LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY

LIGO Laboratory / LIGO Scientific Collaboration

LIGO- T050093-00-K *Advanced LIGO UK* June 2005

Discussion Document for Advanced LIGO Suspension (ITM, ETM, BS, FM) ECD requirements

K.A. Strain

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This is an internal working note
of the Advanced LIGO Project, prepared by members of the UK team.

Institute for Gravitational Research
University of Glasgow
Phone +44 141 330 5884
Fax +44 141 330 6833
E-mail k.strain@physics.gla.ac.uk

Department of
University of Birmingham
Phone +44 (0) 121 414 6447
Fax +44 (0) 121 414 3722
E-mail av@star.sr.bham.ac.uk

Engineering Department
CCLRC Rutherford Appleton Laboratory
Chilton, Didcot, Oxon OX12 0NA
Phone +44 (0) 1235 44 5297
Fax +44 (0) 1235 445843

Particle Physics and Astronomy Research Council (PPARC)

http://www.ligo.caltech.edu/

http://www.eng-external.rl.ac.uk/advligo/papers_public/ALUK_Homepage.htm.

1 Introduction and scope

1.1 Purpose

This document is intended to supplement LIGO E040108-01 and LIGO T040110-01-K by looking at the question of how to extend the "hybrid damping" idea to all necessary degrees of freedom of the main core optics suspensions in Advanced LIGO (ITM, ETM, BS, FM). It does not provide a finished design, but shows that a workable solution exists.

1.2 Scope

The document discusses the damping required for each degree of freedom (for which hybrid damping is required – therefore excluding roll and transverse horizontal), calculates associated noise allowances, and estimates the amount of eddy-current damping (ECD) to be applied. Some results from E040108 and T040110 are reproduced here for convenience.

1.3 Terminology

To avoid confusion: *settling time* is given as the time to settle to 5% of the initial impulse disturbance. This is longer than the damping time, and is a more reliable measure of performance, when there are a multitude of modes each with different Q. It is normally the standard "settling time characteristic" given by the LTI-Viewer in MATLAB (but in a few cases the result has been modified to better represent the long-term settling).

1.4 Acronyms

ISC	interferometer sensing and control
LSC	length sensing and control
OSEM	optical sensor electromagnetic actuator
ECD	eddy current damping
UGP	unity gain point (of servo)

2 References

The document depends on latest versions of

LIGO- T040110, E040108 and T040110, and references therein.

3 Review of requirements and methods

3.1 Damping and modes of operation.

These were defined clearly in LIGO- T040110-01-K and are sumarised here

- **Emergency/Installation/Pre-Alignment.** Possibly the same as acquisition, but requirements for settling time are more obviously appropriate here.
- **Acquisition.** The key requirement is that the fringes are sufficiently slow to allow the ISC controllers to act before the fringe has passed, this does not necessarily correspond to shortest settling.

- **Science mode.** The in-band noise requirements as set out in the DRD must be met in this mode. Additionally it is necessary to restrict the required control-band feedback forces to a reasonable minimum.

The noise requirement in each case is clear (none, control band velocity and 10 Hz noise, respectively). The damping requirements are harder to state. For the Emergency mode the requirement is to allow normal levels of motion to be attained promptly, and we have previously stated a goal of 10s damping time. If the damping provided by hybrid damping, for acquisition mode (see below) is deemed insufficient for situations where the suspension is being disturbed, the damping algorithm can easily be switched to one designed to shorten the settling time.

In acquisition mode the damping goal is minimum steady-state TM motion (velocity or some frequency-weighted measure thereof). Earlier results showed that with active/hybrid damping the corresponding settling time was likely to be somewhat longer than 10s for at least some degrees of freedom.

In science mode the goal for damping is first to meet the noise requirement and second to allow minimum feedback force on the global actuators –where appropriate – at least within a limited band. Often this will be achieved by transferring control of the suspension entirely or dominantly to global control.

3.2 Reiteration of requirements per degree of freedom

Based on that in E040108-01, but rearranged.

Mode	***ITM/ETM***	***ITM/ETM***	***ITM/ETM***	***BS/FM***	***BS/FM***	***BS/FM***
dofs	**l p y**	**v r**	**t**	**l p y**	**v r**	**t**
#osems	3	2	1	3	2	1
Acquisition	Hybrid	Hybrid	Active	Hybrid	Hybrid	Active
Science	Global	Hybrid	Active	Hybrid	Hybrid	Active
Notes	As T040110	As T040110 **r** is active		Perhaps **p** can be active	Check **r** is active	

Table 1: summary of which type of damping is employed in each case. A suggestion for the distribution of ECD is given below.

The algorithm for filling the above table is approximately as follows:

- if the noise requirement in a particular state is met by the standard OSEM then active damping suffices (acquisition and emergency modes are, logically, always the same in this table even if there is a slight gain change needed, the active damping component can always be turned up)
- if global control is available to control the degrees of freedom concerned then it is employed as the primary control of the suspension, (clearly any ECD fitted will still be damping the suspension modes),
- otherwise hybrid damping is needed, with strong low-pass filters in the active part

- the translation (longitudinal or vertical) degree of freedom always presents the strictest requirement, angular control shares the same method.

The optics are well-isolated from the motion of the reaction chains, only the non-ideal actuators and stray magnetic/electrostatic effects introduce significant coupling. There are several motivations for keeping the inter-chain coupling small (such as transfer of radiation forces on the ITM thermal compensator to the ITM). Active damping is adequate for the reaction chains. *There is no requirement for ECD on any reaction chain.*

3.3 Reiteration of the "hybrid damping approach"

Hybrid damping employs a mixture of active and eddy-current damping together on appropriate degrees of freedom of the suspensions.

A practical amount of ECD provides good damping of the higher-frequency suspension modes (those >1 Hz), neither shorting out isolation nor adding too much thermal noise. Active damping is excellent at damping the low frequency modes, but noisy sensors carry the risk of adding noise at 10 Hz, unless sufficiently sophisticated low pass filtering is installed. It is obvious that the design of such low pass filters can be eased if the highest unity gain point of the damping controller is lowered. Damping the modes around 2~4 Hz with ECD allows the highest UGP to be reduced from around 5 Hz to around 1~2 Hz.

If the amount of ECD calculated for a translation degree of freedom is distributed over the appropriate face of the top mass in much the same way as the OSEMs are distributed, adequate damping of angular degrees of freedom should be assured. This leads to a design with several small ECD units on each suspension.

See the appendix for a more detailed description of how the controllers for hybrid damping were developed.

3.4 General note on BS and FM suspensions

The requirements are, mostly, relaxed (compared to the TM suspensions) and although the isolation is reduced there is no difficulty meeting damping time or detection band noise performance targets with hybrid damping. Acquisition velocity requirements are probably much less stringent for these suspensions, as they are effectively in cavities of lower finesse. The performance obtained from hybrid damping is slightly better for the BS and FM suspensions in their latest form, since the relative lengths of the 3 stages are well suited to damping at the top stage, and the suspensions are lighter..

4 Recommendation

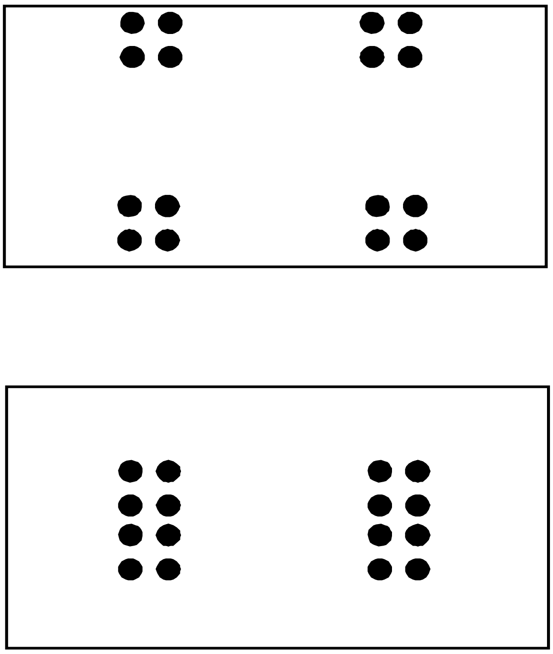
The conclusions in this section are justified in more detail in section 5 and the appendices.

4.1 Estimate of amount and distribution of ECD on ITM/ETM main chains

As was shown in T040110-01, a block of 4x4 magnets (around 27 kg/s) was excellent for vertical damping in all modes. Splitting this into 2 blocks of 4x2 magnets provides roll damping at little cost (although not strictly required it may be wise to do this). If it is more convenient these could be further split into 4 blocks of 2x2 magnets moved towards the corners of the top of the mass. This would also provide pitch damping, useful if the longitudinal dampers cannot easily be positioned to damp pitch (in this case the blocks of dampers should be separated by more than ~120 mm in the longitudinal direction).

The same amount of damping (16 magnets) is sufficient for longitudinal. Again, to provide pitch and yaw damping the ECD should be distributed over the face of the mass (being spaced horizontally by at least ~200 mm and vertically by at least ~120 mm). In this case 4 units of 2x2 seems to be the most reasonable option, and is probably the minimum we require. An alternative would be 3 units of 4x2 suitably arranged to give symmetrical damping (two above 1 on a V-pattern, or the inverse). See the preceding paragraph for the option of providing or enhancing pitch damping using the vertical dampers.

ECD could, optionally, be fitted to transverse horizontal to ensure an increased level of failsafe damping (although the transverse damping provided by 32 ECD magnets on the other degrees of freedom already provides a few kg/s. If the cost is low enough this option should be considered.



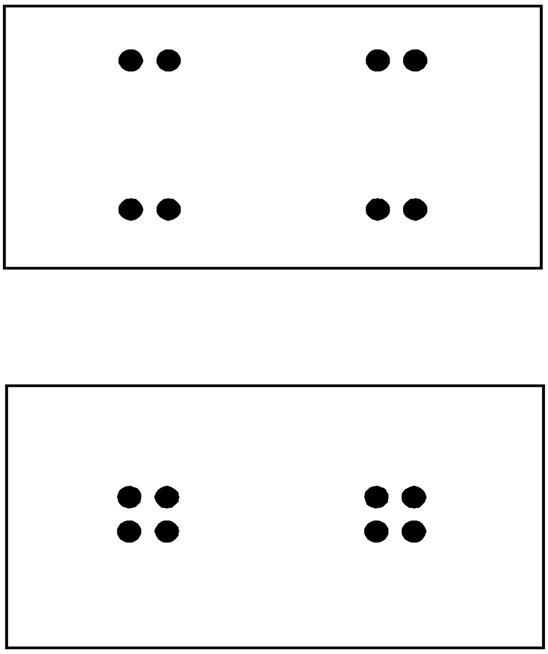
Each black dot represents one damper (magnet) the rectangles represent either plan or face views of the mass (simplified).

The dampers should be at least 200 mm apart trasversely (left-right on the diagram). To obtain pitch damping the dampers should be spread apart by ~120mm or more on at least one of the faces (vertically on the diagram).
(Optionally they can be spread apart by about 80 mm on both faces - none of these figures being very critical.)

Figure 1: minimum requirement for ITM/ETM dampers. The left-right spacing of dampers is only a requirement for those acting longitudinally (yaw damping), those on the top face, acting vertically do not require to be spaced by at least 200 mm, and could be clustered in the centre.

4.2 Estimate of the amount and distribution of ECD on BS/FM main chains

The mass of these triple suspension chains is somewhat less than the ITM/ETM quads. The same amount of ECD as recommended for the quads works fine, but half as much works too, provided the dampers are still distributed so as to damp yaw (and possibly pitch). Since there is an argument to standardize on a common design of damper (probably 4x2 or 2x2) and we are considering almost the minimum number of units (not far from 1 per degree of freedom), it only seems worthwhile to consider full (16 units) and half (8 units) damping.



Each black dot represents one damper (magnet) the rectangles represent either plan or face views of the mass (simplified).

The dampers should be at least 200 mm apart trasversely (left-right on the diagram). To obtain pitch damping the dampers should be spread apart by ~100mm or more on at least one of the faces (vertically on the diagram).
(Optionally they can be spread apart by about 70 mm on both faces - none of these figures being very critical. Indeed it may be possible to remove the need for ECD of pitc h modes.)

Figure 2: minimum requirement for BS/FM dampers. ***Note that the 32-magnet solution for the ETM/ITM case would be perfectly fine for BS and FM too.*** *The left-right spacing of dampers is required for those acting longitudinally (yaw damping), those on the top face, acting vertically do not require to be spaced by at least 200 mm, and could be clustered in the centre (or at any other convenient separation)*

5 Results in detail

Specific results are given for each suspension type and each degree of freedom. These are not necessarily optimum, being hand tuned, but meet requirements. The key requirements are on settling time (or equivalent) and noise performance at and above 10 Hz.

There are no results for transverse or roll (active damping suffices to meet requirements, transverse components of other dampers will give a settling time of no more than a few minutes in the event of failure of the active system). For transverse no ECD units need to be fitted, for roll, vertical dampers will give some roll damping, but there is no requirement to position these to give a particular value of roll damping.

5.1 ITM/ETM

The vertical result is similar to that given in T040110. The other results are new. Noise requirements are given in 3.2.1.2, Table 1 of T010007-01.

5.1.1 Vertical

The "standard" 16 units of damping (27 kg/s) is adequate. This gives a settling time to 5% of 40 seconds, with a gain margin of 6 dB, and a sensor noise coupling at and above 10 Hz of no more than -132 dB (at nominal gain). (There is a further 60 dB "isolation" from the cross-coupling to longitudinal, but this is included in the requirement.) Thus the requirements for all modes are barely met at one gain settling, but the noise coupling can be reduced in science mode to give margin, by turning down the gain (in all cases a sensor noise of 10^{-10}m/Hz$^{0.5}$ is, conservatively, assumed). Alternatively a minor improvement in the low-pass filter design could permit operation at fixed gain with more margin. Note that a slight reduction of the active gain creates additional margin and allows the low pass filter to be made much stronger – so a small gain reduction could lead to a large reduction of the noise coupling.

5.1.2 Longitudinal

The standard 16 units of damping (27 kg/s) is adequate. This gives a settling time to 5% of 45 seconds with a gain margin of 6 dB (although the low pass filter poles require to be optimized when this should be improved). The sensor noise coupling at and above 10 Hz is no more than -178 dB at nominal gain. The gain must be turned down by about 22 dB in lock to reach science mode specification. In this case the settling time to 5% increases to about 200 seconds (of course this is almost meaningless for a suspension under global control, but gives a hint and what would happen immediately after loss of lock). The distribution of the (at least) 16 dampers is determined by the pitch and yaw requirements given below.

5.1.3 Pitch

The total damping is assumed to be the same as for longitudinal, but arranged at some offset from the center of mass so as to give torque damping. The minimum offset that works well with 16 dampers is 60 mm. If a smaller offset must be used the total amount of damping will need to be increased. The dampers should be as far apart as possible. With a spacing of 120mm (60 mm offset) the settling time to 5% is about 15 seconds, with a gain margin of 6 dB. The sensor noise coupling at and above 10 Hz is no more than -189 dB (units are rad/m), safely meeting the design requirements with the standard sensor noise. The dampers on the top of the mass (vertical) could also be positioned to contribute to pitch damping.

5.1.4 Yaw

The total damping is assumed to be the same as for longitudinal, but arranged at some offset from the center of mass so as to give torque damping. The minimum offset that works well with 16 dampers is around 100 mm. If a smaller offset must be used the total amount of damping will need to be increased. The dampers should be as far apart as convenient. With a spacing of 200mm (100 mm offset) the settling time to 5% is about 40 seconds, with a gain margin of 6 dB. The sensor noise coupling at and above 10 Hz is no more than -177 dB (units are rad/m) just meeting the requirement with the standard sensor noise (the gain should probably be reduced a few dB in science mode, or the filter improved a little).

5.2 BS/FM

All of the results are new. Once again there are no particular requirements for roll and transverse modes, damping can be active, with ECD from other degrees of freedom providing a welcome, but inessential, backup. The other results are new. Noise requirements are given in 3.2.3.2 Table 4 of T010007-01.

5.2.1 Vertical

The "standard" 16 units of damping (27 kg/s) is adequate, indeed half as much (two "2 by 2" units) was found satisfactory, and the results are given for this reduced case. This gives a settling time to 5% of 10 seconds (excluding the 9 Hz mode which is almost un-damped), with a gain margin of more than 6 dB, and a sensor noise coupling at and above 10 Hz of no more than -100 dB at nominal acquisition gain, barely meeting the noise requirement at full gain. The noise coupling may be reduced in science mode if required, by turning down the gain or by improving the filter.

5.2.2 Longitudinal

The standard 16 units of damping (27 kg/s) is adequate, but half of this performs nearly as well in acquisition mode. The reason for this is that the settling time is dominated by the 0.45 Hz mode. This needs active damping with a carefully tailored response to reach a settling time to 5% of 25 seconds with a gain margin of 6 dB. The sensor noise coupling at and above 10 Hz is no more than -158 dB at nominal gain. This fails to meet the science mode specification by 14 dB, but the gain may be turned down by at least this much in science mode. In this case the settling time to 5% increases to about 50 seconds (with 27 kg/s) to 80 seconds (with half of the ECD). The distribution of the ECD units is determined by the pitch and yaw requirements given below.

5.2.3 Pitch

The total damping is assumed to be the same as for longitudinal (16 or 8 dampers), but arranged at some offset from the center of mass so as to give torque damping. The minimum offset that works well with 16 dampers is about 50 mm. The dampers should be as far apart as possible. With a spacing of 120mm (60 mm offset) the settling time to 5% is about 50 seconds, with a gain margin of 6 dB. The sensor noise coupling at and above 10 Hz is no more than -198 dB (units are rad/m). This is safely beyond the required isolation. The pitch modes are all very low, and this makes the controller poles and zeros different from the others, it may be far from optimum, as it was obtained through tweaking the standard design. Indeed it may be that a purely active design would suffice for pitch damping (if the modes stay as low as they are in the model at present.) The hybrid approach is still recommended as it provides margin in the event of design changes that increase the pitch mode frequencies.

5.2.4 Yaw

The total damping is assumed to be the same as for longitudinal (16 standard units or 27 kg/s), but arranged at some offset from the center of mass so as to give torque damping. The minimum offset that works well is around 100 mm. If a smaller offset must be used the total amount of damping may need to be increased. The dampers should be as far apart as possible. With a spacing of 200mm (100 mm offset) the settling time to 5% is about 25 seconds, with a gain margin of over 6 dB. The sensor noise coupling at and above 10 Hz is no more than -151 dB (units are rad/m), safely within the requirement.

6 Appendices

This section presents in depth descriptions of the controllers and the models.

6.1 Some notes on the controller design

The active-damping loop was designed intuitively; it would be better if an optimal method could be found, but the correct cost function is not obvious. The aim is to damp the lowest mode in each degree of freedom as strongly as possible (as that determines the settling time in most cases). From previous experience at Glasgow and GEO, the use of a pair of transitional differentiators (one zero, one pole each) providing phase lead around the lowest mode(s) is a good basis. (A single differentiator, with zero and pole more widely spaced, is less good, as the higher modes receive more of an unwanted gain boost for a given phase margin.) It has also been observed, though not quantitatively understood, that the phase at the *lowest* unity gain point has a surprisingly strong effect on the damping behaviour. Further optimisation of the lower corner frequencies of the differentiator could improve the damping somewhat in at least some cases.

The next element of the servo is a notch (or notches with the quad suspension) to suppress the gain at the 3rd (and 4th) modes where the ECD already provides adequate damping. The notch frequency was carefully optimised, but the Q was only set once (very roughly) and not changed, and this could probably be improved.

The whole point of fitting the ECD and notches is to allow the inclusion of aggressive low-pass filters. Experience on GEO (developed from a starting point identified by Roland Schilling) is that a non-elliptic filter of a particular design is a good starting point. The "2-pole plus notch" design module used has its parameters optimised individually (an automated way of doing this has not been found) to give the best attenuation, from sensor to test mass, at all frequencies above 10 Hz. In most cases two stages are employed. A 3rd stage can be added where necessary, but the "optimisation" becomes more tedious. Unlike in the previous (GEO) designs there is no single pole low pass filter between the upper corner of the differentiators and the poles of the filter. The hand-waving justification is that the notch filter brings the gain down over quite a wide band, and so prevents the peak of the low-pass filter response extending too high.

The basic design process starts with the selection of two low-Q poles (1 or 2) as low in frequency as possible, as determined by the acceptable phase margin at the highest intended UGP, followed by pairs of higher Q poles (Q around 5 to 10) placed as low as possible, such that their peaks do not reach unity gain at maximum overall gain (possible because of the attenuation from the other poles and the notch). The peak of the filter response determines the maximum damping that can be achieved at the lower modes, and sets the gain margin. The zeros are then placed around 10 to 12 Hz, as attenuation is provided by the suspension above that. There is, very probably, the possibility to design a much better digital filter to provide the attenuation of sensor noise, but the point is that the simple design, based on a small number of poles and zeros, suffices.

Most of the controllers are satisfactory but all could be improved, some substantially, given some more effort. It is assumed that mode decomposition is employed, and because the controllers are not all the same, this may lead to more trimming to eliminate unwanted cancellations where the decomposition is imperfect.

Note that the controllers are tuned to match the suspensions. In particular the active damping is not, in most cases, stable for the highest mode in a particular degree of freedom. A notch filter is placed at this mode, and in some cases has to be tuned to within about 0.1 Hz (to obtain the 6 dB gain margin stated in section 5).

Although I have tried to remove any errors, there may be misleading comments and unused filter stages in the code below. Note that all of the primitive functions use Hz as the units for input of frequency parameters. Most have as their last parameter a gain factor, here set to unity in all cases (the "gain" variable is used to set the overall gain).

6.2 Quad pendulum models and controller parameters

6.2.1 The generate_simulink file

```
%new hybrid damping script K.A. Strain 05/05
%uses standard quadopt file
%requires new .mdl files and new controller files
%minor changes to B,C,D matrices of ssmakefile
%assumes orthogonalisation at input and outputs of active parts
```

```
%user must ensure consistency of ecd applied to coupled dofs (l,p,y)
global pend
quadopt
ldamp = local4l(20); vdamp = local4v(20);   %in translational units
pdamp = local4p(0.04); ydamp =local4y(0.1); %in torque units
ssmake4e; % better blade modeling from MATHMATICA, Mark Barton (no essential
changes made by KAS beyond a couple of “1”s in C matrices to add outputs)
open pendecd.mdl
```

6.2.2 The longitudinal controller

```
function [damper] = local4l(gain)
%local to be combined with ECD kas 04/04
gain = gain * 10; %was intended to normalize but not actually done yet
[ad1,bd1,cd1,dd1] = transdif(0.2,5,1); %phase lead
[ad2,bd2,cd2,dd2] = transdif(0.3,2,1); %more phase lead
[alp,blp,clp,dlp] = sculte(4.5,2,10.1,40,1); %low pass first part
[alp2,blp2,clp2,dlp2] = sculte(7,10,11.5,20,1); %low pass second part
[an1,bn1,cn1,dn1] = notch(2,10,1); %avoid affecting third mode
[an2,bn2,cn2,dn2] = notch(3.5,10,1); %avoid affecting fourth mode
%start with just gain and then add on other stages
al = 0; bl = 0; cl = 0; dl = gain;
[al,bl,cl,dl] = series(al,bl,cl,dl,ad1,bd1,cd1,dd1);
[al,bl,cl,dl] = series(al,bl,cl,dl,ad2,bd2,cd2,dd2);
[al,bl,cl,dl] = series(al,bl,cl,dl,alp,blp,clp,dlp);
[al,bl,cl,dl] = series(al,bl,cl,dl,alp2,blp2,clp2,dlp2);
[al,bl,cl,dl] = series(al,bl,cl,dl,an1,bn1,cn1,dn1);
[al,bl,cl,dl] = series(al,bl,cl,dl,an2,bn2,cn2,dn2);
damper = ss(al,bl,cl,dl);
```

6.2.3 The pitch controller

```
function [damper] = local4p(gain)
%local to be combined with ECD kas 04/04
gain = gain * 10; %was intended to normalize but not actually done yet
[ad1,bd1,cd1,dd1] = transdif(0.3,5,1); %phase lead
[ad2,bd2,cd2,dd2] = transdif(0.3,4,1); %more phase lead
[alp,blp,clp,dlp] = sculte(4.5,1,10.1,40,1); %low pass first part
[alp2,blp2,clp2,dlp2] = sculte(7,10,11.5,20,1); %low pass second part
[an1,bn1,cn1,dn1] = notch(3,10,1); %avoid affecting third mode
%start with just gain and then add on other stages
al = 0; bl = 0; cl = 0; dl = gain;
[al,bl,cl,dl] = series(al,bl,cl,dl,ad1,bd1,cd1,dd1);
[al,bl,cl,dl] = series(al,bl,cl,dl,ad2,bd2,cd2,dd2);
[al,bl,cl,dl] = series(al,bl,cl,dl,alp,blp,clp,dlp);
[al,bl,cl,dl] = series(al,bl,cl,dl,alp2,blp2,clp2,dlp2);
[al,bl,cl,dl] = series(al,bl,cl,dl,an1,bn1,cn1,dn1);
damper = ss(al,bl,cl,dl);
```

6.2.4 The vertical controller

```
function [damper] = local4v(gain)
%local to be combined with ECD kas 04/04
gain = gain * 10; %was intended to normalize but not actually done yet
[ad1,bd1,cd1,dd1] = transdif(0.2,6,1); %general phase lead
[ad2,bd2,cd2,dd2] = transdif(1,3,1); %more phase lead near 1-2 Hz
[alp,blp,clp,dlp] = sculte(4.5,1,10,40,1); %low pass, first part
[alp2,blp2,clp2,dlp2] = sculte(6,10,11,20,1); %low pass second part
[an1,bn1,cn1,dn1] = notch(2.5,10,1); %avoid affecting second mode
[an2,bn2,cn2,dn2] = notch(4.1,10,1); %avoid affecting third mode
%start with just gain and then add on other stages
al = 0; bl = 0; cl = 0; dl = gain;
```

```
[al,bl,cl,dl] = series(al,bl,cl,dl,ad1,bd1,cd1,dd1);
[al,bl,cl,dl] = series(al,bl,cl,dl,ad2,bd2,cd2,dd2);
[al,bl,cl,dl] = series(al,bl,cl,dl,alp,blp,clp,dlp);
[al,bl,cl,dl] = series(al,bl,cl,dl,alp2,blp2,clp2,dlp2);
[al,bl,cl,dl] = series(al,bl,cl,dl,an1,bn1,cn1,dn1);
[al,bl,cl,dl] = series(al,bl,cl,dl,an2,bn2,cn2,dn2);
damper = ss(al,bl,cl,dl);
```

6.2.5 The yaw controller

```
function [damper] = local4v(gain)
%local to be combined with ECD kas 04/04
gain = gain * 10; %was intended to normalize but not actually done yet
[ad1,bd1,cd1,dd1] = transdif(0.2,6,1); %general phase lead
[ad2,bd2,cd2,dd2] = transdif(1,3,1); %more phase lead near 1-2 Hz
[alp,blp,clp,dlp] = sculte(4.5,1,10,40,1); %low pass, first part
[alp2,blp2,clp2,dlp2] = sculte(6,10,11,20,1); %low pass second part
[an1,bn1,cn1,dn1] = notch(2.5,10,1); %avoid affecting second mode
[an2,bn2,cn2,dn2] = notch(4.1,10,1); %avoid affecting third mode
%start with just gain and then add on other stages
al = 0; bl = 0; cl = 0; dl = gain;
[al,bl,cl,dl] = series(al,bl,cl,dl,ad1,bd1,cd1,dd1);
[al,bl,cl,dl] = series(al,bl,cl,dl,ad2,bd2,cd2,dd2);
[al,bl,cl,dl] = series(al,bl,cl,dl,alp,blp,clp,dlp);
[al,bl,cl,dl] = series(al,bl,cl,dl,alp2,blp2,clp2,dlp2);
[al,bl,cl,dl] = series(al,bl,cl,dl,an1,bn1,cn1,dn1);
[al,bl,cl,dl] = series(al,bl,cl,dl,an2,bn2,cn2,dn2);
damper = ss(al,bl,cl,dl);
```

6.2.6 The simulink diagram (top level and sub-models)

The diagram/model was generated in MATLAB 6.1 and is available on request. The usual pend*.mdl was edited where necessary to include velocity outputs for the top stage for each dof. The ground and sensor noise inputs were added, and the active control renamed to fit the ss names shown in generate_simulink. The ECD was represented by a straight gain (e.g. 27 for 27 kg/s and 0.27 for 27kg/s x 0.1m x0.1m – representing in this case torque damping of yaw).

6.3 Triple pendulum models and controller parameters

The generate_simulink file

```
%new hybrid damping script K.A. Strain 05/05
%uses new triple ssmake file for BS
%requires new .mdl files and new controller files
%minor changes to B,C,D matrices of ssmakefile
%assumes orthogonalisation at input and outputs of active parts
%user must ensure consistency of ecd applied to coupled dofs (l,p,y,r)
global pend
ldamp = local3l(3);  vdamp = local3v(6);
pdamp = local3p(0.011);   ydamp = local3y(0.05);
%transverse and yaw not required.
ssmake3ecd;
open pendecd.mdl
```

6.3.1 The longitudinal controller

```
function [damper] = local4l(gain)
%local to be combined with ECD kas 04/04
gain = gain * 10; %was intended to normalize but not actually done yet
[ad1,bd1,cd1,dd1] = transdif(0.2,3,1); %phase lead
[ad2,bd2,cd2,dd2] = transdif(0.2,3,1); %more phase lead 1 to 2 Hz
[alp,blp,clp,dlp] = sculte(5,1,10.1,40,1); %low pass first part
[alp2,blp2,clp2,dlp2] = sculte(7,7,11.5,20,1); %low pass second part
[an1,bn1,cn1,dn1] = notch(1.7,10,1); %avoid affecting third mode
%start with just gain and then add on other stages
al = 0; bl = 0; cl = 0; dl = gain;
[al,bl,cl,dl] = series(al,bl,cl,dl,ad1,bd1,cd1,dd1);
[al,bl,cl,dl] = series(al,bl,cl,dl,ad2,bd2,cd2,dd2);
[al,bl,cl,dl] = series(al,bl,cl,dl,alp,blp,clp,dlp);
[al,bl,cl,dl] = series(al,bl,cl,dl,alp2,blp2,clp2,dlp2);
[al,bl,cl,dl] = series(al,bl,cl,dl,an1,bn1,cn1,dn1);
damper = ss(al,bl,cl,dl);
```

6.3.2 The pitch controller

```
function [damper] = local4p(gain)
%local to be combined with ECD kas 04/04
gain = gain * 10; %was intended to normalize but not actually done yet
[ad1,bd1,cd1,dd1] = transdif(0.2,1.4,1); %phase lead
[ad2,bd2,cd2,dd2] = transdif(0.2,1.4,1); %more phase lead 1 to 2 Hz
[alp,blp,clp,dlp] = sculte(4,1,10.1,40,1); %low pass first part
[alp2,blp2,clp2,dlp2] = sculte(6,7,11.5,20,1); %low pass second part
[an1,bn1,cn1,dn1] = notch(1.17,10,1); %avoid affecting third mode
%start with just gain and then add on other stages
al = 0; bl = 0; cl = 0; dl = gain;
[al,bl,cl,dl] = series(al,bl,cl,dl,ad1,bd1,cd1,dd1);
[al,bl,cl,dl] = series(al,bl,cl,dl,ad2,bd2,cd2,dd2);
[al,bl,cl,dl] = series(al,bl,cl,dl,alp,blp,clp,dlp);
```

```
[al,bl,cl,dl] = series(al,bl,cl,dl,alp2,blp2,clp2,dlp2);
[al,bl,cl,dl] = series(al,bl,cl,dl,an1,bn1,cn1,dn1);
damper = ss(al,bl,cl,dl);
```

6.3.3 The vertical controller

```
function [damper] = local4v(gain)
%local to be combined with ECD kas 04/04
gain = gain * 10; %was intended to normalize but not actually done yet
[ad1,bd1,cd1,dd1] = transdif(0.5,3,1); %general phase lead
[ad2,bd2,cd2,dd2] = transdif(0.5,3,1); %more phase lead near 1-2 Hz
[alp,blp,clp,dlp] = sculte(5.0,1,10,40,1); %low pass, first part
[alp2,blp2,clp2,dlp2] = sculte(5.5,10,11.2,20,1); %low pass second part
[an1,bn1,cn1,dn1] = notch(4.3,10,1); %avoid affecting second mode
%start with just gain and then add on other stages
al = 0; bl = 0; cl = 0; dl = gain;
[al,bl,cl,dl] = series(al,bl,cl,dl,ad1,bd1,cd1,dd1);
[al,bl,cl,dl] = series(al,bl,cl,dl,ad2,bd2,cd2,dd2);
[al,bl,cl,dl] = series(al,bl,cl,dl,alp,blp,clp,dlp);
[al,bl,cl,dl] = series(al,bl,cl,dl,alp2,blp2,clp2,dlp2);
[al,bl,cl,dl] = series(al,bl,cl,dl,an1,bn1,cn1,dn1);
damper = ss(al,bl,cl,dl);
```

6.3.4 The yaw controller

```
function [damper] = local4y(gain)
%local to be combined with ECD kas 04/04
gain = gain * 10; %was intended to normalize but not actually done yet
[ad1,bd1,cd1,dd1] = transdif(0.3,3,1); %phase lead
[ad2,bd2,cd2,dd2] = transdif(0.3,3,1); %more phase lead 1 to 2 Hz
[alp,blp,clp,dlp] = sculte(4.5,1,10.1,40,1); %low pass first part
[alp2,blp2,clp2,dlp2] = sculte(6,4,11.5,20,1); %low pass second part
[an1,bn1,cn1,dn1] = notch(1.5,10,1); %avoid affecting second mode
[an2,bn2,cn2,dn2] = notch(2.1,10,1); %avoid affecting third mode
%start with just gain and then add on other stages
al = 0; bl = 0; cl = 0; dl = gain;
[al,bl,cl,dl] = series(al,bl,cl,dl,ad1,bd1,cd1,dd1);
[al,bl,cl,dl] = series(al,bl,cl,dl,ad2,bd2,cd2,dd2);
[al,bl,cl,dl] = series(al,bl,cl,dl,alp,blp,clp,dlp);
[al,bl,cl,dl] = series(al,bl,cl,dl,alp2,blp2,clp2,dlp2);
[al,bl,cl,dl] = series(al,bl,cl,dl,an1,bn1,cn1,dn1);
[al,bl,cl,dl] = series(al,bl,cl,dl,an2,bn2,cn2,dn2);
damper = ss(al,bl,cl,dl);
```

6.3.5 The triple pendulum SS models

```
%combination file of triple_BS provided by Norna and modified, old, triple
pendulum ssmake file  - for brevity most of this is cut out and only sections
which were amended are shown:-

First some of the key parameters to allow checking:-
g      =9.81;
 ux     = 0.06;                 %dimensions of UPPER MASS (square)
 uy  =  0.35;
 uz     = 0.15;
 den1   = 4000;         %density (steel with holes)

 ix     = 0.06;                 %dimension of INTERMEDIATE MASS (cylinder)
 ir     = 0.175;
den2    = 2202;                 %density (fused silica)

tx      = 0.06;                 %dimensions of TEST MASS (cylinder)
```

```
tr      = 0.175;
den3    = 2202;                  %density (fused silica)

l1 = 0.55; % reduce to be equivalent to 2.005 m quad ETM length Feb 04
l2 = 0.55;
l3 = 0.60;

 r1     =300e-6;        % radius of upper wire
 r2     =200e-6;        % radius of intermediate wire
 r3 = 113e-6; % gives stress ~770 MPa

[uf,lnb,anb,hnb,stn]  =  opt(mnb,mntb,8e8,0.25,0.065);%changed  by  NAR,  same
length as MC

[uf,lnb,anb,hnb,stn] = opt(mnb,mntb,8e8,0.14,0.028);%changed by NAR to suit
beamsplitter width
% X direction separation
 su     = 0.00;         % 1/2 separation of upper wires
 si      = 0.015;                 % 1/2 separation of intermediate wires NAR
change 19Sep03
 sl     = 0.005;        % 1/2 separation of lower wires


% Y direction separation
 n0     = 0.077; %changed by NAR for similar footprint to MC
 n1     = 0.13;         % 1/2 separation of upper wires at upper mass
 n2     = 0.06;         % 1/2 separation of intermediate wires at upper mass
 n3     =ir+0.0065;     % 1/2 separation of intermedite wires at intermediate
mass
 n4     =tr+0.0015;     % 1/2 separation of lower wires at intermediate mass
 n5     =tr+0.0015;     % 1/2 separation of lower wires at test mass
```

To make checking simple the B and C matrices were changed to include just the necessary inputs and outputs (ground motion, force (torque) on top mass, translation (angle) of top mass, translation (angle) of test mass and velocity (angular velocity) of top mass. “D” matrices were all zero.

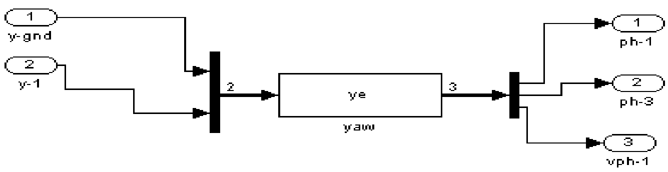
```
B=[0  0     0     0     0     0 -m13*g*d0/I1y/l1/c1 +m13*g/m1/l1/c1   0 0
      0     0
   0  0     0     0     0     0       0    1/m1  0     0         0 0
   0    0    0    0    0    0  1/I1y 0      0 0         0 0
]';

C=[0  1     0     0     0     0     0     0     0     0     0     0
   0  0     0     0     0     1     0     0     0     0     0     0
   1  0     0     0     0     0     0     0     0     0     0     0
   0  0     0     0     1     0     0     0     0     0     0     0
   0  0     0     0     0     0     0     1     0     0     0     0
   0  0     0     0     0     0     1     0     0     0     0     0];

lpe = ss(A,B,C,D);

% yaw

br=[  0     0     0     AA/I1z 0    0
      0     0     0     1/I1z  0    0 ]';

cr=[   1    0     0     0     0     0
         0  0     1     0     0     0
         0  0     0     1     0     0];
```

```
ye = ss(ar,br,cr,dr);

%vertical

bv= [ 0     0     0     h1 /m1  0          0
      0     0     0        1 / m1   0         0
     ]';

cv= [1      0     0     0     0     0
        0   0     1     0     0     0
        0   0     0     1     0     0];


ve = ss(av,bv,cv,dv);

Transverse and roll were not required.
```

6.3.6 The simulink diagram (top level followed by lower level)

The diagram/model was generated in MATLAB 6.1 and is available on request. The usual pend*.mdl was edited heavily to simplify it and include velocity outputs for the top stage for each dof. The ground and sensor noise inputs were added, and the active control renamed to fit the ss names shown in generate_simulink. The ECD was represented by a straight gain (e.g. 27 for 27 kg/s and 0.27 for 27kg/s x 0.1m x0.1m – representing in this case torque damping of yaw).

Note that the ECD gain triangles for longitudinal and vertical accept the damping in kg/s directly, whereas the pitch and yaw dampers require this to be multiplied by the square of the offset from the axis (lever-arm) before entering it in the block. So, for example, longitudinal may have 28 kg/s while yaw may have 0.28 kg/s if the dampers are, on average, 0.1m from the axis. The active gains are normalised so that the nominal operating gain is 1 (at which the settling time figures and noise coupling factors were determined), but the loops should be stable with gain of up to 2 (in some cases not much more).



6.4 Other non-standard functions

To allow the expert reader to find where the poles and zeros were placed. The definitions are meant to be intuitive. The type of notch may be different from used before and may not be optimum.

```
function [a,b,c,d] = notch(notch,qn,dcGain)
zs     = pi*notch*(-1/qn + i*sqrt(4 - 1/(qn^2)));
z      = [conj(zs) zs]';
```

```
k      = dcGain;
ps   = notch;
p    = [-2*pi*ps -2 *pi*ps];
[a,b,c,d] = zp2ss(z,p,k);


function [a,b,c,d] = sculte(peak,qp,notch,qn,dcGain)
%resonant 2-pole low pass filter in state space representaion
%[a,b,c,d] = sculte(peak,qp,notch,qn,dcGain);
%peak             frequency cut (Hz)
%qp               Q factor of resonance
%notch                  notch frequency (above peak)
%qn               Q factor of notch
%dcGain                 dc gain
%Stuart Killbourn (October 95)
z      = pi*notch*(-1/qn + i*sqrt(4 - 1/(qn^2)));
z      = [conj(z) z]';
p      = pi*peak*(-1/qp + i*sqrt(4 - 1/(qp^2)));
p      = [conj(p) p]';
k      = dcGain*(peak/notch)^2;
[a,b,c,d] = zp2ss(z,p,k);


function [a,b,c,d] = transdif(lf,hf,dcGain)
%transistional differentiator in state space representation
%
%[a,b,c,d] = transdif(lf,hf,dcGain);
%
%lf               start differentaition (Hz)
%hf               stop differentaition (Hz)
%dcGain                 dc gain
%Stuart Killbourn (October 95)
z      = -2*pi*lf;
p      = -2*pi*hf;
k      = dcGain*(hf/lf);
[a,b,c,d] = zp2ss(z,p,k);
```