Investigation into blade torsion, blade lateral flexibility, and the effect they have on blade and wire performance.

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1. INTRODUCTION

Following on from the FEA investigations into blade performance with angled wires described in T050259-03 interest has been expressed in the detail of exactly what is happening at the blade tip, specifically the exact interactions between the blade tip and the wire. What follows is an attempt to characterise this behaviour through the use of FEA and analytical methods. Section 2 explains the FE model, section 3 the FEA results, section 4 an analytical study of torsion, and section 5 an analytical study of lateral movement.

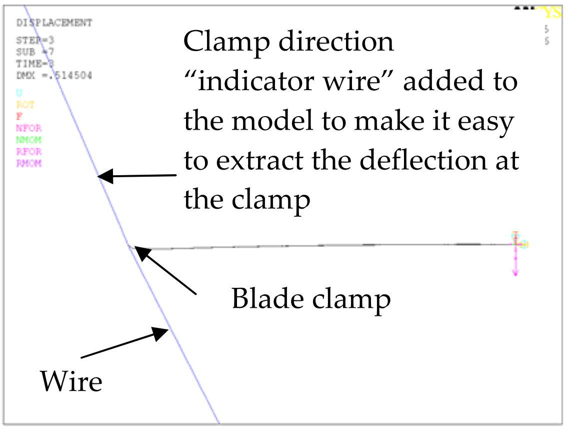
Appendix A has a graphical explanation of a counter-intuitive rotation of the blade tip under lateral loads (see section 3.2).

Appendix B has an email about the stability implications of the lateral flexibility, and appendix C has an email reconciling appendix B with section 5.

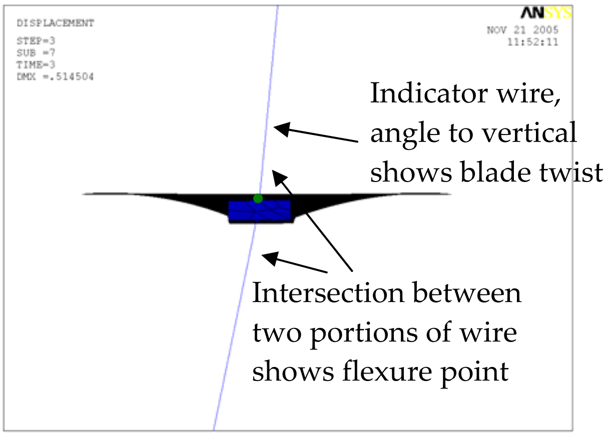
2. BACKGROUND/METHOD: FEA

A simple ANSYS model was developed by Justin Greenhalgh for the frequency analysis of the blades (T040215). This has now been extended to include the wire and wire clamp. Forces can now be excerpted on the wire and the effects on the blades observed. A pair of output plots are shown below in Table 1, these have been labelled to explain the FE model.

Table 1



Side view of loaded blade. Blade clamp portion has very thick elements preventing bending[1]



The green circle shows the break off of the wire. Looking at the intersection between the indicator wire and the break off gives the flexure length.

Various manipulations of the geometry have been tried in an attempt to under stand the blade behaviour. The more useful are described below.

[1] Note; this figure shows a very early model with a poor representation of the wire clamp.

3. OBSERVATIONS/RESULTS OF FEA

It has been known for some time that if a lateral load is applied to the end of a wire, then the wire will bend certain distance below the clamp break off face. It has also been known that the blade will twist a small amount as it retards the force applied to it by the bending wire. In an attempt to quantify this a 100N lateral load was applied to the free end of the wire in the FEA model after the blade had been pulled flat and the wire tensioned. This resulted in the blade twisting as anticipated.

Two things were apparent.

- Firstly, the twisting blade tip will result an alternative d distance (see under 3.1 below).
- Secondly, the blade tip twists in a sense the reverse of what might have been expected (see under 3.2 below)

These are detailed in the following two sections. As required specific FEA analysis will be used as examples.

3.1 "flexure point" distances

Up until recently we believed we had a clear understanding of the wire flexure point, which is to say that point, a small distance below the wire clamp breakoff face, about which the pendulum swings when wire bends a finite distance away from the breakoff face. In "Damping dilution factor for a pendulum in an interferometric gravitational waves detector" by G Cagnoli et al a single wire pendulum is taken and its flexure point defined algebraically as:

$$fl = \sqrt{\frac{EI}{T}}$$

Where:

fl= flexure length

E= Young's modulus

I= 2nd moment of area of the wire

T= wire tension in N

This is the first thing to use the FEA model to verify. A middle blade wire was selected and the wire clamp held statically, the free end of the wire was then pulled sideways by 100N and the flexure distance measured. This provided a flexure length of 0.002277m, using the same inputs for the above equation yields 0.002365m. This is a very encouraging start, and could possibly be improved upon using more elements.

The model can now be extended to look at the blade, the clamp and the wire. This will result in some blade twist as well as the wire bending. In addition there will be a lateral movement of the blade tip. These three things are illustrated in the diagrams below.

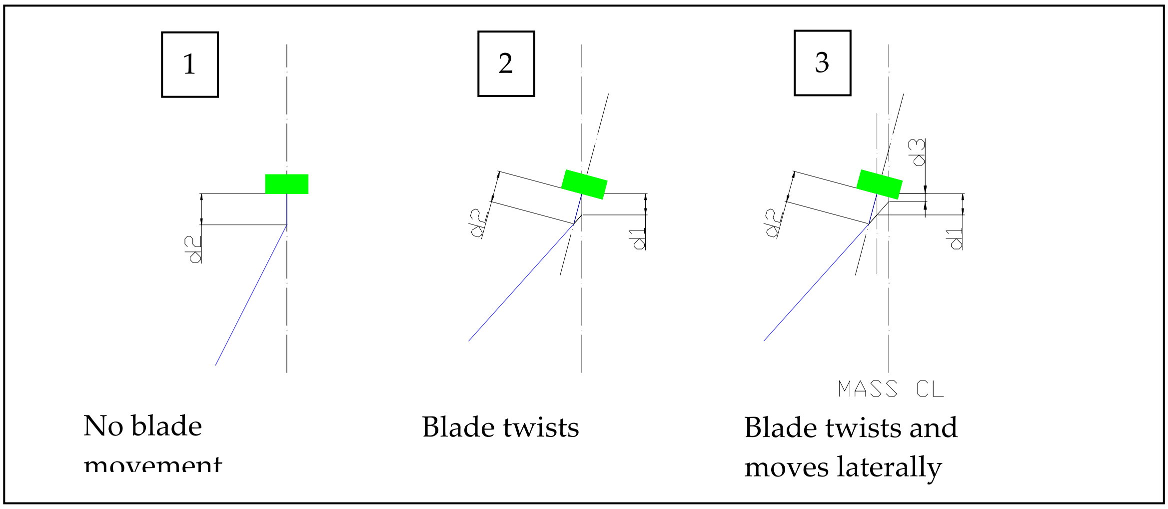
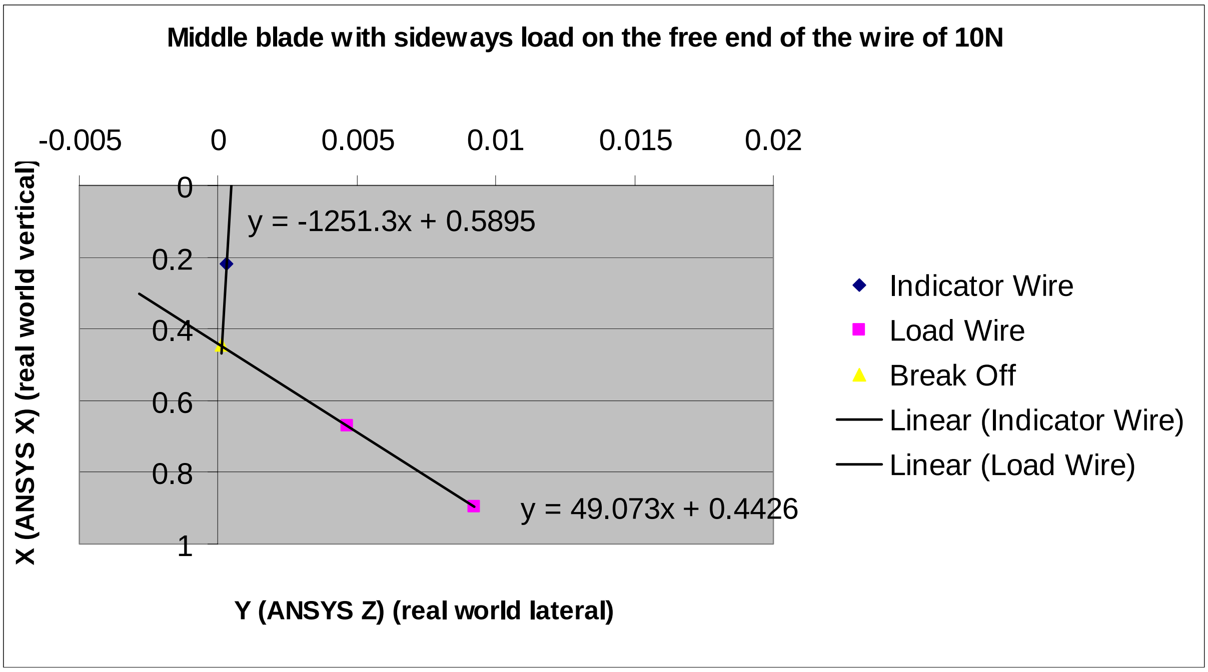


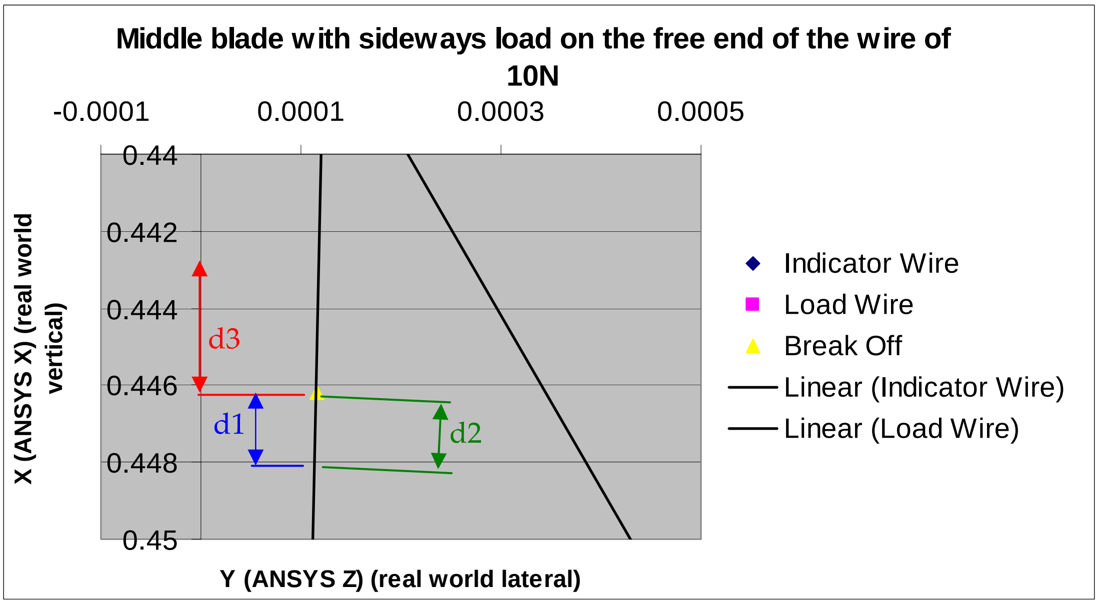
Figure 1
Section 1 of Figure 1 shows a simple statically constrained wire clamp with a wire bending below the wire break off. In fact of course the wire has a finite bending radius but this simple approximation has been applied successfully to date. The distance d2 is the same flexure distance defined by Cagnoli et al. Section 2 of Figure 1 shows a more complex system with the additional variable of the blade tip twisting. It should be noted that this configuration yields an alternative flexure distance d1, this is the wire flexure distance with reference to the local vertical (the untwisted CL of the blade). The third and final section of Figure 1 shows the additional complication of the blade tip having moved laterally due to the forces excerpted by the wire, this has resulted in d3; the d distance on the mass centre line.

3.2 Direction of twist of blade tip.

Now looking at a real blade we can collect some data from a FE analysis and plot where the wire and the blade tip are. The plot below is lifted directly from Excel. It shows the x and y locations of points on the wire and on the indicator wire, in their deflected positions. There are 5 data points, two on the indicator wire, two on the loaded wire and one at the clamp break off. From these it is possible to construct the three flexure points.



Clearly a close up of the centre would be useful. This is shown below.



Using the definitions of d1, d2, & d3 above they can be calculated to be:

d1	d2	d3
0.00232m	0.002072m	-0.004048m

From just these simple results it is apparent that the blade is not behaving like the sketches in Figure 1 suggest. Taking the raw FEA data and sketching out what it means results in a figure a bit like Figure 2. Looking at Figure 2 it is evident that d3 is smaller than either of the other two. Clearly d3 can be negative in this set up, and it is also correct that d1 is bigger than d2.

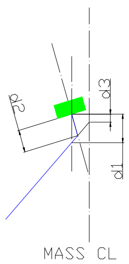


Figure 2

Looking at Figure 2 the blade tip has moved in the same direction as the force. It is slightly disconcerting that the tip has twisted in the opposite direction to the wire. This is because the blade geometry is imperfect, the standard imperfection is that the parallel sided tip kicks up at the end causing the wire break off point to be above the centre line of the majority of the blade, this creates a moment around the blade with the blade kick up providing the lever arm. This is what causes the twist shown in Figure 2. For a graphical explanation see Appendix A.

3.3 Breaking the problem down

At first glance Section 3.3 appears to adequately explain the blade behaviour, however, the lateral offset between the mass centre line and the blade centre line is dependant on the magnitude of the sideways force. So for very small sideways forces induced by pitch the difference between d1 and d3 will be very small.

It is also likely that the pitch of the blade tip is dependant on the magnitude of the sideways load, once again this will mean that d1 and d2 are very similar if the mass pitch is very small.

In an attempt to understand the magnitudes of the various contributing factors we are going to break the problem down into three sections.

a. Does the flexure point vary with sideways force if the wire clamp is fixed? We don't believe it does but it is best to make sure.

b. How far does the blade tip move and twist with varying side loads?

c. With the answers to 'a' and 'b' is it possible to correlate these two behaviours with what is seen on the full wire blade model? What changes are then seen with small sideways forces? Can a pitch tolerance be put on the assembled masses to prevent a problem?

3.3.1 Does the flexure point location vary with sideways load?

In short the answer is no. Running multiple analyses with consistent geometry, only a small movement of the flexure point can be seen as the sideways load increases from 5 to 100N (see figure 4). It was observed that the flexure point varied in position by 0.2mm in ~2.5mm. It should be stated that the element size has a significant effect on the errors in this calculation. We do not plan to pursue this further.

3.3.2 How far does the blade tip move and twist with varying side loads?

Re-configuring the FE setup the blade twist can be simulated; running a series of side loads provides the results shown in Figure 3:

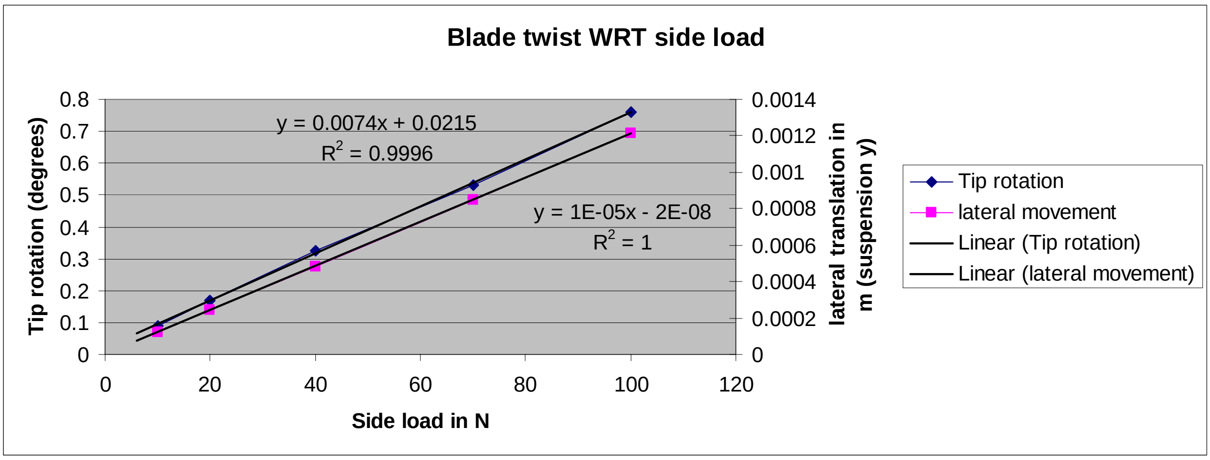


Figure 3

As would be expected the twist and translation of the blade tip is directly proportional to the side load. And as such is zero when zero force is applied. For an advanced LIGO suspension this tells us that the blade tip will remain in its original position, and won't rotate when very small forces are applied.

In principal it should now be possible to simulate a number of blades with wires and hopefully see that d2 is constant for each blade, and that the sideways force then contributes some blade twist and lateral motion.

3.3.3 *Combined wire and blade*

If we have now explained all the phenomena contributing to blade behaviour then with a combined analysis we would expect that d2 would be constant, and the lateral motion and twist of the blade tip would vary with side load. d1 and d3 would thus vary with side load also.

Below is this data graphed for various side loads.

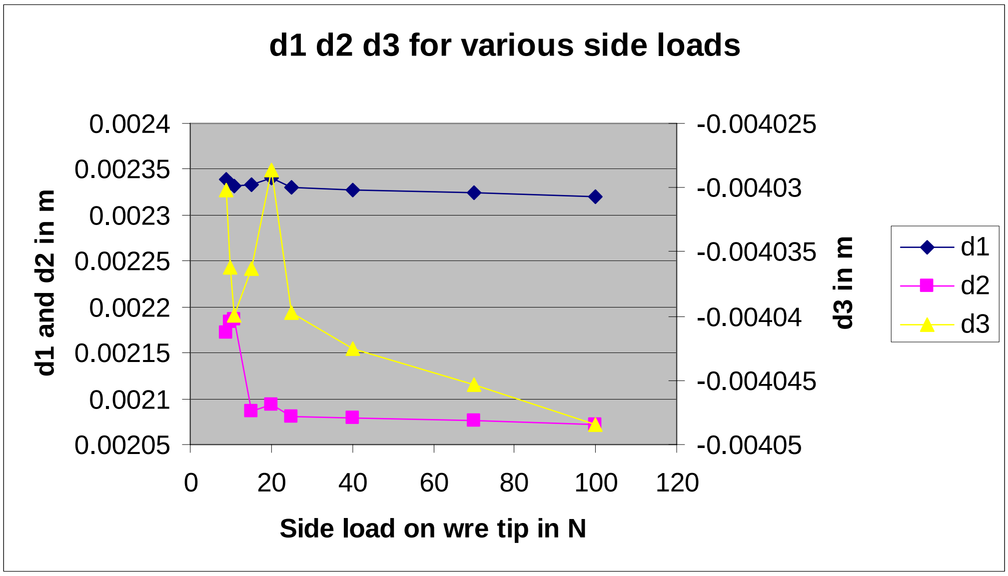


Figure 4

Figure 4 is hard to interpret. It shows d3 is a constant negative number, albeit with some noise (this is a non-linear FE so some noisiness in results is expected). This is interesting and seems to be born out by a very simple look at the geometry (shown in Section 5). What is not clear is what happens at 0 sideways load, and more importantly what happens just to either side of zero.

D2 and d1 are also shown in Figure 4 and are both rather messy data sets. The most useful conclusion at this point is that that D1 and D2 are roughly constant and very similar which is what would be expected for small tip rotations. It may be that there are subtle changes hidden in the noise of this data. Due to their small sizes it is not intended to investigate these small changes further.

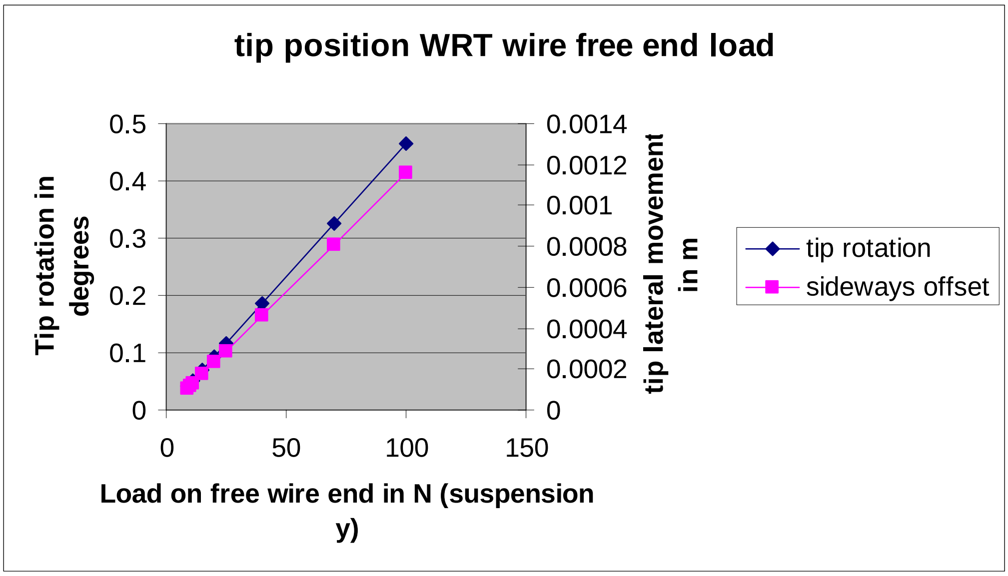
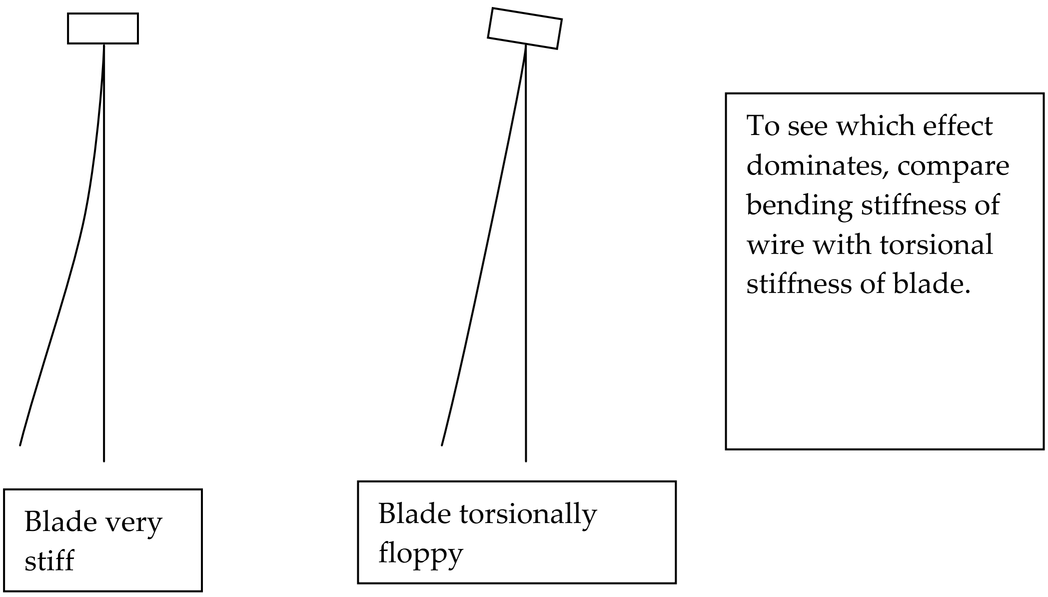


Figure 5

Figure 5 shows the lateral stiffness and torsional stiffness of the middle blades. The most significant, and possibly unexpected, result is that the lateral stiffness of the blade is about 0.01mm per N. This compares with 0.37mm per N vertically giving only a factor of 30 difference in their stiffnesses. This is lower than the factor 80 that was considered as the minimum required in order for lateral stiffness to be ignored.

4. ANALYTICAL APPROACH - TORSION

The aim of this study is to compare the torsional stiffness of the blade with the resistance of the wire to bending.



4.1 Sources

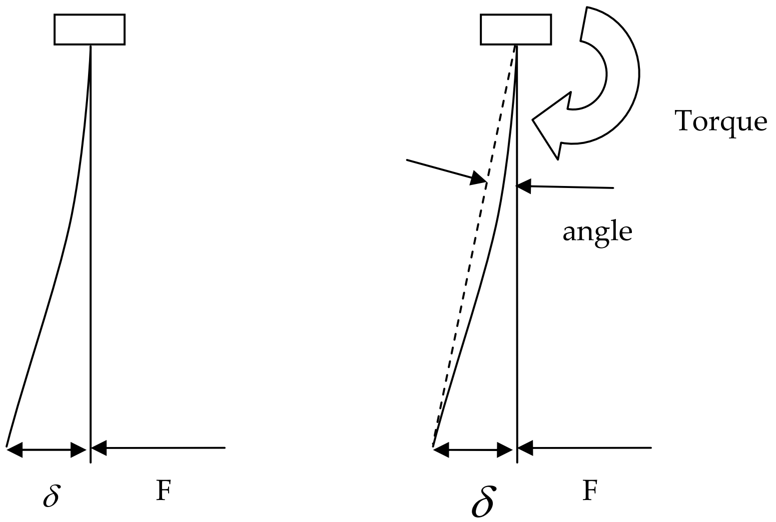
I have used the Cagnoli et al paper Phys Lett A 272 (200) 39-45 plus emailed assistance for the stiffness of a wire under tension, and the paper of Norna Robertson (5 Sept 2002, "Some notes on pitch frequency…") for the torsion of the blade. The stiffness of a wire with no tension (for comparison) comes from Roark.

4.2 Equations

4.2.1 Wire with no tension

Encastered wire loaded sideways at the end; length L; diameter d; Young's modulus E; displacement delta; force F; stiffness K:

$$K = \frac{F}{\delta} = \frac{3EI}{L^3} \quad \text{where } I = \frac{\pi d^4}{64}$$



Considering the force to exert a torque about the fixed point (in engineer's parlance, the encastering moment), and expressing the displacement at the end as an angle subtended at the fixed point, we can get an equivalent torsional stiffness $K_{torsion}$:

$$K_{torsion} = \frac{torque}{rotation} = \frac{FL}{\delta/L} = KL^2 = \frac{3EI}{L}$$

4.2.2 Wire with tension

From Cagnoli A.6, ignoring the gravitational part (See email in appendix 1):

$$K = \frac{T}{L}\left(\frac{1}{\lambda L}\right) = \frac{T}{\lambda L^2}$$

and as above

$$K_{torsion} = KL^2 = \frac{T}{\lambda} \text{ and noting that } \lambda = \sqrt{\frac{T}{EI}} \text{ yields } K_{torsion} = \sqrt{TEI}$$

4.2.3 *Torsion of blade*

The torsional stiffness of the blade is given in Norna's paper Equation 11 as

$$K_{torsion} = \frac{Gt^3}{3\int \frac{dz}{b(z)}}$$ where the integral is evaluated for a given blade profile.

4.3 Numerical results

Using a spreadsheet the results above have been evaluated for a typical wire and blade, the middle blade of the controls prototype. I have ignored the angling of the wire.

WIRE parameters				**BLADE Parameters**		
	diam	0.00071	m	E	1.86E+11	
	I	1.24739E-14	m^4	nu	0.3	
	T	600	N	G	7.15E+10	
	L	0.308	m			
	E	2.10E+11	Pa	B	0.059	middle blades, controls
				L2	0.415	
No preload				L1	0.0661864	
	stiffness	2.69E-01	N/m	b	0.01	
	torsionally	2.55E-02	N.m/rad	t	0.0046	
With preload				Norna equation 11		
Cagnoli (A.6)				integral	19.531495	
	term1	6.78E-03				
	stiffness	13.21555743	N/m			
	torsionally	1.25368064	N.m/rad	K	118.83828	N.m/rad

For reference the torsional stiffness of the other two blades are:
Top blades 90.28 Nm/rad
Bottom blades 115.23 Nm/rad

4.4 Conclusion from the analytical work

As can be seen, the stiffness of the tensioned wire is larger than that of the un-tensioned wire, but both are much less than the stiffness of the blade in torsion. This suggests that the effect of blade twisting will be negligible.

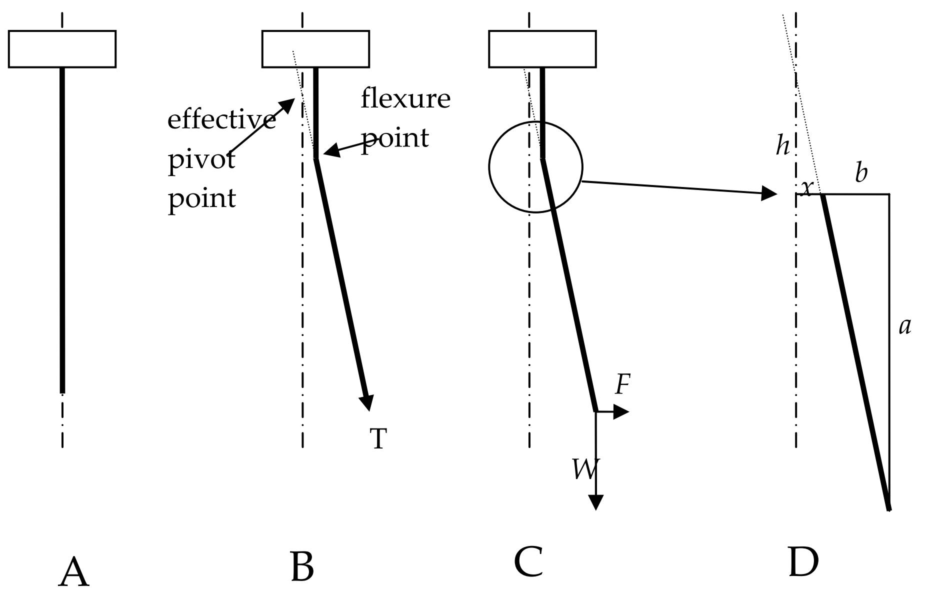
5. ANALYTICAL APPROACH – LATERAL FLEXIBILTY

5.1 Wire flexure with lateral deflection of blade

This section considers the effect that lateral deflection will have on the effective pivot point of a suspension. It follows from some apparently anomalous results from Ian Wilmut's FEA, which in fact are borne out by the analysis below.

5.2 Explanation

In the figure, part A shows the wire and clamp in undeflected state, with only a vertical tension in the wire. B shows what happens when the wire is pulled to the side (as well as down). The wire forms a flexure point in accordance with Geppo's paper, and the clamp and the wire move to one side because of the lateral elasticity of the wire. This will generate an "effective pivot point" on the original vertical centreline of the clamp but higher than the flexure point. C separates the wire tension into lateral and vertical components. In D I have labelled the lateral shift *x*. The effective pivot point is above the flexure point a distance *h*. I have also drawn a triangle of arbitrary size on the wire.



I hope it is clear that by considering similar triangles,

$$\frac{h}{x} = \frac{a}{b}$$

But we know that the deflection *x* is given by the lateral force and the blade's lateral elasticity k_l, and the ratio of *a* to *b* is same as the ratio of *W* to *F*. So

$$h = x\frac{W}{F} = \frac{F}{k_l} \bullet \frac{W}{F} = \frac{W}{k_l}$$

Interestingly, *h* is independent of the lateral force *F*. In the case of the middle blade, the lateral stiffness is close to 0.1 Newton per micron (slope of, eg, figure 5 above), and the vertical load is about 500 N, so the pivot point is higher than the flexure point by about 5000 micron, or 5 mm. Looking at the data it is believable that this is what we are seeing when D3 is graphed as a constant at about -4mm (4mm above the breakoff).

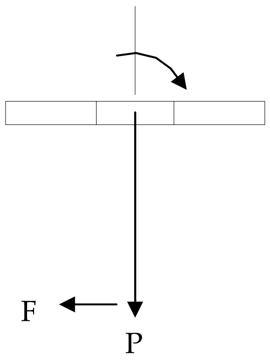
6. OVERALL CONCLUSIONS

This has been an interesting study into blade behaviour. We conclude that

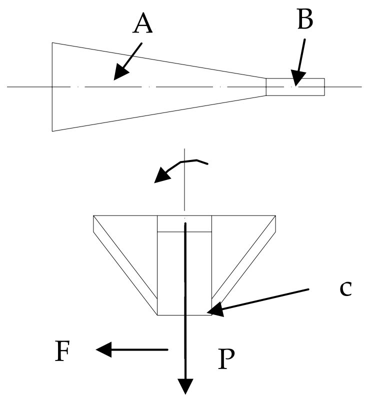
- Blade torsion will not have a significant effect on the wire flexure point.
- The blade is rather less stiff laterally than had been thought and
    - this may affect the pendulum dynamics;
    - this will have a significant effect (possibly dominating all others) on the pivot location and hence on the “d” distances and pendulum stability.

Appendix A

End views on blade: simple, with no end kick-up (left hand side), more realistic, with end kick-up (right hand side)



Blade shown assumed to be drawn flat by P. And showing assumed rotation of tip due to F.



Here is a more realistic situation. Section A of the blade is much softer to torsional loads than section B. Section B thus acts as a lever with F at the top

Appendix B – stability implications of lateral flexibility

Ian, Justin and colleagues

Further to my previous note (and distracted from what i should be doing this afternoon!) I was writing up my thoughts on lateral stiffness and wanted to share this with you to see if it ties up with your findings, which i will read carefully.

Consider the diagram attached. Assume for now that we neglect the stiffness of the wire, and we attach a wire a distance d below the CM line. The mass has been turned through an angle phi. The restoring torque due to the load (let load be m*g) is given by m*g*d*phi for small angles.

Now if the wire is attached to a blade which is not infinitely stiff in transverse direction the action of the component of the weight along the direction as indicated by the blue arrow will move the point of suspension in that direction (as Justin described to me to think about a wire on a ring sliding along). If the lateral stiffness is small enough that this force moves the point of suspension so that it lies directly under the CM, then there is no restoring torque. If it moves further we have gone into instability. Now the magnitude of the force (blue arrow) is m*g*phi, and the distance to move to reach the beginning of instability is d*phi. Thus we have a problem if the transverse stiffness kt of the blade is less than that force divided by that distance i.e. if kt is less than mg/d

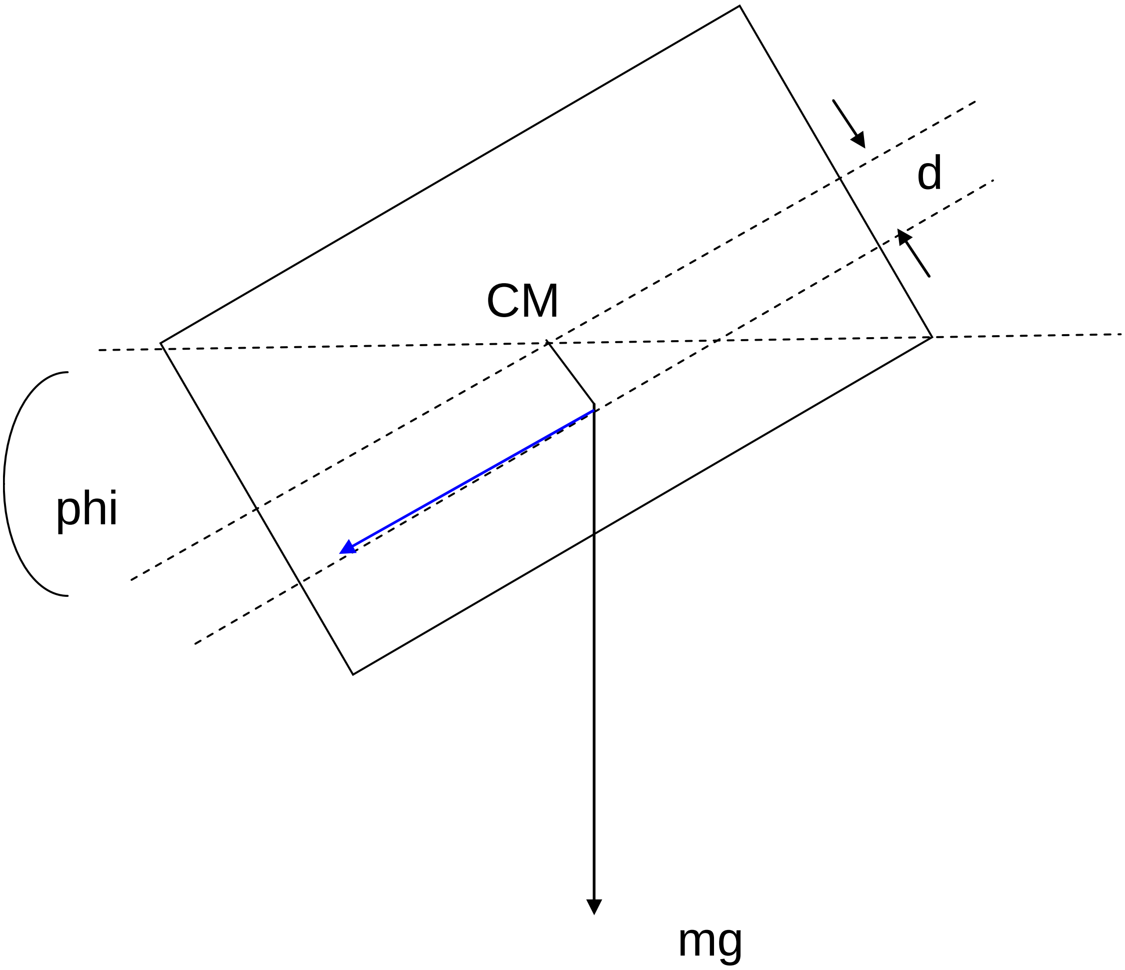
Now in the vertical the stiffness, kv, is given by mg/D where D is the amount the blade deflects when loaded. So we have a problem if kt/kv is less than D/d.

What is D? i am guessing that for the UI blade it is around 150 mm (is that right)? So we have a problem if kt/kv is less than 150/1, assuming d is 1mm. I think i heard someone say that the ratio is in fact 30 to 1. So what we would have to do is make d 5mm instead of 1mm to counteract the non-infinite lateral stiffness of the blade. This seems to tie up with the number I heard spoken today at the SUS telecon.

Note that I have ignored the wire stiffness. It would help to increase the restoring force and thus partially offset this effect.

This has helped me to understand physically what is happening and how we can counteract it - however I may be just repeating what Ian and Justin have done in a different way. Comments/thoughts welcome.

Cheers
Norna


d
CM
phi
mg

Appendix C – note reconciling appendix B with section 5 of paper.

Dear Norna,

Many thanks for this elegant analysis. Ian and I had a talk about it this afternoon and indeed you have arrived at the same result we did.

Looking at the diagram on page 10 in our paper, the sus will become unstable if h becomes greater than d. (d is not shown on the diagram but I think it should be clear why this inequality applies). That would put the pivot point above the cg of the mass. Noting that we found h=W/kt, and noting that the initial deflection (your D) is related to our W (the weight, mg) by D=W/kv, the inequality becomes

for stability h<d
W/kt<d
(D.kv)/kt<d
kv/kt<d/D
or
kt/kv>D/d as you had.

So either we are both right, or both wrong in precisely the same way. Calum asked Ian if we could do the sum for previous suspensions: all we need to know to work out our h is the weight and the horizontal stiffness kt. I am pretty sure that kt can found with a simple linear FEA of the blade in its flat condition, so if someone can give us the dimensions and the suspended weights we can soon do the sum. (Ian will check that for the blade on which he has done the full nonlinear FE, this trick works to give the correct lateral stiffness.) If the h we find is much smaller than the d that was used then it's no surprise that we never saw anything.

Cheers - Justin.