LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY

LIGO Laboratory / LIGO Scientific Collaboration

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Transfer functions of scattered lights in AdvLIGO COC

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1 Introduction

In order to estimate the effect of various fields back scattered into the Advanced LIGO core optics system, transfer functions from the injected field to the dark port carrier audio components were calculated. A DC readout schedule is expected to be used in Adv.LIGO, and only CR is considered in this calculation. By comparing these transfer functions with that of the DARM signal, one can calculate the upper limit of back scattered fields so that these fields would not compromise the sensitivity.

The configuration calculated is one particular configuration of a detuned RSE with the detuning angle of pi/2 * 0.038. When the configuration of the optical system is finalized, it will be necessary to recalculate these transfer functions.

The calculation was done using twiddle, a frequency domain model, and e2e, a time domain model, were used to calculate transfer functions. Simulation using e2e can include the optical spring effect, but a LSC is needed to make the interferometer stable which is under development now (when revision -00- was being written). Because of problem, all calculations were done without activating the optical spring effect. In the next section, the possible effect is discussed.

2 Basics

2.1 DARM signal

The carrier field coming out of the dark port has the following amplitude.

$$\begin{aligned} &A^{+}(\Omega)\cdot Exp[i\cdot\Omega\cdot t]+A^{-}(\Omega)\cdot Exp[i\cdot\Omega\cdot t] \\ &=A_{\mathrm{R}}(\Omega)\cdot\sin(\Omega\cdot t+\varphi_{R})+i\cdot A_{I}(\Omega)\cdot\sin(\Omega\cdot t+\varphi_{I}) \end{aligned}$$

The frequency Ω is the signal sideband frequency, and A^+ (A^-) is the amplitude of the upper (lower) sideband. A_R and A_I are amplitudes of real and imaginary part of the carrier field. The signal is a function of these amplitudes, and it depends on the detection scheme.

The DARM transfer function, $DARM_{R,I}$, is defined to be the real and imaginary amplitude when the L_- is modulated at frequency Ω, in units of √Watt / meter. This is calculated with an input power of 1W, and should be multiplied by √ input power. In Fig.1, the transfer functions, real and imaginary, calculated using twiddle are compared with transfer functions, homodyne phase $\pi/2$ and 0, calculated using the formula by A. Buonanno and Y. Chen, which includes the optical spring effect. Because the twiddle calculation does not include the optical spring effect, the transfer functions behave differently at the optical spring resonance and in the low frequency region, while two calculations match reasonably well at the optical resonance and in the high frequency region.

Strictly speaking, $DARM_R$ and $DARM_I$ are not the signal themselves, but the comparison in Fig.1 indicates that they represent descent variation of the detection scheme. In the following calculation, these real and imaginary amplitudes are used to estimate the effect of scattered field effect.

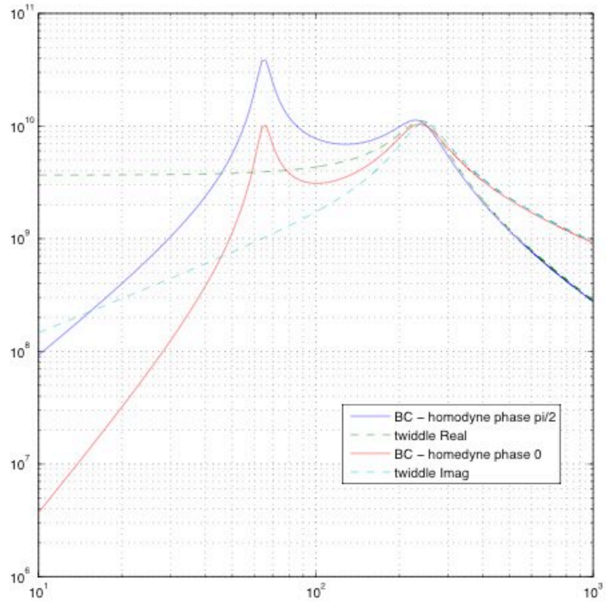

Fig.1 Comparison of transfer functions : twiddle vs B.C formula with different homodyne phase

2.2 Scattered noise transfer function

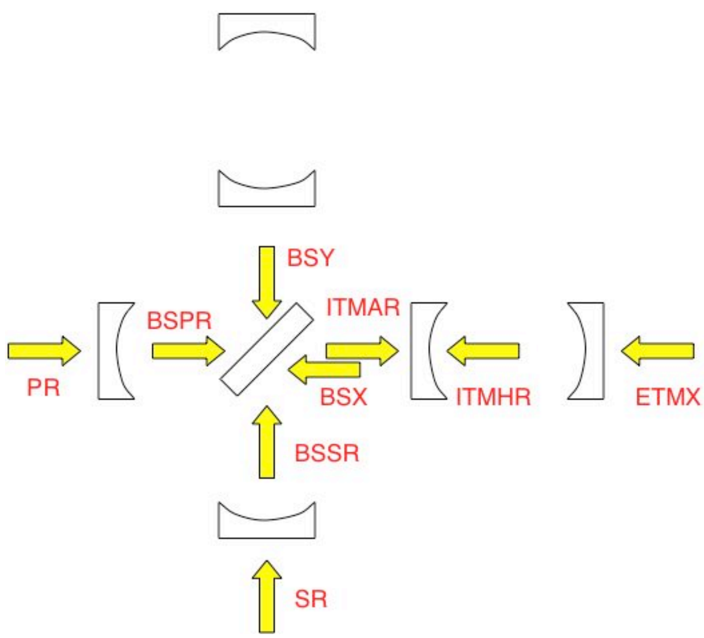

Fig.2 Scattered field injection to Adv.LIGO core optics system

Fields are injected at 9 locations back into IFO as is shown in Fig.2. The field injected is

$$E_{SN} \cdot Exp[i \cdot \delta_{SN} \cdot \sin(\Omega \cdot t)] \approx E_{SN} + i \cdot E_{SN} \cdot \delta_{SN} \cdot \sin(\Omega \cdot t)$$

The transfer function, $SNXXX_{R,I}$, from one source to the dark port signal is defined to be the real and imaginary amplitude of the carrier field in units of √Watt / √Watt or dimensionless.

When a GW signal comes with a strain of h, the DARM length is modulated with magnitude of L (arm length) x h, and the output signal is

$$DARM_{R,I} \times L \times h \times \sqrt{P_{IN}}$$

When a scattered field XXX is injected, the signal size caused by this noise is

$$SNXXX_{R,I} \times E_{SN} \times \delta_{SN}$$

To satisfy the requirement that the noise is smaller than ε of the sensitivity set by other constraints, the following constraint is derived.

$$E_{SN} \times \delta_{SN} < \varepsilon \cdot \frac{DARM}{SNXXX} \cdot L \cdot h \cdot \sqrt{P_{IN}}$$

2.3 Effect of optical spring

In the rest, transfer functions are calculated without optical spring. The effect of the optical spring is twofold as can be seen in Fig.1. One is that the signal is enhanced at the optical spring resonance frequency, while in the lower frequency region, the signal becomes harder to go out to the dark port.

As the noise requirement formula above shows, the smaller the DARM transfer function, the more stringent about the noise upper limit. The scattering noise transfer function can be affected by the optical spring effect, but the effect may not be identical to the DARM transfer function. So, in general, the ratio DARM/SNXXX will be different if you include the optical spring dynamics.

2.4 Transfer functions from all optical DOF

Fig.3 shows transfer functions from all optical DOF to the dark port CR signal.

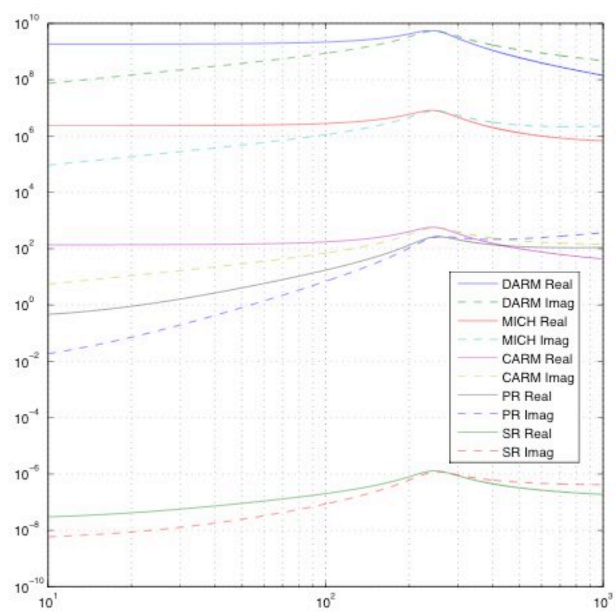

Fig.3 Transfer functions from various DOF

3 Transfer functions

3.1 Numerical results

Fig.4 shows all the transfer functions of scattered field, together with the DARM signal. In each plot, scattered light transfer functions are multiplied by 1e10.

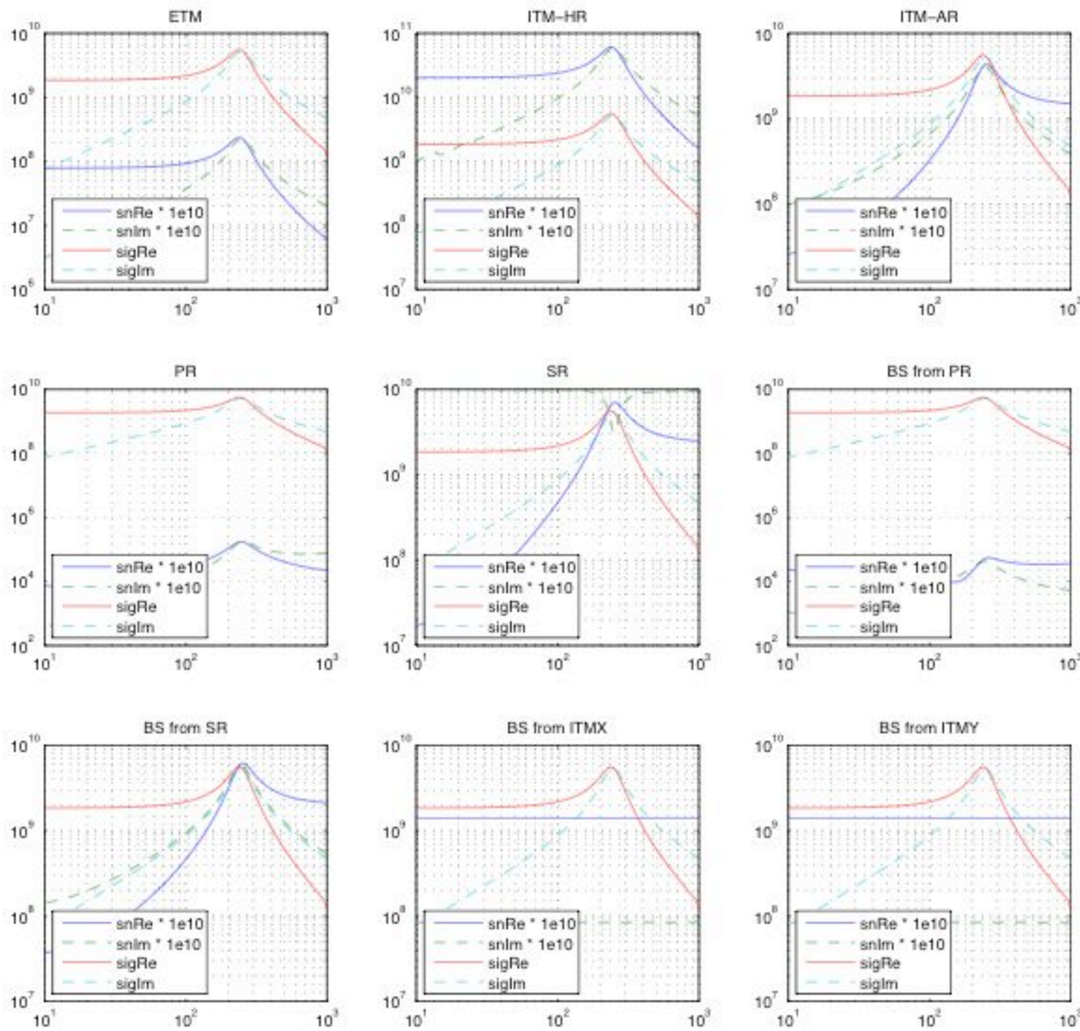

Fig.4. Transfer functions : Absolute values

Fig.5 shows the ratio of the scattered noise transfer function to the DARM signal. This can be substituted the formula to find the requirement.

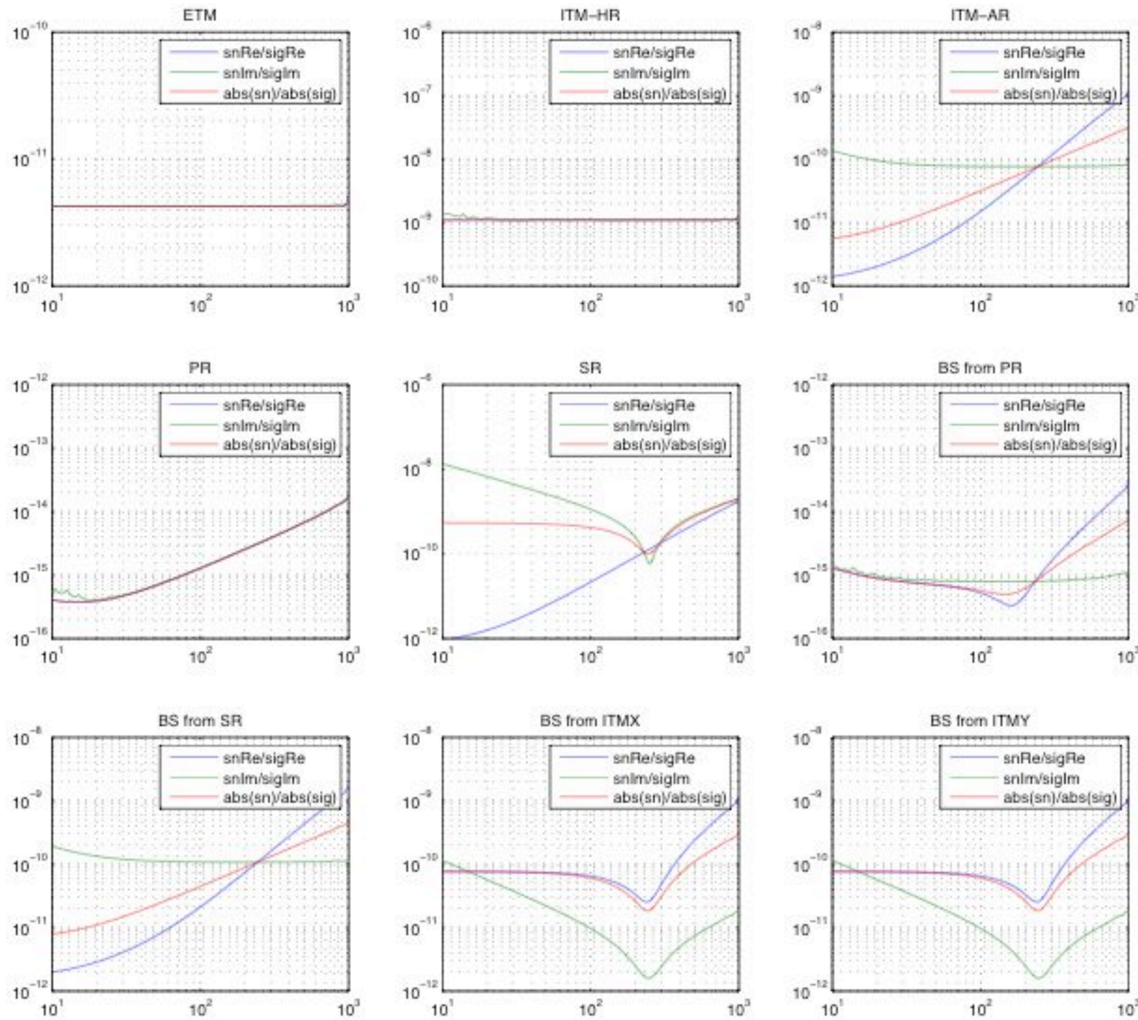

Fig.5. Ratio of transfer functions : SNXXX / DARM

3.2 Which quantity to use

The detection scheme is chosen by various considerations, and the actual signal is not simply the real or imaginary part of the dark port carrier output. The phase of the noise amplitude is random and cannot compare the real part of signal vs real part of noise. The worst (largest) case noise contribution can be estimated by the square root sum of the two noise amplitudes. The best (largest) case DARM signal is also the square root sum. A reasonable choice of the ratio will be the ratio of square root sum of real and imaginary amplitudes.

3.3 Simple case study

The DARM signal generation is equivalent that an external source is injected inside of the ETM with a magnitude of

$$\delta E_{DARM} = i \cdot E_{arm} 4\pi \frac{x}{\lambda} \sin(\Omega \cdot t)$$

where x = L h, and E_{arm} is the field amplitude in the arm.

When the noise is injected from the back of ETM, the magnitude of the field inside of ETM is

$$\delta E_{SNETM}(HR!side!ofETM) = \sqrt{T_{ETM}}\,\delta E_{SNETM}(AR!side!of!ETM)$$

So the ratio of the noise to the signal is

$$\begin{aligned}
&\frac{\delta E_{SNETM}}{\delta E_{DARM}}(HR!side!of!ETM) \\
&= \frac{\sqrt{T_{ETM}}}{E_{arm} \cdot 4\pi / \lambda} \frac{\delta E_{SNETM}(AR!side!of!ETM)}{x} \\
&= \sqrt{\frac{15ppm}{800W}} \frac{1.064\mu}{4\pi} \frac{\delta E_{SNETM}(AR!side!of!ETM)}{x} \\
&= 1.16 \cdot 10^{-11} \times \frac{\delta E_{SNETM}(AR!side!of!ETM)}{x}
\end{aligned}$$

Around half of the noise contribute to differential component, so the ratio is ~0.5×10^{-11}.

The field injected from SR is fully reflected by the mirror at all frequency region except for the optical resonance. The magnitude of the DARM at the low frequency is 2×10^{9} √Watt / meter, and SNSR ~ 1 because of perfect reflection. From this, SNPR / DARM ~10^{-9}.