Thermal Self-Locking in a Fabry-Perot Cavity

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September 23, 2005

Abstract. While investigating new coating materials for advanced LIGO, we have observed an unusual nonlinear phenomenon of high power Fabry-Perot cavities: thermal self-locking. This phenomenon can, under the right conditions, bring the cavity into resonance with the laser and hold it there. This paper will describe conditions required for a stable thermal self-lock and present experimental evidence of thermal self-locking in a short Fabry-Perot cavity.

1 Introduction

1.1 Background

Laser frequency stabilization is an important topic for many areas of physics, chemistry, and astronomy research. Technical applications include high precision clocks for telecommunication and GPS, rotation sensors, and interferometry measurements, for example as in LIGO. Fundamental research applications relate to frequency modulation spectroscopy and locking to an inversion of a medium. Currently the Pound Drever Hall method is the main method of laser frequency stabilization. It uses a Fabry-Perot cavity to measure the laser.s frequency and then sends it back to the laser to get rid of any small frequency changes. In order to differentiate between frequency and intensity fluctuations, this method uses the error signal, which goes as the derivative of the reflected intensity of the cavity. This technique measures and controls frequency fluctuations more quickly than the cavity can respond, and thus it is a very powerful tool [1,2].

Scientists note many different types of thermal self-locking. For example, thermal self-locking of frequency due to changes in mode volume[3], and due to change in refractive index of the crystal in the laser[4]. There is also thermal self mode locking [5,6] and phase locking[7]. The thermal effect described here is, however, uniquely different from all of those because it involves changes in the physical length of the cavity due to thermal expansion.

1.2 Nature and Scope of the Problem Being Investigated

In this project, I was dealing with a particular high-finesse Fabry-Perot cavity with an absorptive gold-coated mirror at one end. The cavity measured about a forearm.s length, not nearly the size of the huge four kilometer arms used in

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LIGO. Power was fairly high with a laser power on the order of a few hundred milliwatts and a circulating power even higher[8]. Thus my setup had the key characteristic that there was enough power along with an absorptive mirror to cause significant expansion of the glass substrate behind the reflective coating. Basically the cavity can adjust its length to maintain lock. In the course of my research I investigated the mathematical basis for thermal self-locking and obtained experimental evidence to support it.

1.3 Rationale for the Work

At the beginning of the summer I set out to measure the thermal coefficients [9] of some newly fabricated titanium doped silica-tantala mirrors under consideration for advanced LIGO. This required that I put together the system to measure them from scratch. The mirrors in question require an absorptive coating in order to measure the coeffients[10]. After mode-matching the probe beam to the Fabry-Perot cavity[11], I worked to get the TEM_{00} mode. I thought I almost had it, but some tricky faint fringes remained around the center bright spot. I called up my mentor for help, then went back to the lab and sat around for a good fifteen minutes until my mentor arrived. The bright spot was still there when he arrived and it was obvious that some then unknown feedback mechanism, now understood as thermal self-locking, must be accounting for it. When the wrong mirrors arrived three weeks into my SURF, I shifted the focus of my project to study this new discovery.

2 Thermal Self-Locking in a Fabry-Perot Cavity

2.1 Summary and Findings

Through my research I described the thermal self-locking phenomenon in a Fabry-Perot cavity, both mathematically and experimentally.

Mathematical formulation of solutions. At the end of my research I provided a mathematical model that indicated points of stable lock in steady state. This required three relations.

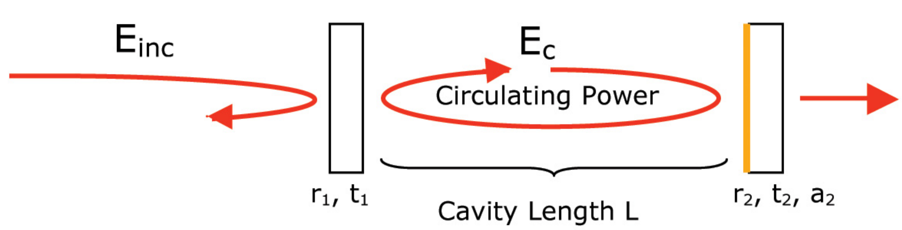

Figure 1: **Circulating Power and Cavity Length.**

1. First I needed the circulating power in the cavity as a function of cavity length (see Fig. 1). Since power is the square of the magnitude of the electric field, I first found the field inside the cavity:

$$E_c = E_{inc} t_1 + E_c \cdot e^{iKL} r_2 \cdot e^{ikL} r_1$$

where E_c and E_{inc} are cavity and incident electric fields, r_1 and r_2 are the reflectivities of the front and back mirrors, t_1 is transmissivity of front mirror, and L is the length of the cavity. Thus,

$$P_c = |E_c|^2 = \frac{|E_{inc}|^2 |t_1|^2}{|E - r_1 r_2 e^{i2kL}|^2} \quad \text{with} \quad k = \frac{2\pi}{\lambda} \quad \text{fixed.}$$

And finally,

$$P_c(L) = \frac{P_{inc} |t_1|^2}{1 - r_1^2 r_2^2 - 2 r_1 r_2 \cos(2kL)}.$$

Basically circulating power remains nearly constant, except around multiples of the free spectral range of the cavity meaning $L = n \cdot \Delta\nu_{\text{fsr}} \frac{2}{c}$. Then light can be transmitted through the cavity, thereby reducing the power inside of it.

2. As a second step I needed to find the thermal expansion of the mirror as a function of the circulating power in steady state. In this case the diffusion equation becomes:

$$\nabla^2 T = \frac{1}{\kappa^2} \frac{\partial T}{\partial t} \to 0.$$

Then we have the change in temperature as

$$\Delta T = \frac{P_c a_2 \cdot \Delta x}{\kappa A},$$

where a_2 is the square of the absorption, kappa is thermal conductivity, and A is the area of the gold coated mirror. Integrating gives:

$$T(x) = T_{\text{room}} + \frac{P_c a_2}{\kappa A} x.$$

Therefore the expansion of the mirror is:

$$\Delta \ell = \int_0^{\ell} \alpha \frac{P_c a_2}{\kappa A} x \cdot dx = \frac{\alpha P_c a_2}{\kappa A} \frac{\ell^2}{2},$$

where α is the thermal expansion coefficient (see Fig. 2).

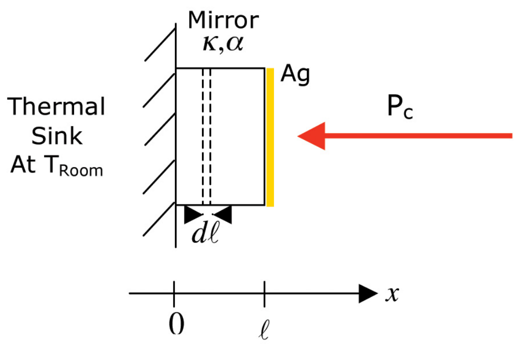

Figure 2: **Change in Cavity Length.**

And the change in cavity length is given as:

$$\Delta L = -\Delta \ell$$

$$\Delta L = -\frac{\alpha P_c a_2}{\kappa A}\frac{\ell^2}{2}.$$

3. Combining the two relations from above yields the final relation of length of cavity versus expansion of the mirror (or change in cavity length):

$$\Delta L = L - L_0$$

$$\implies L - L_0 - \frac{\alpha a_2}{2\kappa A}\ell^2 \left[\frac{P_{inc}|t_1|^2}{1 + r_1^2 r_2^2 - 2r_1 r_2 \cos(2kL)}\right]$$

This final relationship can be plotted to obtain a graphical depiction of the solutions using known cavity parameters, as Fig. 3. Here, the solutions are where the black line representing $L - L_0$ intersects the series of Lorentzian peaks given by

$$-\frac{\alpha a_2}{2\kappa A}\ell^2 \left[\frac{P_{inc}|t_1|^2}{1 + r_1^2 r_2^2 - 2r_1 r_2 \cos(2kL)}\right].$$

Fig. 4 is a close-up on the solutions for different incident powers. The blue curve has lowest power while the red curve has highest power.

Additionally, based on the solutions I deduced a relation between cavity parameters and the minimum power needed for a stable thermal self-lock:

Assume the peaks can be approximated by points. Then let LP be position of last peak less than initial cavity length:

$$LP = \frac{n\lambda}{2} \quad \text{where} \quad n = \left\lfloor \frac{L_0}{\lambda/2} \right\rfloor,$$

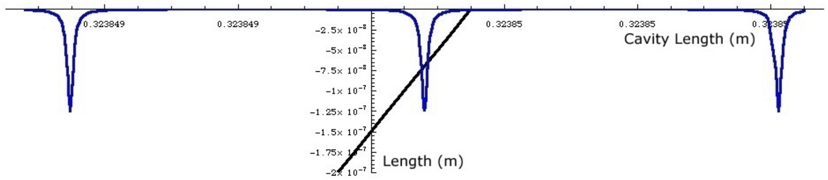

Figure 3: **Solutions for Thermal Self-Locking.**

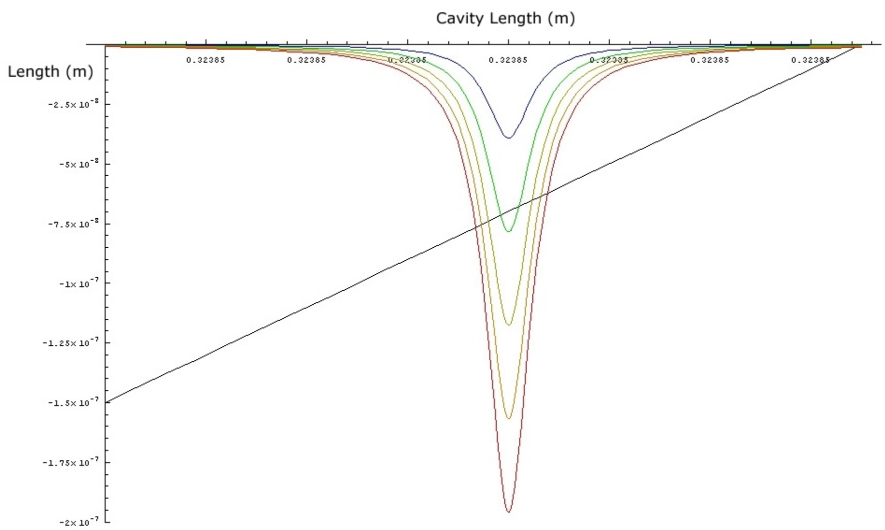

Figure 4: **Thermal Self-Locking with Varying Incident Powers.**

with the height of the Lorentzian peaks given as

$$\text{Peak amplitude} = \frac{\alpha a_2 \ell^2}{2\kappa A} \frac{P_{inc}|t_1|^2}{(1 + r_1^2 r_2^2 - r_1 r_2)}.$$

Then, for thermal self-locking, we must have

$$\text{Peak amplitude} \geq L_0 - LP$$

$$\Longrightarrow \text{Peak amplitude} \geq (L_0 - LP) \cdot \frac{\alpha a_2 \ell^2}{2\kappa A} \frac{|t_1|^2}{(1 + r_1^2 r_2^2 - r_1 r_2)},$$

provided $(L - L_0) < -(\alpha a_2/2kA)[P_{inc}|t_1|^2/(1 + r_1^2 r_2^2 - 2r_1 r_2 \cos(2kL))]$ at $L = L_0 - \epsilon$. Otherwise, this condition must be satisfied:

$$\text{Peak Amplitude} \geq L_0 - NLP \quad \text{where} \quad NLP = (n-1)1/2.$$

Thus the necessary incident power can be found for any cavity length L_0.

2.2 Experimental Evidence

I also obtained experimental data that demonstrated thermal self-locking was indeed occurring in this particular Fabry-Perot cavity. In order to study a signal, I tapped the beam coming out of the Fabry-Perot cavity using a polarizing beams splitter with a quarter wave plate on one side facing the cavity. Then I sent this beam into a photodetector (see Figs. 5 and 6).

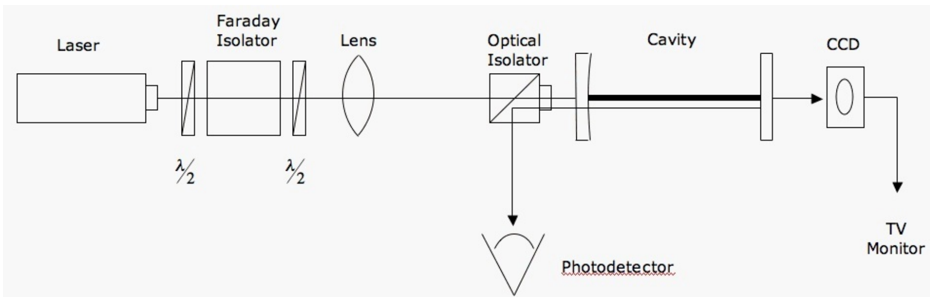

Figure 5: **Schematic Diagram of the setup.**

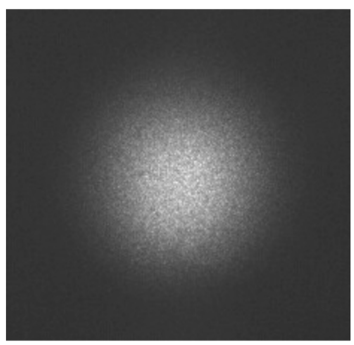

Figure 6: **The TEM_{00} mode** as seen by a CCD closed-circuit video camera and displayed on a television monitor (from http://laser.physics.sunysb.edu/∼alex/tmodes/tem00.jpg).

In particular I took data by dithering the laser at different frequencies and measuring the amplitude of oscillations in reflected power while the laser was locked to the TEM_{00} mode. It is very important that the amplitude of the driving signal be much less than the width of the resonant peak so as not to take the system completely out of lock.

The reflected power was measured by looking at the DC out signal. This gave me a plot logarithmic in character. Fig. 7 is the log log plot of amplitude oscillations in reflected power obtained by dithering the laser at various frequencies. The reflected power was measured by looking at the DC out signal. Frequency was read from the signal generator.

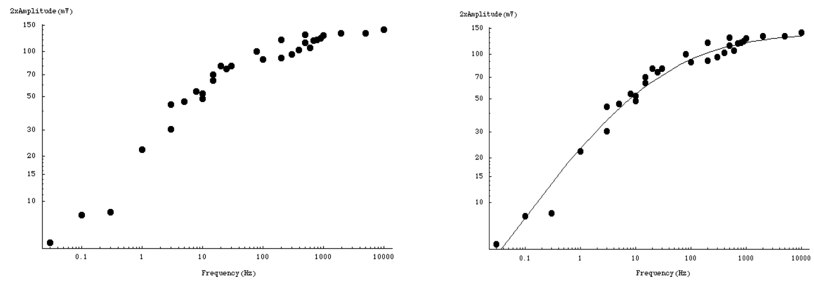

Figure 7: **Reflected Power Oscillations v. Frequency.** Data (left) and a possible fit (right)—the equation $2 \cdot \text{amplitude} = \frac{140}{1+\frac{5}{f^{1/2}}}$ is a fairly good approximation of the curve.

For this range of frequencies, the amplitude of oscillations in reflected power increases when dithering the laser at higher frequencies. Eventually these oscillations appear to asymptote toward a maximum amplitude. Reliable data could not be obtained for frequencies above and below those on the table. At the lowest frequencies the signals could not be distinguished from noise. More interestingly, for higher frequencies than those shown here, oscillations showed a decrease. A possible explanation could be that the laser came out of lock too quickly at such high frequencies that accurate signals could not be obtained.

2.3 Discussion of Results

As the long steady bright spot on the TV indicated, there was indeed a thermal phenomenon occurring to induce self-locking.

The oscilloscope showed locking to a point near the maximum of the error signal, indicating that the cavity locked to a side of the resonance peak rather than at the center as with Pound Drever Hall locking. The power signal would oscillate at higher frequencies while under lock, while the error signal would remain constant. Then as the cavity goes out of lock, the reflected power signal would go constant while the error signal would develop growing oscillations. A plot of the solutions (Fig. 8) illustrates the basis for a stable thermal self-lock.

The stability of the two solutions can be described by the combined effects of the circulating power in the cavity and the induced change in cavity length. First let's look at the stable point slightly to the left of resonance. Upon approaching this point from the left, more and more power is being transmitted through the cavity. This reduces the power in the cavity and causes the mirror to contract and the cavity to expand. Thus if the system is in this region, it will be pushed closer toward the stable solution. Approaching from the right of this point, less and less power is being transmitted, so the mirror will expand and the cavity will shrink, thus again being pushed back toward the stable solution. At the other fixed point, if the system is on either side it will be moved away making this point unstable. Approaching this point from the left, less light is being

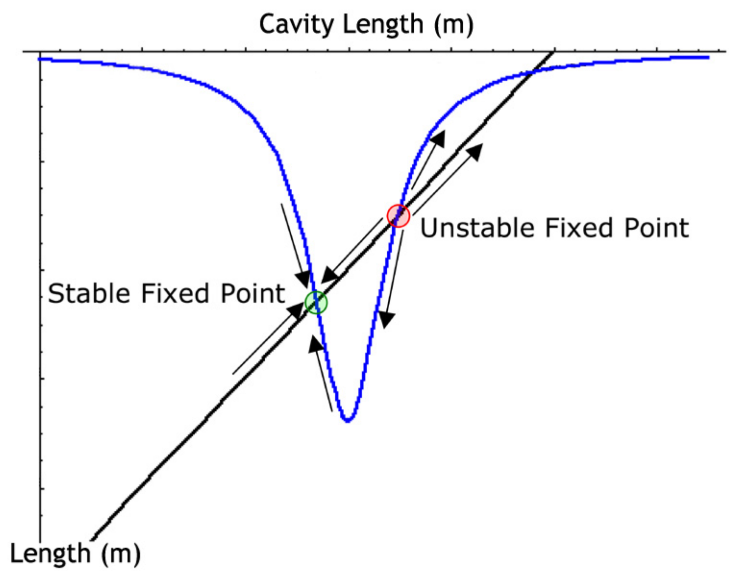

Figure 8: **Solutions for thermal self-locking.**

transmitted, so the mirror will expand and the cavity will shrink, whereas on the right side the cavity will be encouraged to expand. These two opposing behaviors do not converge.

Based on this result, we can understand the experimental data of the previous section. By dithering the laser at some frequency, the system is moved off the stable fixed point. However, the changing amounts of transmitted light force the system back to the stable solution. The amplitude of the oscillations in reflected power indicate how far off the system moves from the stable point. The greater the amplitude of the driving signal, the larger the oscillations in reflected power will be.

3 Conclusions and Future Work

3.1 Implications of the research

This research demonstrates that with thermal self-locking lasers can be reliably locked without electronic feedback through the servo controller. Also, techniques such as Pound Drever Hall laser locking could be used in conjunction with an absorptive mirror to create an even more robust system with lower laser frequency noise for longer periods of time. This technique would be especially useful in high energy cavities, since then the length would not have to be as precise in order to insure thermal self-locking. This research also serves as a warning for cavity locking using high powers. If thermal self locking is not taken into account, the dynamics of a high-power system could be significantly richer than the naive might expect, and it is not inconceivable that a system which is stable at low powers could become unstable at high power.

3.2 Relation to other work in lab

Laser frequency noise always remains a subject of concern in LIGO research. As in this study, the observatories also utilize very high energy Fabry-Perot cavities in the detector arms. However, the phenomenon of thermal self-locking cannot be directly applied to present gravitational wave detectors. In order to arrive at a very stable thermal self-lock, the cavity must contain an absorptive mirror, while the mirrors for advanced LIGO must have an extremely high reflectivity in order to obtain the best signal and to keep thermal noise as low as possible. Still, all mirrors are at least partially absorptive. Thermal self-locking could occur in LIGO at sufficiently high powers. A good future project would be to do this calculation.

3.3 Future Research Directions

One very important step in understanding thermal self-locking focuses on creating a dynamical model in order to reliably deduce the stability of the various fixed points of the system. This paper has only set forth a steady state model which can be used to arrive at stabilities intuitively. Another direction to pursue would be to take more data over a larger frequency range to help come up with a mathematical description of the system. Low frequency signals could not be distinguished from noise, while high frequency signals quickly came out of lock. Thus both ends of the data require more careful study.

3.4 Acknowledgements

Eric Black
Akira Villar
Katherine Dooley, Richard Kirian, Chinyere Nwabugwu
Kenneth Libbrecht
LIGO
SURF
NSF

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5 Appendix

5.1 Mathematical Equations

- Power insde cavity v. cavity length
  $E_c = E_{inc}t_1 + E_c \cdot e^{ekL} r_2 \cdot e^{ikL} r_1$
  $E_c = E_{inc}t_1 + E_c r_1 r_2 e^{i2kL}$
  $E_c = \frac{E_i nct_1}{1 - r_1 r_2 e^{i2kL}}$
  $P_c = |E_c|^2 = \frac{|E_{inc}|^2 |t_1|^2}{|1 - r_1 r_2 e^{i2kL}|^2}$
  $P_c = \frac{P_{inc}|t_1|^2}{(1 - r_1 r_2 e^{i2kL})(1 - r_1 r_2 e^{-i2kL})}$
  $(1 - r_1 r_2 e^{i2kL})(1 - r_1 r_2 e^{-i2kL}) = 1 + r_1^2 r_2^2 - 2r_1 r_2 \cos(2kL)$
  where
  $$k = 2\pi/\lambda \text{ fixed}$$
  giving
  $$P_c(L) = \frac{P_{inc}|t_1|^2}{1 - r_1^2 r_2^2 - r_1 r_2 2cos(2kL)}$$
- Thermal expansion of mirror v. incident power
  $\nabla^2 T = \frac{1}{K^2} \partial T \partial dt \to 0$
  $$I = \kappa A \frac{\Delta T}{\Delta x}.$$
  $I = P_c x \cdot a_2$ where a_2 is mirror absorption and where
  $$t_2^2 + r_2^2 + a_2 = 1$$
  $\kappa = 0.8 \frac{\mathrm{W}}{\mathrm{m \cdot K}}$
  A = area of mirror ($= \pi \cdot 1$ inch^2)
  Δx = mirror thickness
  $\Delta T = \frac{P_c a_2 \cdot \Delta x}{\kappa A}$
  Expansion
  $d\ell = \alpha[T(x) - T_R]dx$
  $\Delta \ell = \int_0^\ell d\ell = \int_0^\ell \alpha[T(x) - T_R]dx$
  $$\Delta \ell = \int_0^\ell \alpha \frac{P_c a_2}{kA} x \cdot dx = \frac{\alpha P_c a_2}{\kappa A} \frac{\ell^2}{2}$$
  $\Delta L = -\Delta \ell$, thus
  $$\Delta L = -\frac{\alpha P_c a_2}{\kappa A} \frac{\ell^2}{2}$$

- Length of cavity v. expansion of mirror

$$\Delta L = -\frac{\alpha a_2}{2\kappa A}\frac{\ell^2}{2}\left[\frac{P_{inc}|t_1|^2}{1 + r_1^2 r_2^2 - 2r_1 r_2 \cos(2kL)}\right]$$

Since $\Delta = L - L_0$,

$$L = L_0 - \frac{\alpha a_2}{2\kappa A}\frac{\ell^2}{2}\left[\frac{P_{inc}|t_1|^2}{1 + r_1^2 r_2^2 - 2r_1 r_2 \cos(2kL)}\right]$$

$$L - L_0 = -\frac{\alpha a_2}{2\kappa A}\frac{\ell^2}{2}\left[\frac{P_{inc}|t_1|^2}{1 + r_1^2 r_2^2 - 2r_1 r_2 \cos(2kL)}\right].$$