Coherent Targeted Searches for Gravitational Bursts using LIGO

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One class of sources LIGO is searching for consists of short duration gravitational wave bursts of *a priori* unknown waveform. Potential sources include supernovae core collapse and the merger phase of coalescing binary black holes. To date, existing burst search algorithms have concentrated only on all-sky searches for such events. We have developed an algorithm that allows for maximum sensitivity to a desired sky location, by forming a linear combination, a coherent sum, of data from a network of detectors. To accomplish this a search is performed over a two dimensional parameterization of signal space. This algorithm was implemented as a part of the Q-pipeline data analysis package, and was tested for various types of simulated data for Hanford and Livingston detectors. A substantial gain on SNR (signal-to-noise ratio) was observed in most cases depending on the incident direction of the gravitational wave; moreover, it was found that for detection purposes it is sufficient to iterate the sum over all possible sky positions. In addition, statistical tests were developed in order to distinguish between glitches and gravitational waves, especially for the two Hanford detectors.

The effects of gravitational waves, the squeezing and stretching of matter, is staggeringly small, compared for instance to that of electromagnetic waves. Detecting this effect experimentally is the ultimate objective of the LIGO Scientific Collaboration [1]. Gravitational waves, as predicted by Einstein's theory of general relativity, are generated whenever movement of matter results in a time-changing mass quadruple moment [2]. Our focus will be on gravitational bursts, those of duration of less than one second which fall in the 50 –2kHz frequency bandwidth of ground-based detectors. Signal from gravitational bursts are expected to fall very closely to the noise level of the detectors, and consequently very hard to detect [3]. Considerable work has been completed regarding theoretically expected waveforms that can be detected through matched-filtering approach. In this article, however, we deal with gravitational wave signals that do not have well known *a priori* form. We primarily focus on the time-frequency representation approach used for bursts detection, in particular on the Q transform [4], which was originally designed and implemented to process data from different detectors independently. Only in the post-processing stage could candidate events be verified through comparison across different detectors in the network. However, this incoherent approach is not very efficient, and requires that the single detector response meet the minimum criteria for a candidate event. We have developed and implemented an algorithm that enables us to coherently combine the response of all detectors, and then threshold for candidate events on the single coherent response. This modification makes use of currently existing detector software, such as Q transform, and is implemented as an extension. We expect the coherent approach to bring better detection efficiency, a higher signal-to-noise ratio, and depending on the number of detectors, other capabilities such as detection of source location and ability to distinguish between glitches and gravitational waves.

In this article, we present our algorithm, which follows closely from the work of Julien Sylvestre [3]; furthermore, the implementation of the algorithm is also discussed, followed by its application to a simulated gravitational burst. More detailed aspects of the algorithm and the required computational costs are also looked at closely. Lastly, focusing on the LIGO detectors alone, a method of consistency checks using only the collocated Hanford detectors is proposed.

Q Transform

We use the method of time-frequency approach in order to detect gravitational waves. The data is first projected onto a basis of waveforms covering the targeted signal space, and then thresholded on energy for selection of candidate events. In particular, the Q transform [4] is used: the basis waveforms are approximately Gaussians windowed sinusoids defined by a central time, central frequency, and Q value. Q is defined as the ratio of central frequency to the frequency width (f_c / σ_f), which roughly corresponds to the number of oscillations in the wave.

Detector response is whitened and projected onto the space of time-frequency-Q, and in turn searched for any statistically significant events that could qualify as a gravitational wave. The projection corresponds to assigning of a complex coefficient to any particular tile in the target space. The magnitude of these coefficients, known as the Z value, is related to the signal to noise ratio (SNR) through the relationship:

$$SNR = \sqrt{2 \times (Z - 1)} \quad \textbf{(0)}$$

Algorithm for targeted searches

In order to achieve maximum sensitivity to a desired sky location, we have designed the following algorithm. It is based closely on the work of Julien Sylvestre [3]. The underlying approach, aside from the trivial time shift, is to reduce the *a priori* knowledge of the wave for the evaluation of detector response to only two parameters: the ratio of the power of the two independent polarizations, and their inner product.

In the general case, we assume a network of N interferometers is used for GW detection. The response of detector i will be denoted as x_i, and modeled as:

$$x_i = F_i^+(\theta,\phi,\psi)T[\Delta_i(\theta,\phi)]h^+ + F_i^\times(\theta,\phi,\psi)T[\Delta_i(\theta,\phi)]h^\times + \eta_i \quad \textbf{(1)}$$

h^+ and $h^\times$ denote the intrinsic components of the two polarization of the propagating wave. $F_i^+(\theta,\phi,\psi)$ and $F_i^\times(\theta,\phi,\psi)$ are detector specific antenna pattern functions. $\theta,\phi,$ define the incident direction of the wave; ψ is the polarization angle and is arbitrarily taken as zero in this algorithm, its definitions in turn fixes the polarization components of the wave. η_i represents the additive noise of the given detector. We construct a linear combination of all detector inputs that will result in highest sensitivity to the desired sky location. This response will take the form for N detectors:

$$X = \sum_{i=1}^{N} a_i T(\delta_i) x_i \quad \textbf{(2)}$$

A set of real coefficient a_i are selected, denoted as vector $\vec{a}$, to achieve highest SNR (for the given direction) in the combined response. $T(\delta_i)$ is the time shift operator. Before forming a coherent detector response from the network input, it is imperative that we correct for the different arrival time of the gravitational wave burst. Gravitational waves propagate at the speed of light; consequently, the geographical separation of the detectors results in time-shifted responses. Given the direction of incidence, we can trivially correct for this by shifting all data to the same reference, for instance the center of the earth.

From here on, x_i will denote the time shifted response of detector i, which accounts for the different arrival time of the gravitational wave. The power of combined response is defined as $P = X \cdot X$. We separate the signal and noise components of this power.

$$P = \xi + \eta \quad \textbf{(3)}$$

The signal power is written as,

$$\xi = \sum_{i,j} a_i a_j \left[F_i^+ F_j^+ h^+ \cdot h^+ + (F_i^+ F_j^\times + F_j^+ F_i^\times) h^\times \cdot h^+ + F_i^\times F_j^\times h^\times \cdot h^\times \right]. \quad \textbf{(4)}$$

Similarly the stochastic noise power is formulated as,

$$\eta = \sum_{i,j} a_i a_j \left[(F_i^+ h^+ + F_i^\times h^\times)\eta_j + \eta_i \cdot \eta_j \right]. \quad \textbf{(5)}$$

Taking the expectation value of the noise power, and assuming white Gaussian noise, it can be shown that

$$E[\eta] = \sum_i a_i^2 \sigma_i^2, \quad \textbf{(6)}$$

where σ_i denotes noise variance of detector i. The problem is then reduced to maximizing ξ with $E[\eta]$ kept constant. To do so, we use the Lagrange multiplier method. We also define SNR ratio ρ as,

$$\rho = \xi / E[\eta]. \quad \textbf{(7)}$$

Since ξ is defined as a function of a_i s, we evaluate the gradient of ξ over the space of these parameters.

$$\frac{d}{da_i}\xi = \sum_j a_j \left[F_i^+ F_j^+ h^+ \cdot h^+ + (F_i^+ F_j^\times + F_j^+ F_i^\times) h^\times \cdot h^+ + F_i^\times F_j^\times h^\times \cdot h^\times\right]. \quad \textbf{(8)}$$

Imposing the condition of fixed noise power, we have

$$\lambda a_i \sigma_i^2 + \sum_j a_j \left[F_i^+ F_j^+ h^+ \cdot h^+ + (F_i^+ F_j^\times + F_j^+ F_i^\times) h^\times \cdot h^+ + F_i^\times F_j^\times h^\times \cdot h^\times\right] = 0. \quad \textbf{(9)}$$

The above expressions is divided by $|h^+||h^\times|$, this will reduce the number of parameters, and lead to a redefinition of λ. Moreover, we define parameters $\Lambda_{+/\times}$ and $\Lambda_{+\cdot\times}$ as $\frac{|h^+|}{|h^\times|}$ and $\frac{h^+ \cdot h^\times}{|h^\times||h^+|}$ respectively. We are required to search over this two-dimensional parameter space for a given target position in order to find the optimum combination of detector response. We have reduced the problem of completely unknown waveforms to that of an unknown power ratio and direct product of two polarizations, enabling feasible computational searches over the parameter space for directional searches. We can express the modified version of equation **9** in its matrix form. We define matrix S_{ij} as:

$$S_{ij} = \frac{1}{\sigma_i^2}\left[F_i^+ F_j^+ \Lambda_{+/\times} + (F_i^+ F_j^\times + F_j^+ F_i^\times)\Lambda_{+\cdot\times} + F_i^\times F_j^\times \frac{1}{\Lambda_{+/\times}}\right]. \quad \textbf{(10)}$$

Eigenvectors and eigenvalues of matrix S are easily determined from equation $S.\vec{a} = 0$. Each eigenvalue corresponds to the Lagrange parameter scaled in terms of the energy of the two polarizations. The eigenvectors, corresponding to vector of scaling coefficients, are plugged back into equation 4, ξ is calculated, and the eigenvector corresponding to largest possible ξ is maintained. This vector is the scaling coefficients that are to be used for the specific sky position.

The final algorithm that implements the above with the Q transform takes the following form:
1. A sky location is selected (θ, ϕ), time corrections are applied to each detector response
2. Search is conducted over 2D parameter space $\Lambda_{+/\times}$ and $\Lambda_{+\cdot\times}$
3. For each $\Lambda_{+/\times}$ and $\Lambda_{+\cdot\times}$ matrix S is computed, its eigenvectors and eigenvalues are determined
4. The N eigenvectors are plugged back into equation 4; the vector that maximizes the signal power is selected
5. This vector is used to calculate the combined response; the Q transform is performed on combined response, SNR of the combined signal is determined.
6. Step two is repeated, and the process continues until highest SNR is found.

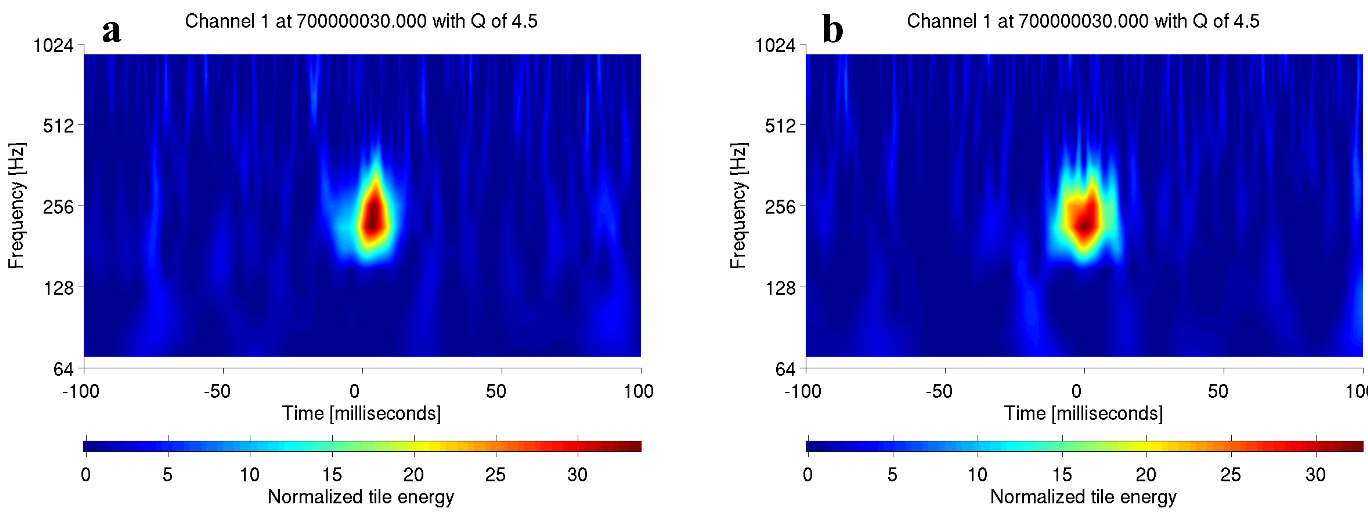
The final eigenvector gives us the optimal way of combining the detector responses for the given sky location. Moreover, parameters $\Lambda_{+/\times}$ and $\Lambda_{+\cdot\times}$ give us two parameters (polarization power ratio and cross product) of the wave originating from given location. Studies ought to be conducted on the systematic errors originating from effects of signals from other locations which might interfere with what is scaled and detected. Consistency tests (especially using time differences) are necessary to avoid such scenarios.

Implementation: Coherent Sum

The software implementation of the above algorithm –referred to as the coherent sum, broke down into two major components. The first step was the time shifting of the detector data appropriate to the desired sky position implemented using Matlab (qtimeshift.m). This was primarily built upon existing code from the collaboration's library [4]. The second component was the calculation of the sum coefficients, which we chose to separate into two distinct pieces of software (qcoherent.m, qcoherentsub.m). In qcoherentsub.m the calculations relating to the Lagrangian maximization are performed. All parameters, including the two describing the intrinsic waveform, are

passed to this Matlab function, which in turn returns a coherent transform corresponding to the linear combination that maximizes the SNR. The maximization at this stage, i.e. with the two waveform parameters and sky position set, is only between the different eigenvectors (which is numerically equivalent to number of independent detectors) derived from the Lagrange approach; the theoretical value of signal power, as describe in pervious section's equation 4, is used to select the eigenvector (corresponding to set of scalar coefficients) that would lead to maximum SNR. Numerical evaluation of the SNR of the combined response is not required at this stage to select the appropriate coefficients when all the parameters are fixed. The theoretical values that are dependent on the waveform parameters calculated will suffice for this purpose, since at this stage the parametric values are not varied. In order to search over the parameter space of feasible waveforms –the two parameters corresponding to the ratio of the powers and the normalized inner product of the two polarizations, we coded and implemented Matlab function qcoherent.m. In this function a search over the 2D parameter space is conducted. Furthermore, at this stage, we are required to determine the SNR of the combined response in order to select the particular choice of parameter that is believed to correspond most closely to the true waveform. The Q transform software is used for this purpose [3]. Keeping in mind, that our goal is detection and not reconstruction, we do not closely associate the selected parameters with a true waveform, but rather think of them as phenomenological parameters. However, it is of interest to be able to perform consistency checks based on how realistic a set of parameters might be.
We will describe the application of the coherent sum method to a simulated Sine-Gaussian wave with frequency of 235Hz (SG235) originating from the galactic center added to simulated detector noise; moreover, we only use the two LIGO detectors (Hanford and Livingston) for this analysis. It has to be noted that the results observed below are reflective only of this particular case, in general gains on SNR and detection capability is dependent on the waveform, direction of propagation, and the detectors used. A general quantification of increase in detection capability using the coherent approach is also presented at the end of this article.

As can be observed in figure 1, the single detector maximum Z value is around 33.7, the coherent Z however is close to 60.0, this corresponds to a gain of approximately 78 percent in the maximum Z value (the figures are auto-scaled to maximum Z value). This is a significant gain and will have tangible implications in detection, especially for cases where the signal SNR is very close to the threshold level.



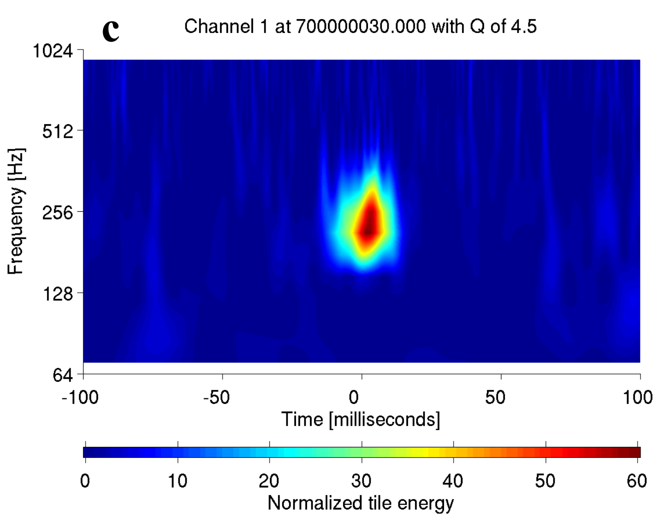


Figure 1| a, *Q spectrogram of Hanford response, Q = 4.5 is displayed. A maximum Z value of around 34 is observed. Source is simulated Sine-Gaussian wave with frequency of 235 Hz from the galactic center.* ***b,*** *Q spectrogram of Livingston response to the same simulated GWB. Approximately same maximum Z value is observed.* ***c,*** *Q transform spectrogram of the coherent sum (i.e. linear combination of single detector responses that would result in maximum SNR). A maximum Z value of approximately 60 is observed for the coherent sum. Note that the figures are auto-scaled to their maximum Z value. The colorbar below each subfigure reflects the observed Z values.*

Parameter Space

As was discussed in the derivation of optimization algorithm, it is possible to determine the optimal combination coefficients based on a minimum knowledge of two parameters about the intrinsic form of the propagating gravitational wave. We plot the variations of maximum Z on a two dimensional contour plot. The Y axis corresponds to the normalized inner product of the two polarizations on a linear scale, and in turn the X axis corresponds to ratio of power between the plus and cross polarizations in a logarithmic scale. The limits of this infinite parameter space are set according to theoretical expectations and empirical observations; for the ratio of powers logarithmic limits were set at -2 and 2, below and above which the wave has almost all its power in one of its polarization components and the coefficient stabilize asymptotically. The inner product limits are set at -1.5 and 1.5 on a linear scale, although the limits might seem counterintuitive, they were selected empirically to capture any interesting behavior and major fluctuations in the parameter space. It is also of importance to recognize that these parameters are phenomenological by nature and the connection to power is one that is established arbitrarily; it can be argued that these two parameters correspond to some physical parameters relating some property of the two polarizations (in case of the inner product), and some property of the wave to itself (in case of single powers), that is a generalized kernel can be introduced in the definition of these parameters –refer to eq.1. For our purposes, it suffices to think of these parameters in terms of energy ratios and inner product of two functions normalized in terms of their energies. Fig 2 presents the variation of the Z value over the waveform parameter space for our example of SG235 simulated burst. Due to the explicit use of two parameters in describing the waveform, the results become independent of the polarization angle of the gravitational wave. We arbitrarily choose an angle of zero as our polarization angle, but for demonstration purposes we present the same parameter space (for same input) but with the polarization angle changed to $\pi/8$ (Figure 2).

In both cases the same value of maximum Z is achieved, however the parameter values corresponding to this maximum change by modifying the polarization angle; moreover, the overall shape of the parameter space variation also changes. This is expected since a change in polarization angle corresponds to a redefinition of the two polarizations components h+ and h×, resulting in a modified variation pattern.

We can focus on the special case of linear polarization by choosing ψ such that $h^{+} = h\cos(2\psi)$ and $h^{\times} = -h\sin(2\psi)$. Then in the limit of $\psi \rightarrow 0$ one can evaluate the two parameters $\Lambda_{+/\times}$ and $\Lambda_{+\cdot\times}$ for the case of linear polarization. It is found that $\Lambda_{+\cdot\times}$ = -1 for such waveforms, with the negative originating from our arbitrary definition; furthermore, $\Lambda_{+/\times}$ can be related to ψ as $\left|\cos(2\psi)/\sin(2\psi)\right|$. This value changes from 0 to infinity. To be solely sensitive to linearly polarized waves then one can fix as $\Lambda_{+\cdot\times}$ -1 and conduct a one-dimensional search over $\Lambda_{+/\times}$.

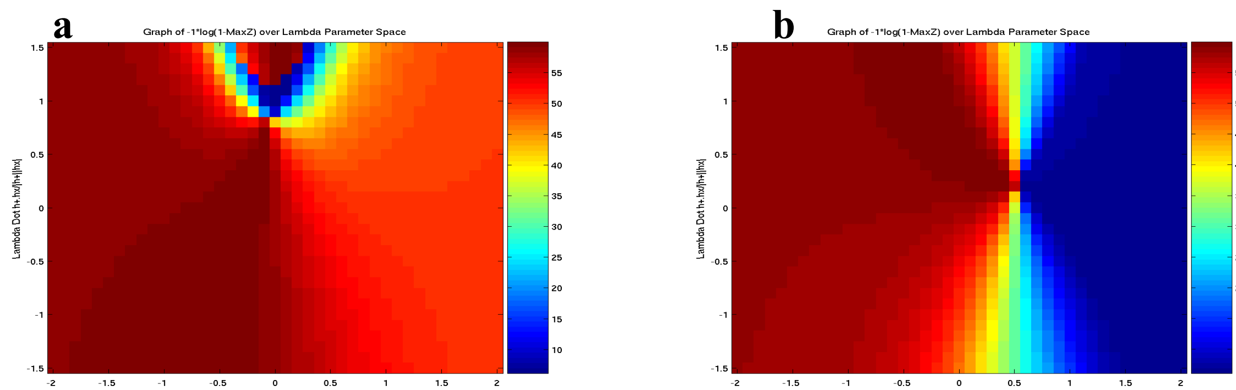


Figure 2| a, *Parameter space for coherent sum of SG235 simulated wave originating from the galactic center. The x axis corresponds to variation of* $\Lambda_{+/\times}$ *logarithmically from –2 to 2. Y axis corresponds to variation of* $\Lambda_{+\cdot\times}$ *from –1.5 to 1.5. Plotted are the Z values for each given choice of parameters.* ***b****, Same plot but with varied polarization angle, psi = pi/8. We observe a change in the pattern as expected from the redefinition of each polarization component. The same maximum value of Z is however observed. The arbitrarily choice of psi then does not alter the maximum Z value that is achievable through formation of a linear combination.*

Sky Maps

In practice, the assumption of known origin of the gravitational wave does not stand, we are required then to search over all sky positions, and apply the above algorithm of coherent sum to every single location. In theory, we expect the highest SNR to be achieved at the true position of the wave or equivalent positions that are indistinguishable, in practice however, we observe that this is not the case due to statistical limitations and systematic errors. However, it has to be noted that for the purposes of detection, it suffices to simply find any position of high SNR, in which case, we remain careful and do not use the derived information of the waveform as of those corresponding to that of the true wave.

First, to demonstrate the regions of equivalent time shift, which play an important role in the coherent sum and also in the analysis of glitches, we plot the time shift difference between the two detectors as a function of position over all sky. The spherical plot (Figure 3a) uses Galactic coordinates, and plots each value on the surface of a sphere. As is expected, we observe circles of equivalent time shifts along an axis that corresponds to the line connecting the two detectors in space. The maximum amount of time shift occurs at the poles of this imaginary coordinate system. If we were then to redefine our latitude and longitude, so that the axis connecting the two detectors would correspond to the reference axis, the time shifts would be independent of longitude and only a function of this redefined latitude.

For the case of SG235 data, we have conducted a complete sky search with resolution of five degrees. The resulting sky map is plotted as figure 3b with the true direction of propagation of the wave (the galactic center) marked with a white dot. We can clearly observe regions of similar coherent sum Z, which correspond to circles of equivalent time shift. The absolute highest Z value does not correspond to the true wave position, and regardless is not statistically significant enough to be of any information. For detection purposes finding a high region of SNR without knowledge of the direction propagation is sufficient. However, this has to be accompanied with strict consistency checks to ensure that the observed signal was a result of a gravitational wave as opposed to detector glitches.

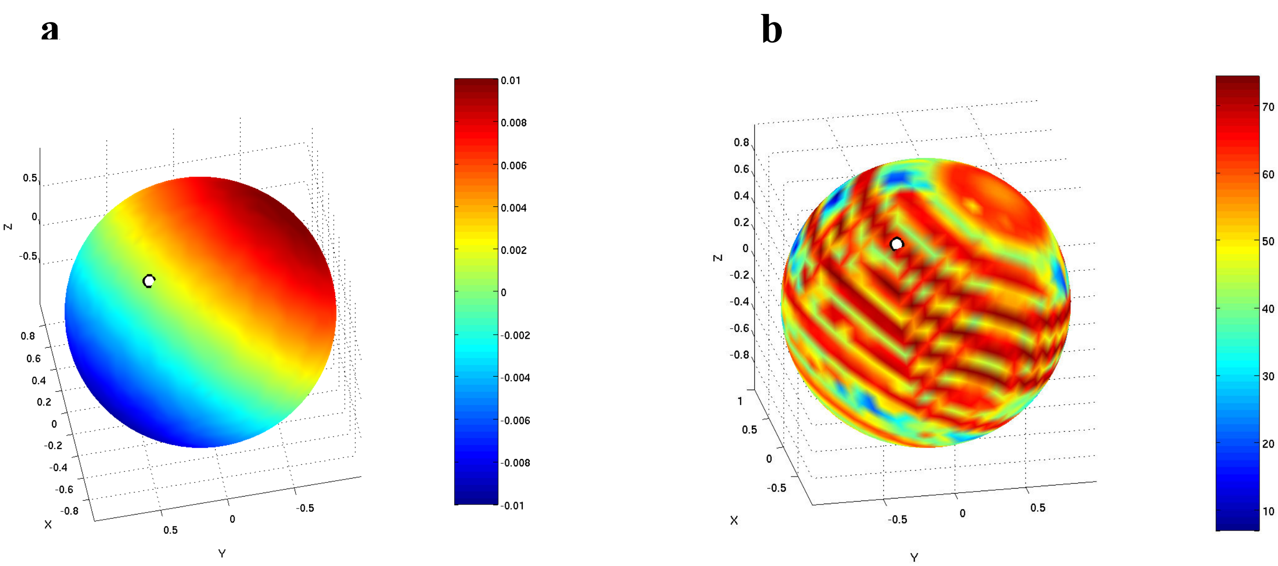


__Figure 3| a__, Regions of same time shift. Sphere corresponds to a map of the sky in galactic coordinates. Each color corresponds to the needed time shift to transform the detector response to an imaginary detector located at the center of earth. These regions correspond to circles with circumfrance points that are equidistant to a given detector. The imaginary axis corresponds to the line connecting to the two detectors (Hanford and Livingston detectors in this case). __b__, All sky search applied to the simulated GW SG235. The color shading corresponds to highest Z value (and in turn SNR) that was possible through formation of the coherent sum. The white circle marks the true direction of incidence (i.e. the galactic center) of simulated wave. The true origin does not corresponds to highest Z, circular regions of high Z, coincident with regions of same time shift are observed.

Distinguishing glitches from gravitational waves

In line with our simulated experiment in burst detection using only the LIGO network of detectors, a consistency check needed to be developed to distinguish between glitches and gravitational waves. If no assumption is made of the expected form of the wave –and this tends to be the case for bursts, it is theoretically impossible to categorize a set of responses as either glitches or waves. In the three detector case, however, it is possible to look for a linear combination of responses (analogous to the coherent sum), that would result in the termination of the signal. This approach, known as the Null Stream approach, is currently being explored [2], but requires a third independent detector, such as VIRGO located in Italy.

If restricted to the LIGO network, it is possible to perform consistency checks using only the two Hanford collocated detectors H1 and H2. The H2 detector is analogous to H1 but has half the arm length, and consequently greater noise than H1. It is expected that a true gravitational wave would result in an analogous response in both these collocated detectors, i.e. same transform coefficients ought to be observed in frequency, time, and Q space. The amplitude of the responses must also be related, and hence a subtraction of the signals in H1 and H2 should leave only the background noise if the signal was caused by a gravitational burst as opposed to a glitch. An event that passes the threshold then, is ran against a consistency check, where the Q transform of the signal of H1 and H2 detectors are subtracted from each other. The resulting difference is tested over a region of the time-frequency plane for consistency with noise. The region of interest is empirically selected to contain mostly signal while minimizing the contribution of background noise, and the test is performed using a modification of the well known Kolmogorov-Smirnov (KS) statistical test. The distribution of the subtracted coefficients should be Gaussian both in both their real and complex components. Figure 4 shows the resultant subtracted transform of a simulated glitch and a simulated gravitational wave. For comparison purposes, the distributions of the real component of the two resultant transforms are also presented. Only the simulated gravitation wave would pass the KS statistical test; in the case of the glitch the difference transform contains large coefficients that show at both ends of the distribution distorting it from a true Gaussian shape.

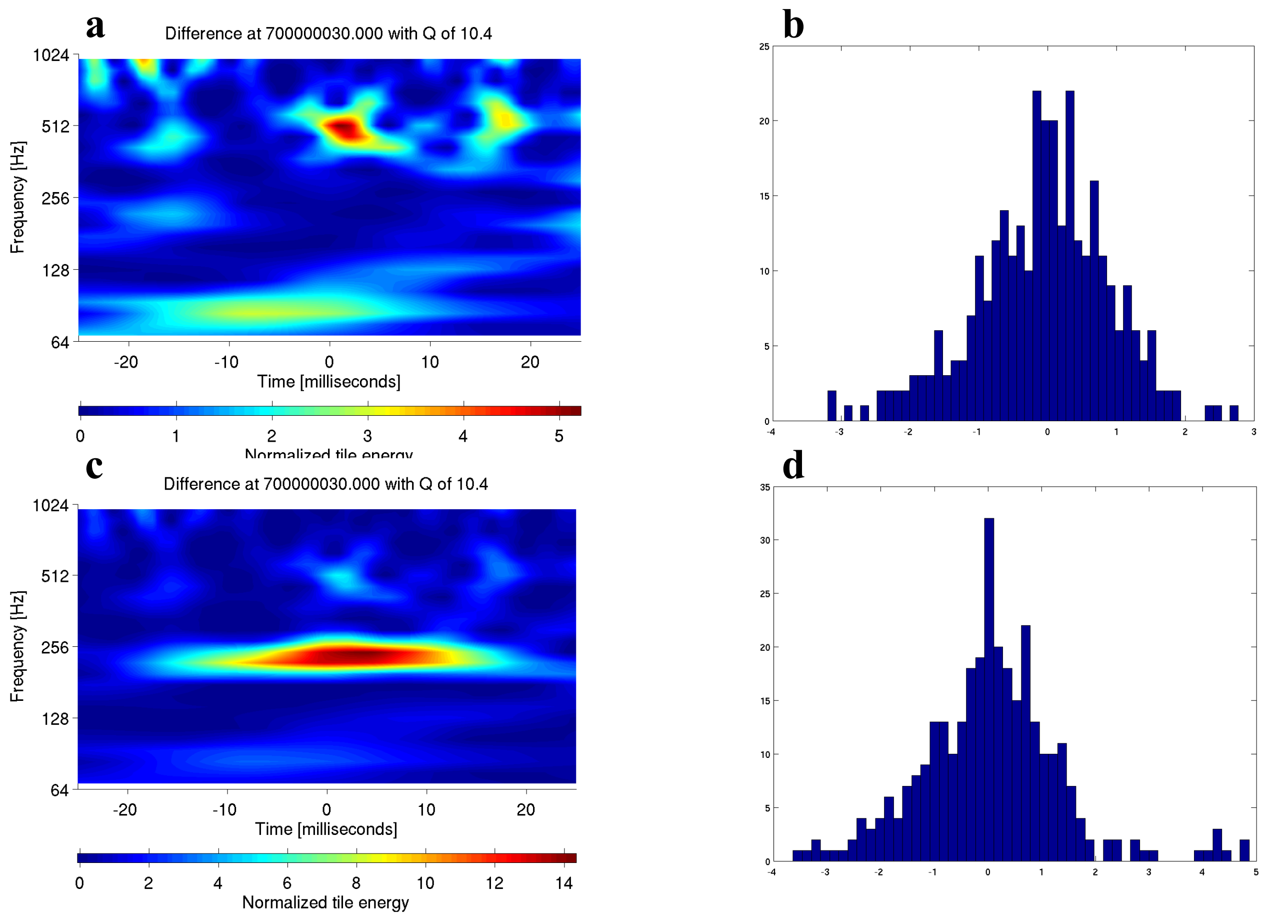


Figure 5| ***a**, Q spectrogram of H1 – H2. The two detector response transform coefficients are directly subtracted from each other without any weighing. The simulated response in this case corresponds to a true gravitational wave.* ***b**, The distribution of real part of the transform coefficients of a. As theoretically expected the distribution is a Gaussian and passes the KS statistical test. The x axis ranges from –4 to 3.* ***c**, Q spectrogram of H1 – H2. In this case the detector responses were due to a simulated glitch.* ***d**, Distribution of real part of the transform coefficients of c. The distribution is not Gaussian, note the excess signal on each side. The x axis ranges from –4 to 5. This distribution fails the KS statistical test. We reject the hypothesis of occurrence of true gravitational wave.*

Discussion

In summary, we have developed an algorithm and implemented it as a part of Q transform data analysis package that enables us to combine responses from a network of detectors, increasing detection capability for gravitational wave bursts. In order to find the coefficients that would result in maximum SNR, it was necessary to conduct a search over a two dimensional parameter space corresponding to the intrinsic wave-form; one parameter corresponds to energy ratio and the other to the inner product of the two polarizations of the propagating wave. This approach assumes a known direction of propagation for gravitational wave; however, it is possible to conduct all sky blind searches by iterating the search over all sky locations. This method was demonstrated for the case of Sine-Gaussian simulated waveform originating from the galactic center and incident upon the two LIGO detectors. An improvement of 50% was observed in SNR when a coherent sum was formed. In general, however, the gain is dependent on the direction of incidence, the location and number of detectors.

Furthermore, we have developed a consistency check method that would allow us to better distinguish between glitches and gravitational waves when restricted only to the LIGO network. This method made use of the two collocated detectors at the Hanford site. This consistency check works concurrently with the coherent sum algorithm and prevents the incorrect combination of two glitches that would result in a false detection.

As the next step, we plan to perform more systematic testing on the coherent sum method, for purposes of validation and comparison to existing approaches. It is also our goal to run this algorithm on real data from the LIGO network and look for candidate events.

METHODS

Software. All software implementation was conducted using Matlab R14.

Coherent Sum. The coherent algorithm makes the assumption of white noise, i.e. it is expected that the variance of detector noise would remain constant. However, this is not true in practice, the detector noise is colored and greatly varies over its frequency bandwidth. To get around this problem we divide the frequency bandwidth into distinct frequency bins. Each bin is assumed to have constant noise variance; computation of the coefficients then is conducted in each bin separately. This forces us to make the assumption that the waveform parameters are the same in all given frequency bins.

Selection of coefficients resulting in maximum SNR. The intervals of time, frequency, and Q that are of interest are passed to qcoherent.m, the function then searches for the parameters that would result in the highest possible *single* value of Z in the desired region. A coherent transform corresponding to this highest value of Z is returned to the parent function. Note that the approach of maximizing a single value of Z is a very reasonable one for the purposes of detection. However, in order to make sure that we are not susceptible to false detection consistency checks on possible detections at a post-processing stage are required. For now, the statistical test we use is simply the selection of the highest possible SNR value in the region of interest.

Parameter Space Resolution used. Empirical tests were performed on what kind of resolution was necessary for the parameter space. This was especially computationally significant when the search over the parameter space had to be iterated over all sky locations. The figures in this article have much higher resolution than what was used in all sky searches. In most cases we reduced the number of points sample to around 50. Qualitatively, this made us less susceptible to waves with more sophisticated waveforms.

Testing Inverse Reconstruction. For testing purposes, we were able to do rough noisy reconstruction of the gravitational wave form. It suffices to calculate the inverse matrix of the detector responses, and simply find the solution to the linear set of equations that have resulted in the response functions. We use the linearity property of Fourier transform, and conduct this search in the frequency domain. A residual noise term equal to the inverse matrix multiplied by the detectors noise spectra remains, then in order to see the signal we take the q transform of the noisy reconstructed wave forms; this gives as a rough idea about the wave form expected, and the reconstruction powers of the algorithm. Later, we hope to use this technique in our studies on distinguishing gravitational waves from glitches.

Q Difference. It is possible to extend the coherent sum algorithm to coherent differences where linear combinations are formed to minimize the SNR, i.e. eliminate the signal. In the three detector case, this can be used rigorously for distinguishing glitches and gravitational waves. We expect that it should be possible to use a coherent difference algorithm –a trivial extension of the current algorithm, as a form of a consistency check in the two detector case.

Consistency check using collocated detectors. In order to perform this consistency check we go through each frequency band, since the detector noise is colored but also well understood and modeled as a combination of seismic, thermal, and shot noise. Subtraction of coefficients (which are complex Gaussian distributed due to the central limit theorem) results in a new distribution for the noise, which is also Gaussian distributed with a variance equal to the summation of all single detector variances. In order to maintain the white nature of the noise, this process is conducted in each frequency band first, and the bands are later normalized to reflect white noise distribution. A key issue that remains is the dependence of tiles. We account for this roughly by using a modified number of independent values in the KS statistical test. Matlab's KSTest routine was modified for this purpose. It remains, however, that for statistically significant results a region in frequency, tile, and Q planes should be selected that would reflect the signal and not that of the noise. This selection process is empirical at the moment but is to be extended further in future. Statistical KSTest2 is a standard test where two empirical distributions are compared to each other. Modified number of independent tiles, approximated, is used as the number of independent points in each distribution. A second theoretical Gaussian distribution is formed with the same mean and standard deviation as the data and ran against the empirical distribution of the data. The systematic calibration error between the two collocated detectors can be ignored due to the extensive statistical error present in the case of H1 and H2 detectors.

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