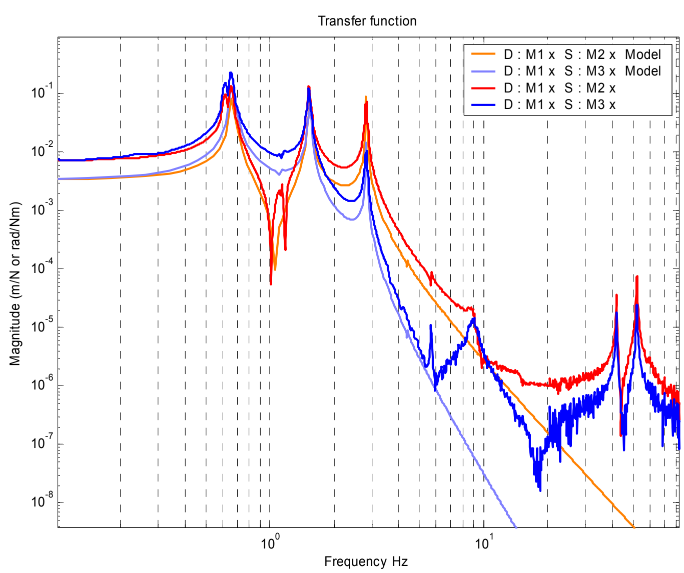
So we can't plot the roll and pitch transfer functions, but we can plot the yaw :



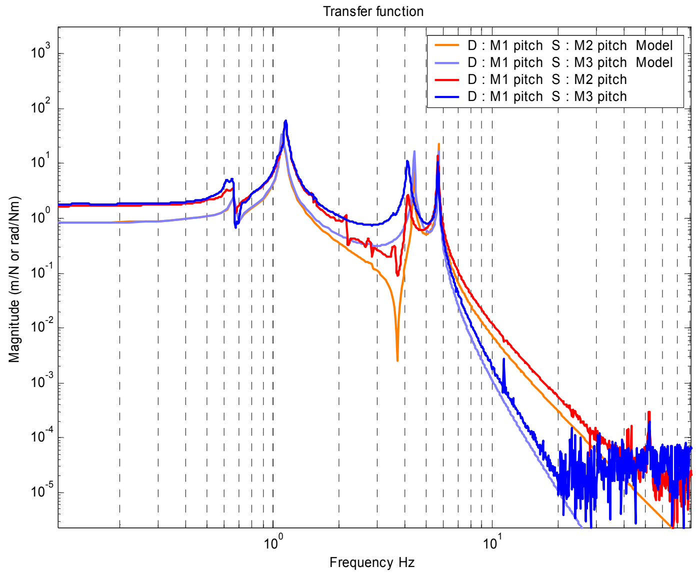
Transfer function from M0 X (frame motion in X) to mass1 X
Matlab model (blue)
Measurement (green)

3.9.4 Results on the 3 masses

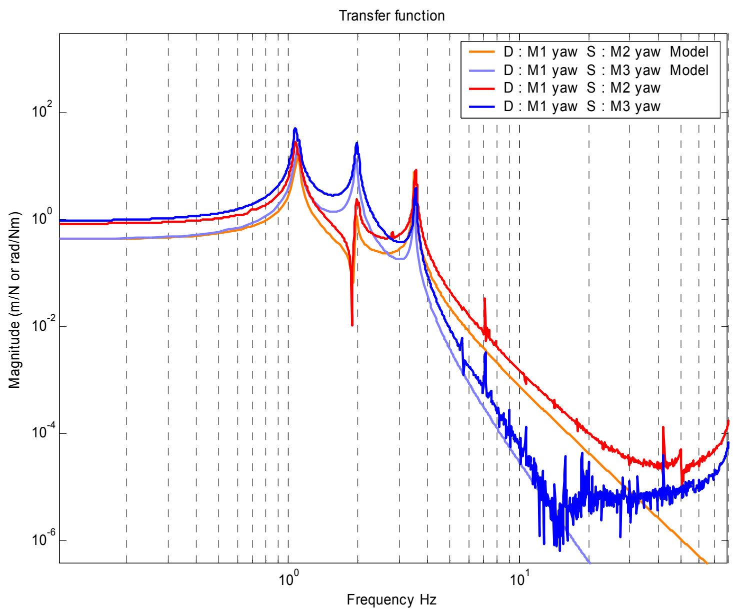
We have focused on the first mass of the pendulum above, but we can also plot some transfer functions for the 2 other masses in some dof (x, pitch and yaw) :



Transfer function between Mass1 X drive and …
Mass2 X : Matlab model (orange)
Mass2 X : Measurements (red)
Mass3 X : Matlab model (purple)
Mass3 X : Measurements (blue)



Transfer function between Mass1 Pitch drive and …
Mass2 Pitch : Matlab model (orange)
Mass2 Pitch : Measurements (red)
Mass3 Pitch : Matlab model (purple)
Mass3 Pitch : Measurements (blue)



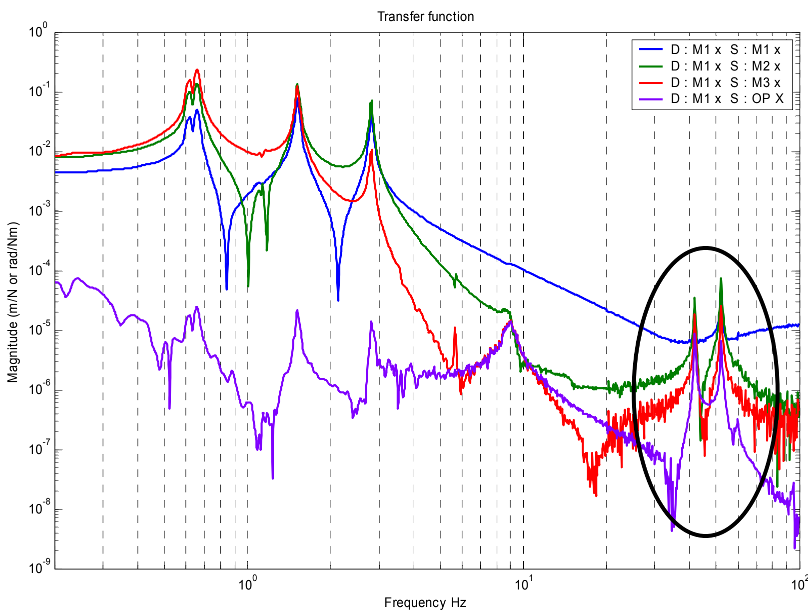
Transfer function between Mass1 Yaw drive and …
Mass2 Yaw : Matlab model (orange)
Mass2 Yaw : Measurements (red)
Mass3 Yaw : Matlab model (purple)
Mass3 Yaw : Measurements (blue)

As we can see, the dynamic behavior is very close to the one calculated by the model. In high frequencies, the motion of the pendulum becomes so small that we start to see the motion of the frame+optical table+external platform in the measurement.

As you can see on the plot in X direction, the transfer function shows 2 very strange resonances at 41Hz and 52Hz.

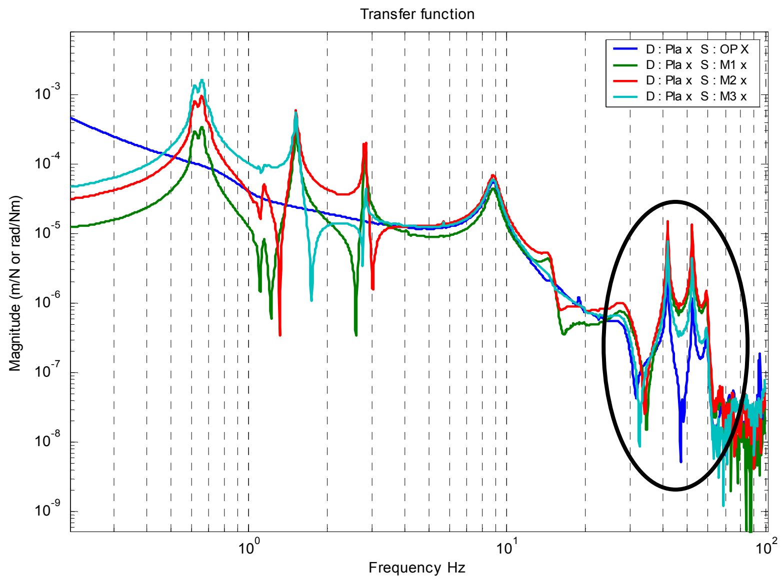
3.9.5 Frame resonances

The first interesting thing to do is to see what the geophones measure and if the 2 resonances depend on the drive :



Transfer function between Mass1 X and …
Mass1 X : measurements (blue)
Mass2 X : measurements (green)
Mass3 X : measurements (red)
Optical table X : measurements (purple)

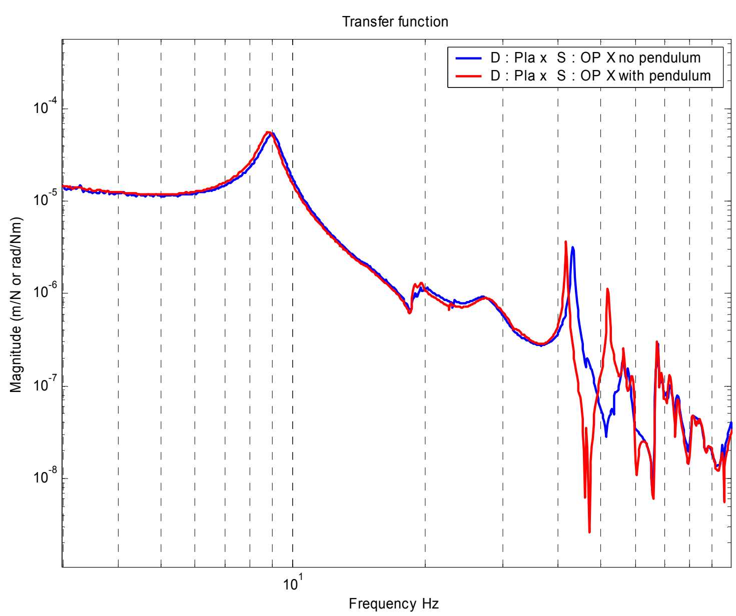
As we see on this plot, it is very surprising to see that the geophones on the optical table are measuring these resonances too, which means that the optical table is resonating when the osems are driving.



Transfer function between HEPI X and …
Mass1 X : measurements (blue)
Mass2 X : measurements (green)
Mass3 X : measurements (red)
Optical table X : measurements (purple)

As we see here, the same thing happens when we drive HEPI, the resonances can be measured on the pendulum or on the optical table. Let's not forget that our osems measure the motion of the pendulum minus the motion of the frame, and so a motion of the optical table can be measured with the osems.

Since we had to remove the pendulum from the tank, we took this opportunity to carry out a system ID of the optical table with no pendulum on it and we compared it to the one with the pendulum sitting on the table.



Transfer function between HEPI X and …
Optical table X with no pendulum on it: measurements (blue)
Optical table X with pendulum on it : measurements (red)

The results show that the resonance is present with or without the pendulum on the table, but that the dynamical behavior of the table (or HEPI structure) is modified in these frequencies by the frame. It is not very clear if the HAM resonance is coupled with a frame resonance or not.

Later, we took data on the pendulum sitting on a granite table outside the tank to check those frequencies, we discovered that the frame also has a strange behavior around 40 Hz, which could confirm the fact that there was a kind of coupling between the frame on the HEPI structure. But since the pendulum's frame and the HAM configuration will be completely different for advanced LIGO, we decided to not pursue this experiment but keep it in mind for later tests on the advanced LIGO structure.

4 CONCLUSION

Most of the tests have been carried out with success in LASTI. The installation has been done smoothly and no major problem has been found.

However, we found that whenever the pendulum was moved, for example from outside to inside the tank, the OSEMS required realignment. This was difficult in the confined space inside the tank. This problem will be worse when there is more instrumentation on the table inside the tank. This should be considered when planning in what order to carry out installation of instrumentation.

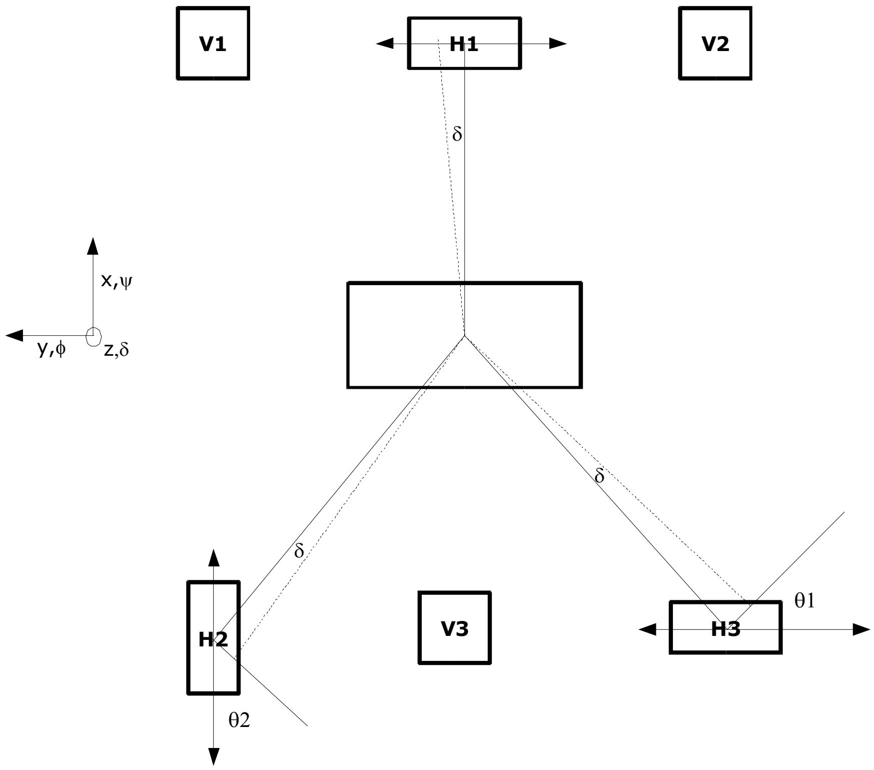
The characterization enabled us to create new methods and new software for the acquisition and calculation of the transfer functions, these methods will be used for the quadruple pendulum. Comparing the results with the model showed us 2 little mistakes that we corrected in the model for the roll transfer function and the wire diameter. Even though it took us a long time to figure out this second trouble, it also enabled us to learn more about the pendulum, and especially to be more confident with the ability to perform actions and changes on the pendulum on the optical table or even how to remove the pendulum from the tank.

We also have been able to test the active damping and checked that it worked perfectly on the pendulum.

Unfortunately, we were unable to use HEPI as well as we would have thought, driving the entire platform in rotation is close to impossible and the results or not easy to get.

Appendix I : *OPTICAL TABLE COORDINATE SYSTEM*

This section describes how to change the coordinate system in order to get the 6 dof motion of the table with the 6 geophones measurements.



The distances used in calculation are absolute distances and the angles of rotation of the table are considered very small so that we can write $\sin(\alpha)=\alpha$

$$H_1=y+l_{h1}\dot{\delta}+h_{h1}\dot{\psi}$$
$$H_2=x+l_{h2}\dot{\delta}\cos(\theta_2)+h_{h2}\dot{\phi}$$
$$H_3=y-l_{h3}\dot{\delta}\cos(\theta_3)-h_{h3}\dot{\psi}$$

$$V_1=z-lx_{v1}\dot{\phi}+ly_{v1}\dot{\psi}$$
$$V_2=z-lx_{v2}\dot{\phi}-ly_{v2}\dot{\psi}$$
$$V_3=z+lx_{v2}\dot{\phi}+ly_{v3}\dot{\psi}$$

$$
\begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & h_{h1} & 0 & l_{h1} \\ 1 & 0 & 0 & 0 & -h_{h2} & l_{h2}\cos(\theta_2) \\ 0 & 1 & 0 & h_{h3} & 0 & -l_{h3}\cos(\theta_3) \\ 0 & 0 & 1 & ly_{v1} & -lx_{v1} & 0 \\ 0 & 0 & 1 & -ly_{v2} & -lx_{v2} & 0 \\ 0 & 0 & 1 & ly_{v3} & lx_{v3} & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ \Psi \\ \Phi \\ \delta \end{bmatrix}
$$

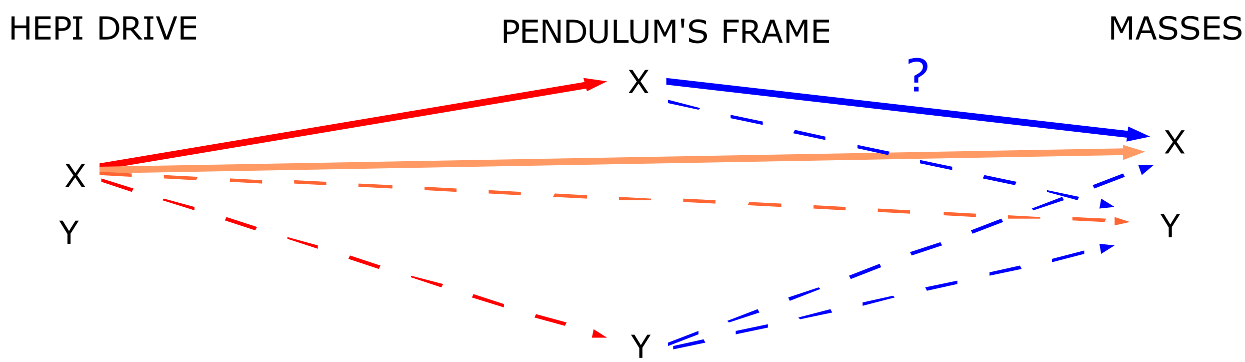
Then we just need to invert this matrix to find the matrix of coordinate basis change.
Numerical data are given below :

l_{h1}	0.7366 m
l_{h2}	0.9912 m
l_{h3}	0.9893 m
$\cos(\theta_2)$	0.6402
$\cos(\theta_3)$	0.7446
lx_{v1}	0.7366 m
lx_{v2}	0.7366 m
lx_{v3}	0.7366 m
ly_{v1}	0.635 m
ly_{v2}	0.635 m
ly_{v3}	0.0254 m
h_{h1}	0.825 m
h_{h2}	1.04 m
h_{h3}	1.05 m

Appendix II : Calculating TF without a pure drive

As we have seen above, the drive with HEPI system can not be considered as perfectly pure, driving HEPI in X doesn't move the frame (the point where the pendulum is attached) only in X because of different reason such as rotations and flexibility. It is especially true for the rotations of HEPI, a HEPI drive in rotation doesn't produce only rotation at the top of the frame, it actually also and mainly produces translations.

The idea we had was so to measure all these transfer functions and to use them to extract the pure drive from it. Let's see this schematic we showed above again and let's focus on a 2 dof system called X and Y.



The drive with HEPI is not pure
Additional path of the motion in dashed lines
We want to calculate the blue transfer functions

If we write the equations :

$$Tf_{x1x3} = Tf_{x1x2}.Tf_{x2x3} + Tf_{x1y2}.Tf_{y2x3}$$
$$Tf_{x1y3} = Tf_{x1x2}.Tf_{x2y3} + Tf_{x1y2}.Tf_{y2y3}$$
$$Tf_{y1x3} = Tf_{y1x2}.Tf_{x2x3} + Tf_{y1y2}.Tf_{y2x3}$$
$$Tf_{x1y3} = Tf_{y1x2}.Tf_{x2y3} + Tf_{y1y2}.Tf_{y2y3}$$

$$\begin{bmatrix} Tf_{x1x3} & Tf_{x1y3} \\ Tf_{y1x3} & Tf_{x1y3} \end{bmatrix} = \begin{bmatrix} Tf_{x1x2} & Tf_{x1y2} \\ Tf_{y1x2} & Tf_{y1y2} \end{bmatrix} . \begin{bmatrix} Tf_{x2x3} & Tf_{y2x3} \\ Tf_{x2y3} & Tf_{y2y3} \end{bmatrix}$$

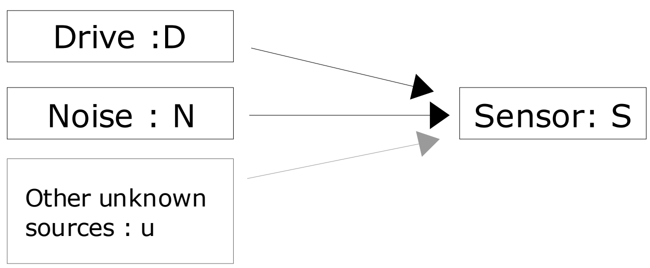
Appendix III : NOISE REDUCTION FOR WHITE NOISE DRIVE

Based on paper : Automatic cross-talk removal from multi-channel data
Bruce Allen, Wensheng Hua, Adrian C.Ottewill

The goal of this method is to calculate the transfer function between the drive D and the sensors without being affected by the noise N coming from the ground. In order to do that, we will measure the motion on the floor N during the acquisition. Then we can

- Calculate the transfer function by removing the part of S that is coherent with the noise (N) measurement
- Calculate the "total coherence", a combination of the coherence between D to S and N to S (this is not a simple addition, you can't sum coherence). If the "total coherence" is 1, that means the other sources don't exist and you know everything about your system.

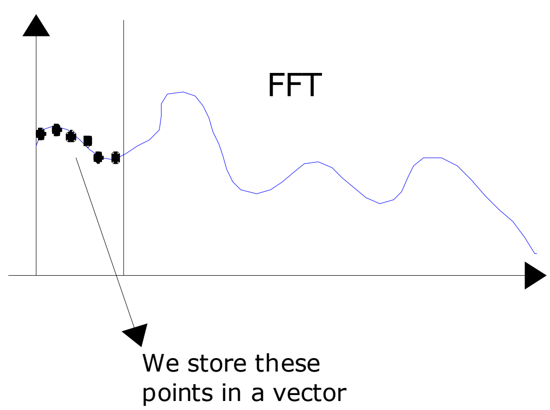
We will only show the main step in this annex.



We first introduce a cartesian inner product, it is useful to express the correlation between 2 channels:

$(X_1, X_2) = \sum_{1<k<k\max} X_1(k).\overline{X_2(k)}$ (where the bar means complex conjugate)

We start by calculating FFT of the 3 channels and we split these FFT into many parts, gathering points of each parts into vectors (each vector correspond to a frequency range). We will calculate the coherence for each vector.



For each vector (each vector correspond to a frequency band), we want to write the vector S as :

$S = Tf_d.D + Tf_n N + u$

Where Tf_d and Tf_n are scalar and correspond to the transfer function and where u (a vector) is unknown and is an indication of the coherence (u is 0 if the "total coherence" is 1).

The goal is to find Tf_d and Tf_n so that we minimize u . Which means that we assume the best transfer functions are the one that minimize the norm $N = (u,u)$

This leads to

$$(D,u) = \left(D, S - \overline{Tf_d}D - \overline{T_{fn}}N\right) = 0$$
$$(N,u) = \left(N, S - \overline{Tf_d}D - \overline{T_{fn}}N\right) = 0$$

With the properties of the dot product :

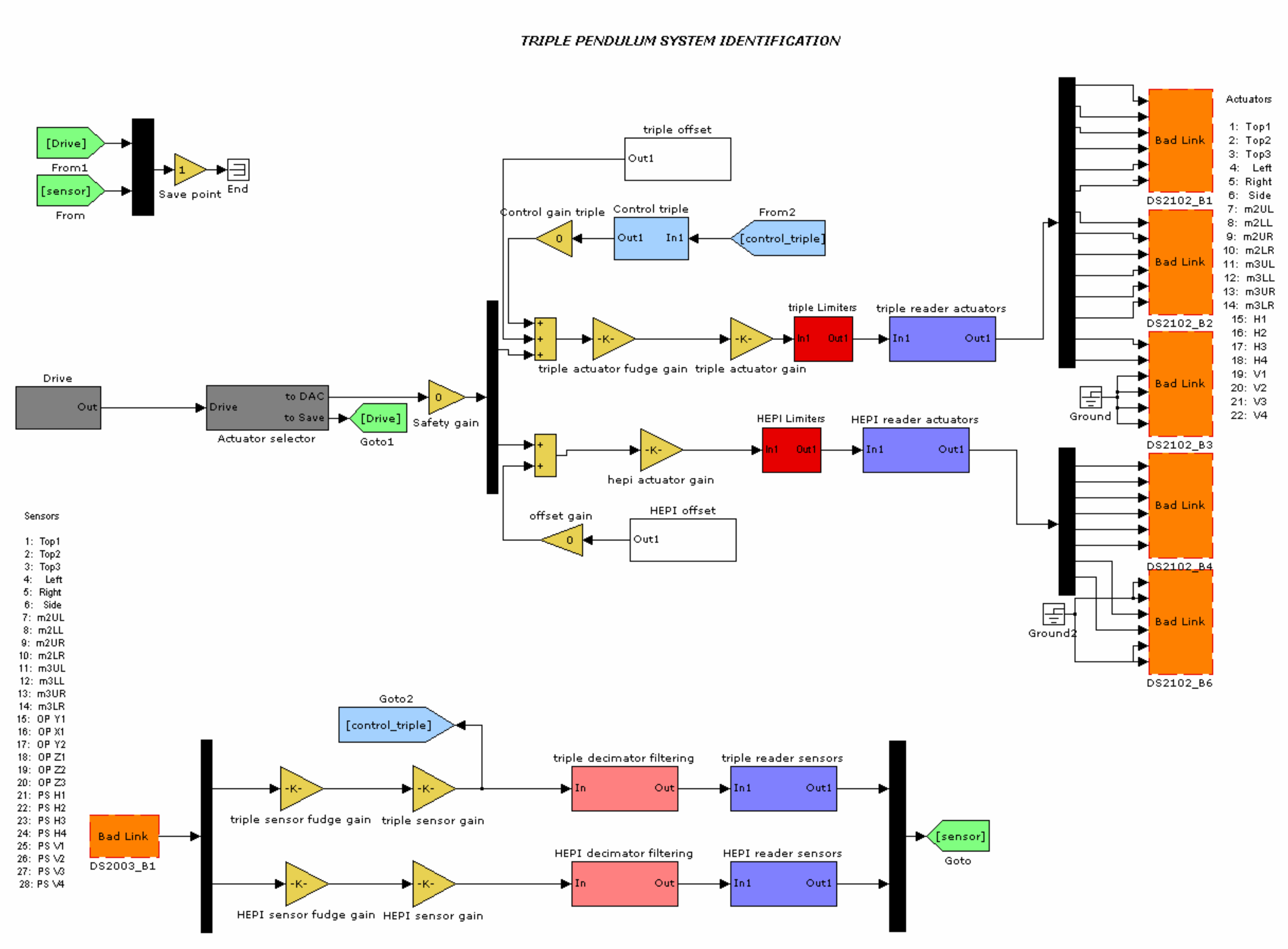
$$(D,S) - \overline{Tf_d}.(D,D) - \overline{Tf_n}.(D,N) = 0$$
$$(N,S) - \overline{Tf_d}.(N,D) - \overline{Tf_n}.(N,N) = 0$$

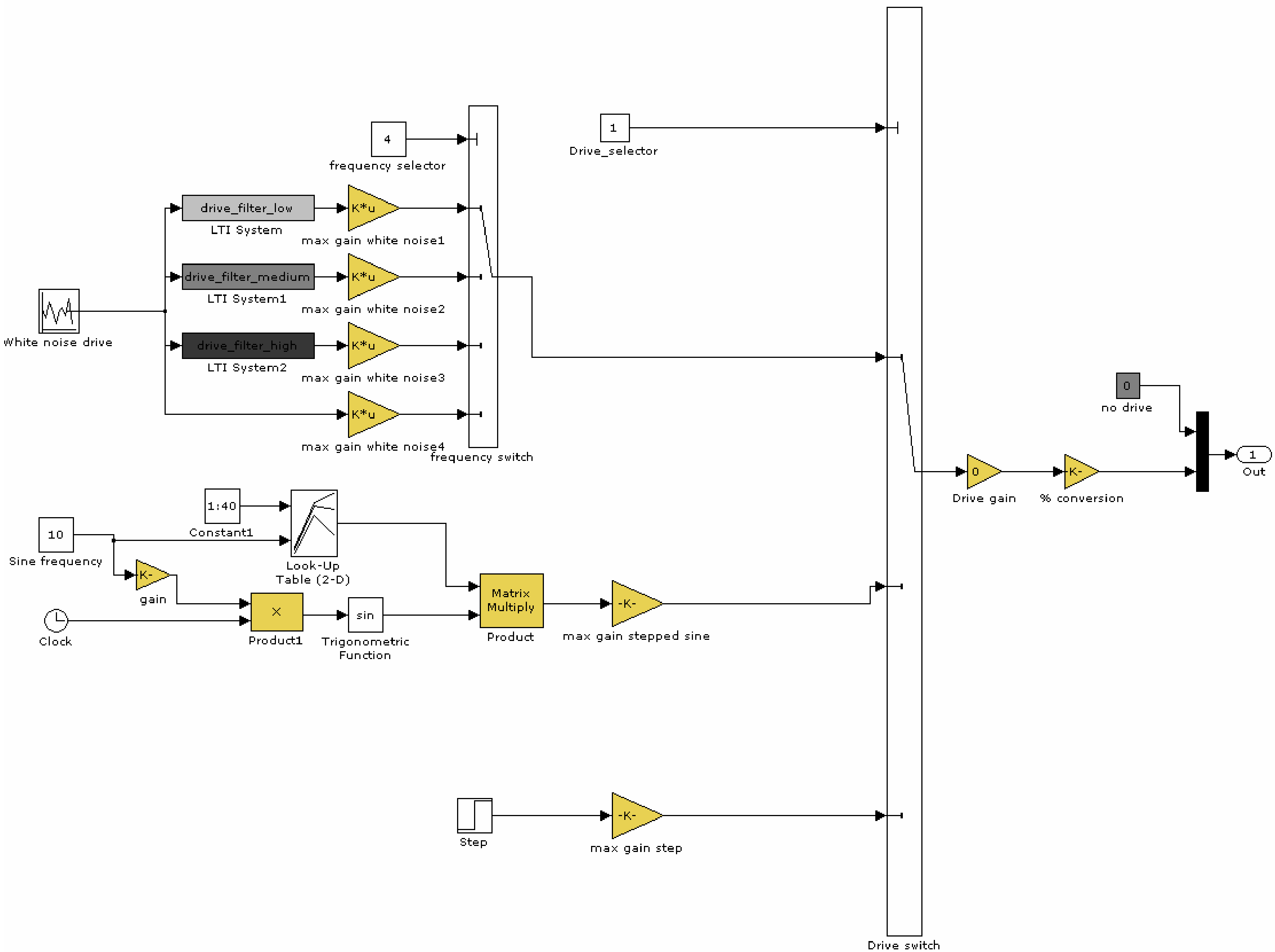
$$\begin{bmatrix} Tf_d \\ Tf_n \end{bmatrix} = \begin{bmatrix} (D,D) & (D,N) \\ (N,D) & (N,N) \end{bmatrix}^{-1} \begin{bmatrix} (D,S) \\ (N,S) \end{bmatrix}$$

And you can also calculate u with $S = Tf_d.D + Tf_n N + u$ and get the total coherence:

$$\rho = \sqrt{1 - \frac{(u,u)}{(S,S)}}$$

Appendix IV :   SIMULINK DIAGRAMS




4
frequency selector
1
Drive_selector
drive_filter_low
LTI System
K*u
max gain white noise1
drive_filter_medium
LTI System1
K*u
max gain white noise2
White noise drive
drive_filter_high
LTI System2
K*u
max gain white noise3
K*u
max gain white noise4
frequency switch
0
no drive
0
Drive gain
-K-
% conversion
1
Out
1:40
Constant1
10
Sine frequency
Look-Up
Table (2-D)
K-
gain
Clock
Product1
sin
Trigonometric
Function
Matrix
Multiply
Product
-K-
max gain stepped sine
Step
-K-
max gain step
Drive switch