LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
– LIGO –

LIGO Laboratory / LIGO Scientific Collaboration

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Considerations regarding magnet strengths for the Advanced LIGO test mass quadruple suspensions

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1 OVERVIEW

The quadruple-stage suspensions for the test masses in Advanced LIGO include coil-magnet actuators on the top three stages (all but the test mass stage). The choice of magnet strength on these stages involves a trade-off between the desire for large actuation force capability and the need to limit the coupling of external magnetic fields to noise forces on the test masses. This document spells out these considerations. The magnetic field coupling estimates lead to maximum magnet strengths of:

- Upper intermediate mass (UIM): 0.11 A-m^2
- Penultimate mass (PM): 3.6 mA-m^2

These are the allowed individual magnet strengths, specifically for the magnets aligned along the optic axis. Magnets aligned in orthogonal directions could be (several times) bigger, if there is a reason to do that. (There is no practical limit to the magnet size on the top-most mass, since the test mass is isolated from it at 10 Hz by a factor of ~100 compared to the UIM.) Unfortunately, the forces obtainable with such magnets provide little or no headroom compared to the estimated forces required for interferometer control: a larger force capability is desired for both stages. This leads to the following recommendations:

- Use the following ‘round’ numbers for the magnetic moments, which provide a bit more force, but are essentially the same as far as the magnetic field coupling is concerned, given the level of certainty of that calculation: ***UIM*, 0.15 A-m^2 ; *PM*, 5 mA-m^2**.
- Provide the required range of ±0.5 mrad of DC angle bias (pitch and yaw) on the top stage (or some place other than the longitudinal actuators of the UIM stage) of the suspension, so that none of the UIM force range is taken up by the angle bias.
- Investigate the coil design to optimize the force for a given magnet size. This includes considering the current handling capability of the coils, and how much spatial non-uniformity in the force could be allowed.
- Investigate further the magnetometer data to see, e.g., whether the channels that show noise at $10^{-11}\mathrm{T}/\sqrt{\mathrm{Hz}}$ are indeed measuring magnetic field noise, or whether this is due to magnetometer motion. There is a possibility that the $10^{-11}\mathrm{T}/\sqrt{\mathrm{Hz}}$ level used in the calculations is an over-estimate, by a factor of 2-3.
- Consider shielding the magnets to reduce the magnetic field, thereby allowing larger magnets.
- Given the extra isolation of the upper intermediate mass from the test mass, we should consider increasing the UIM magnet size, even in the context of the coupling estimates given here. For example, if the magnet strength were increased ~3×, to 0.5 A-m^2, the frequency at which magnetic field noise is at the technical noise limit increases from 10 Hz to only 13 Hz or so. This may well be an acceptable compromise given the 3× larger force this would provide.

2 EXTERNAL B-FIELD COUPLING

External magnetic fields, those present naturally in the environment and those produced by the detector instrumentation, will exert forces on the suspension magnets, as the *B*-fields will inevitably have gradients. The coupling from external fields to net force on a test mass has been measured for the initial LIGO test masses by Robert Schofield, in Nov-Dec of 2005, by producing a magnetic field frequency comb near each test mass chamber. The results of these measurements are shown in Fig. 1.

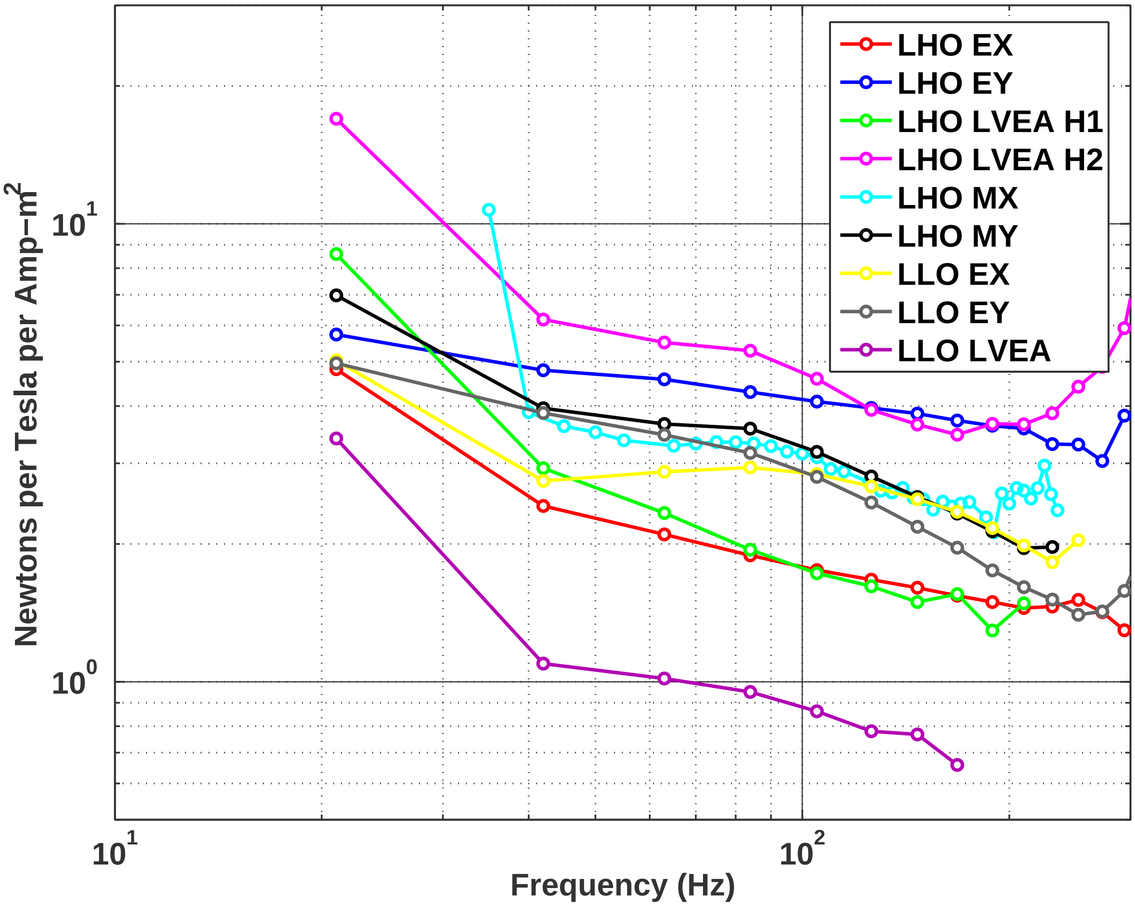

Figure 1: R. Schofield's measurements (made in Nov-Dec of 2005) of the coupling of applied magnetic fields to the net force on the initial LIGO test masses, normalized by the magnetic moment of an individual test mass magnet (7 mN-A^2).

There is some roll-off in the response due to sheilding from the chamber. In AdLIGO, because the transfer functions of force-to-displacement for forces on the PM and UIM to displacement of the test mass roll off as f^{-4} or faster, we only need to consider the coupling at 10 Hz. Since this is below the above measurement band, we need to account for the chamber shielding factor. R. Schofield has made some measurements of the chamber shielding, reported in LIGO-G990079-

29. This data show that the shielding starts at about 10 Hz, and specifically that there is a factor of 1.3× more shielding at 20 Hz than at 10 Hz. *Therefore, we extrapolate an average coupling factor at 10 Hz of* ***10 Newtons/Tesla/A-m^2***.

It is interesting to compare this to a rough estimation of the coupling. The four magnets are arranged with alternating polarity, so that the net dipole moment of a stage is significantly smaller than that of an individual magnet. The resulting force on a stage may be estimated as:

$$F = \frac{\mu B}{l}\varepsilon$$

where μ is the magnetic dipole moment of one magnet, B is the magnitude of the magnetic field at the magnets, l is the length scale over which the magnetic field is varying around the suspension, and ε is the fractional difference in the force magnitude between adjacent magnets (due either to differing magnetic moments or differing field gradients). The dipole moments of the magnets in initial LIGO are matched to about 5% ($\varepsilon = 0.05$). The field gradient scale l is more difficult to predict, but a reasonable guess is $l \sim 10$ cm (the rough scale of the suspension structure elements). This gives a force estimate of $F \approx 0.5\mu B$, neglecting chamber shielding, a factor of 20× smaller than the above 10 Hz estimate. This suggests that the net force arises not from magnet strength variations, but rather from spatial variations of the field gradients, and that better magnet matching would not help. And since the magnet separation in AdLIGO will be very similar to that for the initial LIGO test masses (16 cm magnet-magnet), the B-field coupling is expected to be very similar to R. Schofield's measurements.

3 AMBIENT MAGNETIC FIELDS

The figures below show spectra of magnetic field noise at the observatories, as measured by: the coil magnetometers at LHO (home-made coils, mounted in a vault located at X=1000m, Y=300m); the Bartington flux-gate magnetometers located in the LVEA and VE areas at LHO and LLO. *Given these data, we establish what appears to be a conservative value of the potential magnetic field noise at 10 Hz as:* $10^{-11}\ \mathrm{T}/\sqrt{\mathrm{Hz}}$.

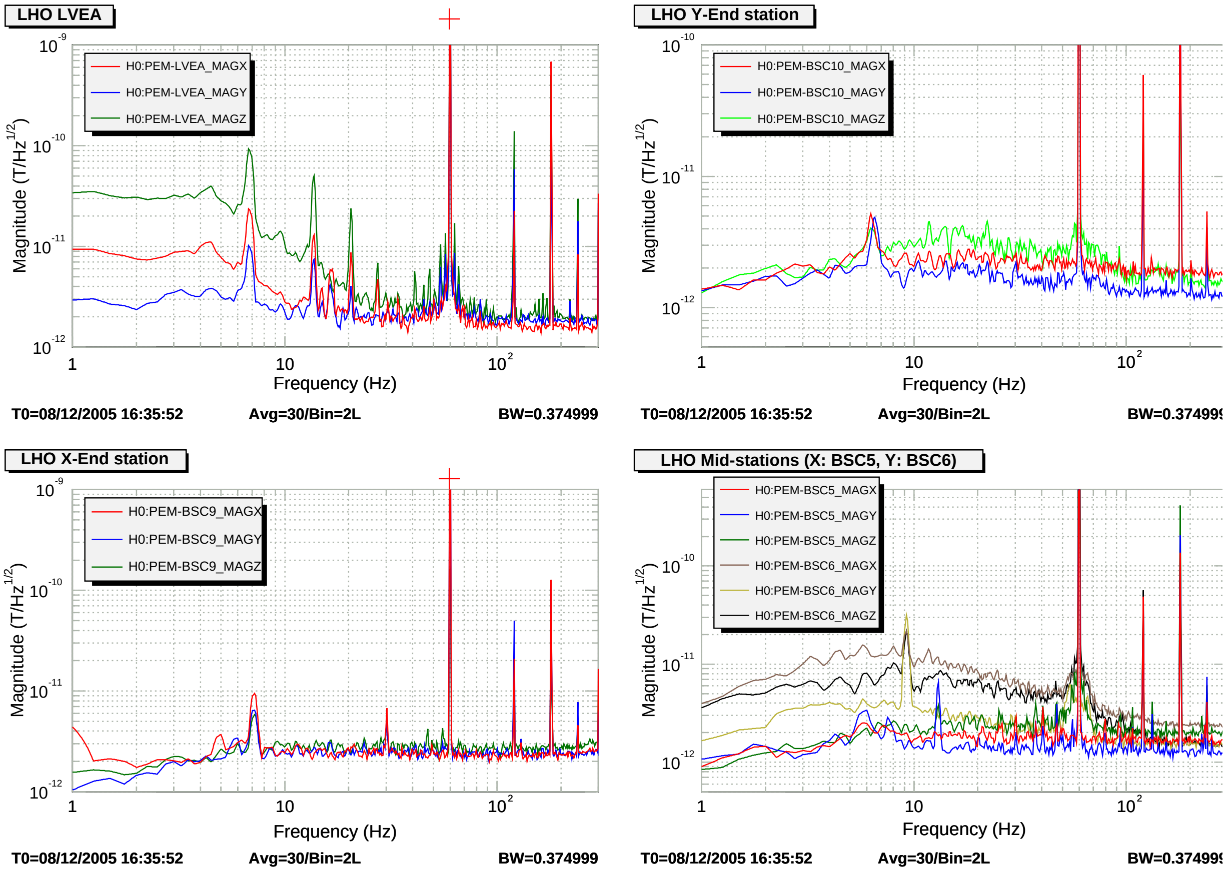

Figure 2: Bartington flux-gate magnetometer spectra for the 5 LHO equipment stations. The noise floor of the sensor is 2-3 pT/rtHz, above a few Hertz. Many of the spectra are at the sensor noise floor, though the LVEA spectra are mostly above it around 10 Hz. It is possible that these LVEA signals are due to motion of the sensor in a static, but spatially varying magnetic field, rather than fluctuations of the field itself.

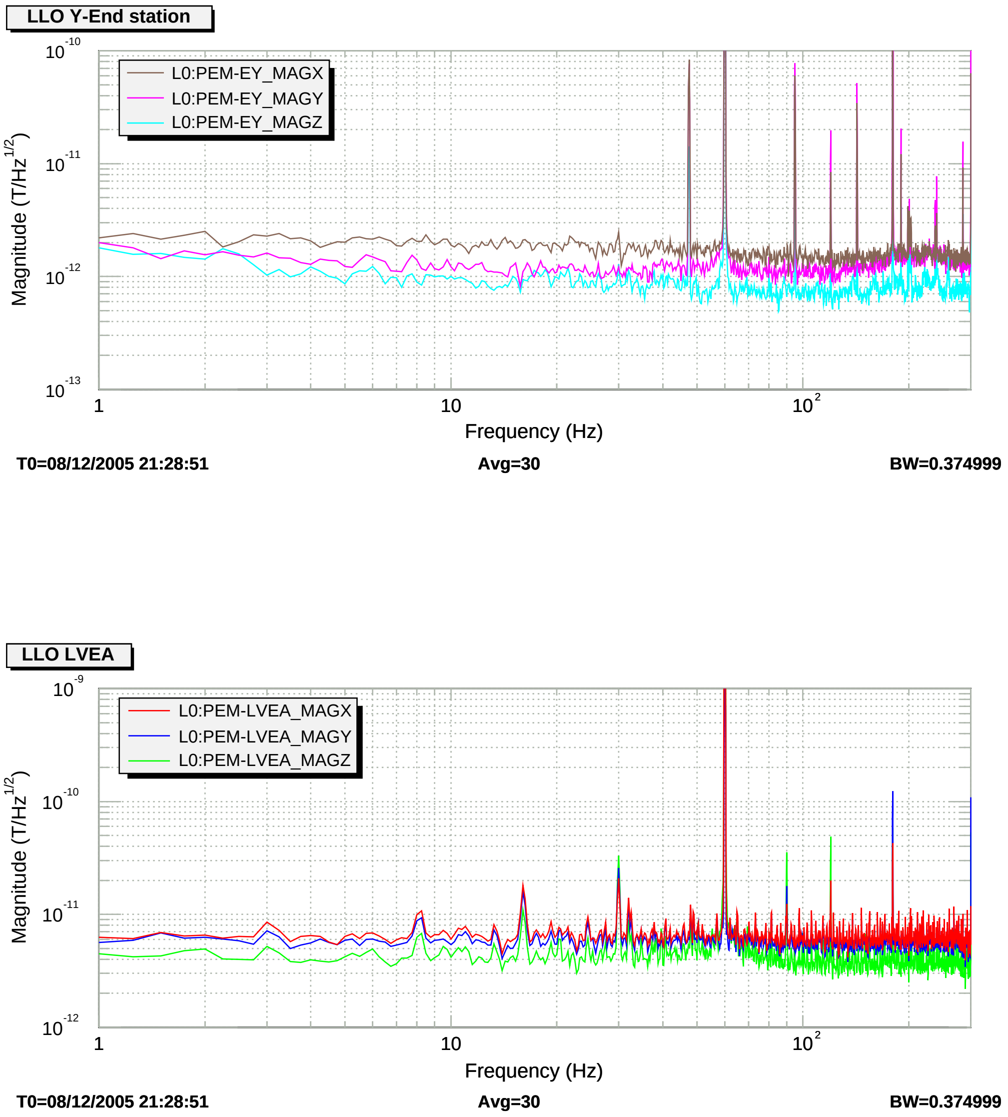

Figure 3: Bartington flux-gate magnetometer spectra for the LLO LVEA and EY stations (the EX sensors are limited by DAQ noise, since they have 10× less gain before the DAQ, and so are not shown). The noise floor of the sensor is expected to be 2-3 pT/rtHz, above a few Hertz. These signals have poorer pre-DAQ signal conditioning than the LHO sensors; this may explain why the LVEA channels are 2-3× above the expected sensor noise level. It is not clear why the EY channels appear to be below the expected sensor noise level.

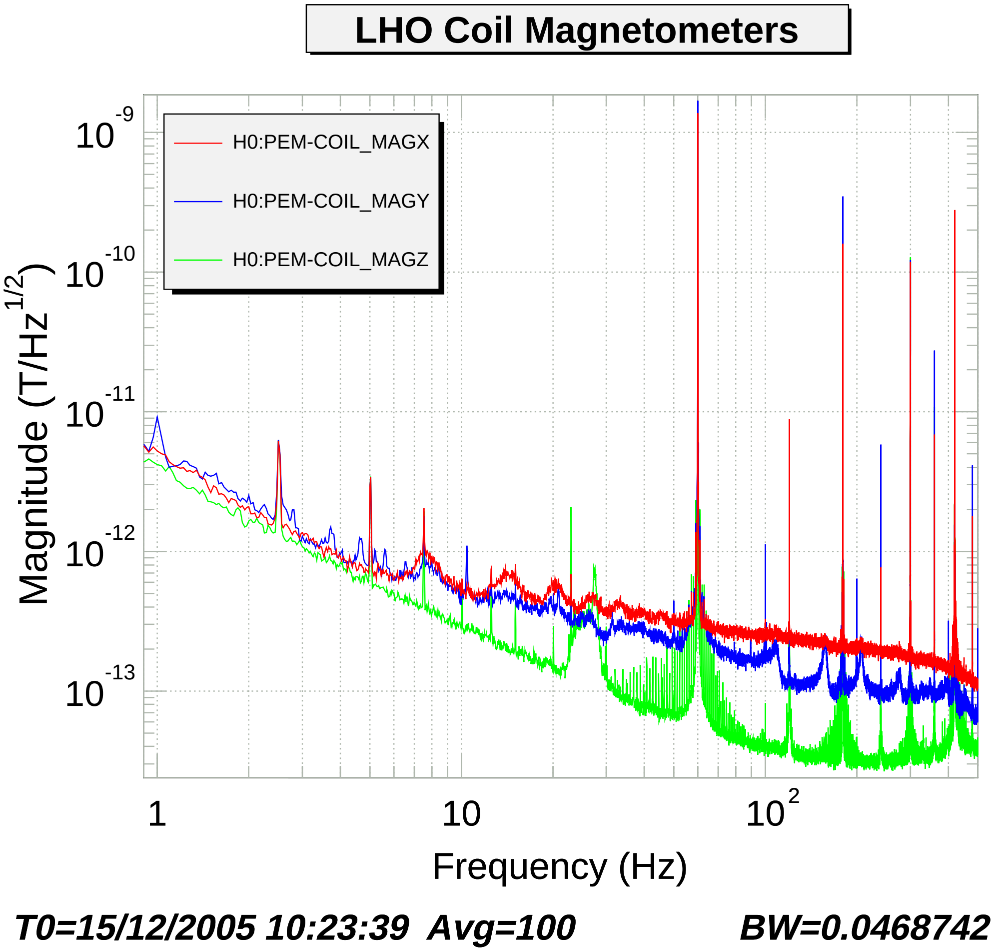

Figure 4: Typical magnetic field spectra from the LHO coil magnetometers. The strong ~26 Hz peak in the vertical axis (z-axis) is a Schumann resonance, as presumably are the weaker peaks seen in the horizontal axes, starting at around 7.5 Hz.

4 IMPLICATIONS FOR MAGNET SIZE & ACTUATION FORCE

To estimate the effect of magnetic fields on the test mass motion, we need the transfer function of force applied to the upper stages to displacement of the test mass. From the Mathematics quad suspension model (numbers courtesy of K Strain), these are:

$$PM_{\text{force}} \rightarrow TM_{\text{disp}}: T(10\ \text{Hz}) = 2.8\times10^{-8}\ \text{m/N}$$

$$UIM_{\text{force}} \rightarrow TM_{\text{disp}}: T(10\ \text{Hz}) = 8.9\times10^{-10}\ \text{m/N}.$$

The technical displacement noise limit of $10^{-20}\ \mathrm{m}/\sqrt{\mathrm{Hz}}$ for a test mass at 10 Hz can then be applied to each stage, to find the maximum individual magnet strength μ for that stage:

$$10\ (\mathrm{N/T/A\text{-}m^2}) \cdot \mu \cdot 10^{-11}\ \mathrm{T}/\sqrt{\mathrm{Hz}} \cdot T(10\ \mathrm{Hz}) < 10^{-20}\ \mathrm{m}/\sqrt{\mathrm{Hz}}$$

This gives:

Upper intermediate stage (UIM)	max μ: 0.11 A-m^2
Penultimate stage (PM)	max μ: 3.6 mA-m^2

For comparison, the initial LIGO test mass magnet strength is 7 mA-m^2.

To estimate the actuation force available with such magnets, scale from the initial LIGO actuator, whose coefficient is about 2.2 N/A/A-m^2. With four axial magnets on each stage, and taking a maximum RMS coil current of 0.1 A, the maximum RMS forces would be:

Upper intermediate stage (UIM)	0.1 N-rms
Penultimate stage (PM)	3.2 mN-rms

5 CONTROL FORCES

Here we make some estimates of the forces needed to control the interferometer.

5.1. Orientation bias

The largest forces are those for the orientation bias. During installation, the suspension is to be aligned to the local surveying instrument(s) to within 10 microradians. Once the system is under vacuum, the suspension must have enough range in the orientation bias to be able to bring the optics to the right global alignment. The range needed for this is best estimated by looking at the experience with the initial LIGO optics, since the local installation surveying accuracy will be the same. In initial LIGO, most of the applied angle biases are less than 100 microradians. However, a few of the optics require larger biases, approximately 250 microradians. To allow for this, with a bit of margin, the suspension alignment bias range (after installation) should be ±500 microradians, in both pitch and yaw. To estimate the force needed to provide this range, we use the DC torque-to-angle coefficients calculated by the Mathematica quad model (numbers provided by M Barton). Not knowing the exact arrangement of magnets on the upper stages, the force calcuated assumes a lever arm of 0.1 m. Furthermore, the forces given below are the total force (at the 0.1m lever arm), not the force per magnet.

Stage	*Pitch*		*Yaw*	
	Torque-to-angle, DC	*Force for 0.5mrad*	*Torque-to-angle, DC*	*Force for 0.5mrad*
Top mass	0.20 rad/N-m	25 mN	0.015 rad/N-m	0.33 N
Upper intermediate mass	0.21 rad/N-m	24 mN	0.037 rad/N-m	0.14 N

5.2. Length feedback control

Rana Adhikari has recently done some modeling of the length controls for the AdLIGO interferometer. He calculated the actuator forces required for control of a ‘noisy state’, one where the shot noise is much higher than the final low-noise state, and the seismic isolation is not yet at the design level. The noise levels chosen for this state are a bit arbitrary, but at least they provide a starting point. The RMS forces he found were needed for this state are:

Upper intermediate stage (UIM)	0.24 N-rms
Penultimate stage (PM)	2.5 mN-rms