Some Notes on Gas Damping in Small Gaps as a Potential Noise Source for Advanced LIGO Suspensions

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1. Introduction.

The baseline design for the suspension of the end test masses (ETMs) in Advanced LIGO calls for a gap of 5 mm between the back face of the test mass and the front face of the reaction mass which has a patterned gold coating for electrostatic actuation. The choice of this size of gap is based on an extrapolation of the electrostatic design currently used in GEO 600, in which a smaller gap of 3 mm is used. The choice of gap size depends on several factors. A larger gap requires more drive to achieve the same force. However mechanical installation and alignment issues are eased. We have chosen 5 mm as the baseline design since we believe we can apply the required level of force across this gap with electronics similar to that used in GEO. We should not require such large forces in Advanced LIGO as in GEO, despite the heavier masses, due to the effect of the active seismic isolation platforms which significantly reduce the residual motion of the masses.

The NSF review committee has raised the issue of the potential for increased gas damping and noise forces from gas in such a gap over and above the expected level due to the average residual pressure in the vacuum chamber. We summarise here findings from the literature that have a bearing on this topic, and draw some conclusions for Advanced LIGO.

2. Gas damping in Advanced LIGO

Currently in LIGO the average pressure as read on a discharge gauge is typically 10^{-8} torr of H_2, or equivalently 1.33 x 10^{-6} N m^{-2} of H_2 (ref e-mail from R Weiss, 2 Aug 2005). We note that the residual pressure may be limited by outgassing from viton which will not be present in Advanced LIGO. However we will assume the above pressure also applies to Advanced LIGO. Using this value we can estimate the amount of gas damping and hence the contribution to the suspension thermal noise due to gas damping.

Firstly we apply Christian's model [1] as quoted in Bao et al [2] for the quality factor of a plate oscillating in a low pressure gas, which is derived using a free molecular model. In this model the resistive damping force is found by considering the momentum transfer from the vibrating plate to the surrounding gas due to collisions between the plate and the molecules of gas. A Maxwell-Boltzmann distribution for the gas velocity is assumed in the derivation. The quality factor is given by

$$Q = \left(\frac{\pi}{2}\right)^{3/2} \rho H f_0 \sqrt{\frac{RT}{M_m}} \frac{1}{P} \qquad (1)$$

where ρ is the specific mass of the plate, H is the thickness of the plate, R is the gas constant, T is the temperature, f_0 is the oscillating frequency of the plate, M_m is the molar weight of the gas and P is the pressure.

We substitute some numbers for the baseline suspension design. The final stage of the quadruple pendulum is 0.6 m long giving an uncoupled resonant frequency f_0 of 0.644 Hz. The specific mass or density is 2200 kg m^{-3} (fused silica) and the thickness of the mass is 0.2 m. Taking the pressure quoted above, a temperature of 300 K and a molar mass of 2×10^{-3} kg for H_2, we calculate $Q \sim 4.7 \times 10^{11}$.

We can now estimate what the residual motion due to suspension thermal noise would be if the pendulum was damped only by gas damping. Assuming a viscous damping model, the mean squared motion (in m^2/Hz) due to thermal noise, above resonance, is given by

$$x^2 = \frac{4kT\omega_0}{mQ\omega^4} \tag{2}$$

where k is the Boltzmann constant, $\omega_0 = 2\pi f_0$ and m is the mass of the oscillator. Substituting $Q = 4.7 \times 10^{11}$ and $m = 40$ kg and solving for the motion at $\omega = 2\pi \times 10$ rad s^{-1} we find

$$x \sim 1.5 \times 10^{-20} \text{ m}/\sqrt{\text{Hz}} \tag{3}$$

This should be compared to the target displacement noise level due to suspension thermal noise of 10^{-19} m/√Hz at 10 Hz for Advanced LIGO. The displacement noise due to the effect of gas damping is ~6.5 times smaller than the target noise level. This can be presented in another way - if gas damping were the only damping mechanism, the Q could be ~ 40 times smaller than the value calculated above, or around 1.1×10^{10}.

Damping due to the suspension itself is assumed to be the largest loss mechanism. We can put a limit to how much extra damping from gas damping we could tolerate by considering for example that the gas damping should increase the displacement noise by no more than 10% from the target noise level. The mean squared thermal motion due to a sum of losses is given by:

$$x^2 = \frac{4kT\omega_0^2\phi_{tot}(\omega)}{m\omega^5} \tag{4}$$

where the total loss is given by $\phi_{tot} = \phi_{sus} + \phi_{gas}$. Here ϕ_{sus} is the loss associated with the suspension and is assumed constant. ϕ_{gas} is the loss associated with the residual gas and $\phi_{gas} = \phi_{0gas} \times \omega / \omega_0 = \phi_{0gas} \times f / fo$ where ϕ_{0gas} is the loss at the resonance. We note $\phi_{0gas} = 1/Q_{gas}$.

Consider $f = 10$ Hz. Since x is proportional to the square root of ϕ_{tot}, a value of ϕ_{gas} which is 20% of ϕ_{sus} will increase x by 10%. Using (4) we find that the suspension loss ϕ_{sus} which corresponds to $x = 10^{-19}$ m/√Hz is 1.44×10^{-9}. Hence we can tolerate a maximum ϕ_{gas} at 10 Hz of $\sim 0.2 \times 1.44 \times 10^{-9}$. Since $Q_{gas} = \omega/(\omega_o\phi_{gas})$ this corresponds to a minimum Q_{gas} of 5.4×10^{10}. This value is almost an order of magnitude less than the estimated Q_{gas} at the current LIGO pressure, and thus the estimated damping due to the current LIGO gas pressure does not present a noise problem.

3. Limitations of the Christian model and the effect of nearby walls.

As Christian himself notes in [1] his theory assumes the oscillating mass is situated in an infinitely large volume. He notes that if the surrounding walls are close, the number of molecules striking the oscillator would be greater due to reflection from the walls, resulting in an increased damping factor or reduced Q. The effect of nearby surfaces has been considered by those working with micro-resonators. In these cases the gap distance is typically much smaller then the typical dimensions of the resonator. For example Bao et al [2] have derived a relationship for Q in the so-called squeeze-film regime (eqn. 24 in [2]) which states

$$Q(\text{Bao et al}) = Q\,(\text{Christian}) \times 16\pi d/L, \tag{4}$$

where d = gap and L = length of oscillating plate.

This equation has been derived by considering the energy transfer from the oscillating plate to the surrounding gas and accounts for the effects of the walls and dimensions of the oscillator. For the case of Advanced LIGO where $d = 5$ mm and $L \sim 340$ mm (where 340 mm is the diameter of the silica mass) the term $16\pi d/L = 0.74$. Thus the Q calculated using this squeeze-film model of gas damping would be 1.4 times smaller than the simple model.

Hutcherson and Ye [3] have also considered squeezed-film damping and have pointed out some limitations of Bao et al's model. Firstly they point out that it assumes a constant particle velocity when calculating the number of collisions it makes during its period of interaction, and that this can lead to an underestimation of the damping. In [3] they allow for a variation in their molecular dynamics simulation code, and for a particular micro-resonator they predicted a Q a factor of 2 less than Bao et al would predict for the same conditions. There is not sufficient information given to extrapolate that finding to the dimensions of the Advanced LIGO mass and gap. However the typical ratio of gap to size of oscillator (d/L) Bao et al discuss is 1/200, compared to our case of 5/340. So we would expect that this extra effect is not so pronounced for our case.

We note that Hutcherson and Ye demonstrate that their model is in good agreement with experimental data taken on microbeam resonators where the gap ratio was ~1/200 at pressures which were in the free molecule regime, although not nearly as low as the pressure in LIGO (good agreement was seen over the range 0.08 to 3 torr). We can assume that the pressure used in their calculations was the ambient pressure in the vacuum tank, not the pressure in the gap, since that would not have been easy to measure. Thus if outgassing were raising the pressure in the gap compared to the ambient pressure its effects were not seen.

Hutcherson and Ye also point out another two assumptions in Bao et al's analysis.

1) the typical time for a gas molecule staying between the resonator and the wall is much smaller then the oscillating cycle, and
2) the amplitude of oscillation of the resonator is much smaller than the gap.

Both of these are valid for our situation (note that the rms velocity of a hydrogen molecule at 300 K is ~ 2 km/s).

In conclusion it appears that taking a conservative estimate based on these two papers, the squeeze-film damping effect could degrade the Q by a factor of ~3 (~1.4 x 2). This is not a problem assuming the current residual gas pressure of 10^{-8} torr.

4. Outgassing and Adsorption.

In the squeeze-film damping model it has been assumed that the pressure in the gap is the same as the ambient pressure in the surroundings. This may not be the case due to local outgassing from the surfaces. There are two issues that we should consider.

i) Outgassing could increase the amount of damping if the mean pressure in the gap is increased.
ii) Random bursts of outgassing could cause impulsive noise events.

Some knowledge of rate of outgassing would help to identify which of these is more problematic. As Ken Strain has noted, (e-mail correspondence) if the "burstiness" is mainly on a short enough timescale it could be dealt with as a modified pressure. Bursts on the ms to s timescale would look more like impulsive noise events.

Before considering i), we note that the opposite process will also occur – molecules impinging on the surface can be adsorbed, and this will also affect the damping characteristics. Work on gas-surface interactions include studies of what is termed the normal momentum accommodation coefficient (NMAC) which characterises momentum transfer efficiency in the interaction of a gas and solid surface. See for example [4]. The value of the NMAC coefficient α_n is zero if no adsorption takes place and its maximum value is 1. In this paper Polikarpov et al investigated the motion of a silicon plate moving in an ultra-high vacuum (~10^{-7} Pa) under varying gas pressure and composition, measuring the damping coefficient of free oscillations. They also derive an expression for the damping coefficient as a function of the NMAC coefficient, α_n, given by

$$\beta = \frac{2-\alpha_n}{\rho H} P \sqrt{\frac{2m_m}{\pi k T}} \qquad (5)$$

where ρ, H, P, T and k are as defined earlier, and m_m is the molecular weight of the gas molecules. Noting that in their paper β is equal to $\omega_0 /2Q$, and that $m_m/k = M_m/R$ we can rewrite this in terms of Q, as follows

$$\begin{aligned} Q &= \frac{\rho H}{2-\alpha_n} \frac{\omega_0}{2P} \sqrt{\frac{\pi R T}{2M_m}} \\ &= \left(\frac{2}{2-\alpha_n}\right)\left(\frac{\pi}{2}\right)^{3/2} \rho H f_0 \sqrt{\frac{RT}{M_m}} \frac{1}{P} \qquad (6) \end{aligned}$$

We can compare this to equation (1) above, and we see the only difference is in the first term in the brackets. The interesting point to note is that a non-zero value of α_n leads to *lower* damping (higher Q).

In [4] the authors obtain experimental values of α_n for various mixtures of gases, and deduce that in their particular set-up the NMAC value for a mixture with a high content of hydrogen is in the range 0.33 to 0.71, a result which they note "does not contradict the known data". These values correspond to an increase in Q as calculated by equation (1) by a factor of ~1.2 to ~1.6 for the silicon plate discussed above. It should be noted however that NMAC results are typically measured under non-equilibrium conditions and so this may not be applicable to the LIGO situation.

We see that adsorption can decrease the gas damping effect. If outgassing leads to a higher pressure than we have assumed in our calculations then it will increase the gas damping effect. One might assume that equilibrium is reached between these two processes, leading to some residual pressure level that may be higher than the ambient gas pressure as measured by a discharge gauge in the tank

The squeezed-film effect might be regarded as equivalent to a raised pressure in the gap. Since Q_{gas} is inversely proportional to pressure, the squeeze-film effect as analysed in [3] could be regarded as equivalent to an increase in pressure of ~3. If in addition outgassing raises the pressure in the gap by a further factor of ~3, we would still only be contributing to raising the suspension thermal noise by ~10% assuming the case of 10^{-8} torr ambient pressure as currently in LIGO. If the pressure in the gap were 10 times the ambient pressure and we also allow for the squeeze-film effect, so that in total the equivalent pressure increase is a factor of 30, the Q_{gas} would be ~1.6×10^{10} and hence ϕ_{gas} at 10 Hz = 9.9×10^{-10}. This is ~70% of the value of the ϕ_{sus} which corresponds to the target displacement noise. Hence the displacement noise would be increased from the target figure by ~ 35% - or in other words it would be 1.35×10^{-19} m/√ Hz at 10 Hz.

ii) As regards random bursts causing impulsive events, firstly we note that these are the type of events which lead us to carry out coincidence experiments. The rate at which two such events in different detectors occur at or close to the same time is much smaller than the rate at which individual events might occur. We can also do a back-of-the-envelope calculation on how much gas needs to be released in 10 ms (say) to move a mirror by a factor of 5 above the expected sensitivity level at 100 Hz, assuming the gas is released at right angles to the surface. For a single mass the residual motion at 100 Hz is expected to be ~ 6×10^{-21} m/rt Hz. Consider a 100 Hz bandwidth. The total motion is thus ~ 6×10^{-20} m. If a burst of mass M of hydrogen molecules is released with velocity ~ 2 km/s (rms velocity of H_2 at 300 K), then the recoil velocity V of the 40 kg mass is given by V = M x 2000/40 = 50M m s^{-1}. The recoil displacement is V/ω. Let this equal $5 \times 6 \times 10^{-20}$ = 3×10^{-19} m. Thus V = $3 \times 10^{-19} \times \omega$ = $3 \times 10^{-19} \times 2 \times \pi \times 100$. Hence M = V/50 = ~ 4×10^{-18} kg. *Is this a reasonable argument? How does this amount compare to typical outgassing rates from a silica surface? (More info needed here)*

5. Conclusions

As shown in section 2, assuming a standard model of gas damping as in Christian [1], ignoring effects due to small gaps, the gas damping due to the current LIGO pressure leads to a Q_{gas} an order of magnitude larger than the value that would increase the suspension thermal noise by 10%, and hence assuming the standard model its effect can be ignored.

With a small gap between the suspended mass and the reaction mass several new factors come into play as outlined in section 3. Firstly there is the squeeze-film effect as modeled by Bao et al [2], which for the Advanced LIGO parameters would suggest a decrease in Q_{gas} of ~1.4 from the value predicted by the standard model. Applying a refinement of the Bao et al analysis due Hutcherson and Ye [3], we conclude that Q_{gas} may be further reduced by a factor less than 2 (their model predicted a factor of 2 for a more extreme ratio of gap to face size). Thus conservatively the squeeze-film effect may reduce Q_{gas} by a factor of 3 from the standard model value. This increase is not significant.

We now consider that outgassing from the surfaces in the gap may raise the local pressure in the gap and hence further increase the gas damping over that due to the squeeze-film effect (which in itself could be considered as equivalent to a localised increase in pressure). As shown in section 4, if the outgassing rate were such that the localised pressure was a factor of three higher than the ambient residual pressure, Q_{gas} would be such that the suspension thermal noise would be raised by 10% from its target value. A localized pressure ten times ambient would raise the suspension thermal noise by 35%.

It is worth noting here, as mentioned in section 3, that Hutcherson and Ye found good agreement between their refined squeeze-film model and some experimental data taken at pressures that were in the free molecule regime. Since we can reasonably assume that the pressure used in their calculations was the ambient pressure in the vacuum tank, this agreement suggests that in their case outgassing was not significantly increasing the pressure in the gap. Admittedly the pressures were much higher than we have in LIGO and so making deductions from their results may not be warranted.

We should also take into consideration that adsorption may be occurring at some level and its effect would tend to counteract that of outgassing.

The above arguments refer to outgassing which is on a timescale much faster than the timescales of interest for gravitational wave detection. Bursts on ms to s timescales could look like impulsive events. However for such events, coincident analysis will greatly reduce the likelihood of their misinterpretation as astronomical events.

References

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