LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
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CALIFORNIA INSTITUTE OF TECHNOLOGY
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Technical Note **LIGO-T060162-01-Z** 2006/07/20

Comments on Anisotropic Stochastic Background Searches

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1 Definition of the Stochastic Background Spectrum

The standard plane-wave expansion for a gravitational wave is

$$h_{ab}(\vec{r},t) = \sum_{A=+,\times} \int_{-\infty}^{\infty} df \iint d^2\Omega_{\hat{n}}\, h_A(f,\hat{n})\, e_{A\,ab}(\hat{n}) \exp\left(i2\pi f\left[t - \frac{\hat{n}\cdot\vec{r}}{c}\right]\right) \tag{1.1}$$

where $\{\overleftrightarrow{e}_A(\hat{n})\}$ are the usual transverse traceless basis tensors, normalized to obey

$$e_{A\,ab}(\hat{n})\, e^{ab}_{A'}(\hat{n}) = 2\delta_{AA'} \tag{1.2}$$

The spectrum $H(f)$ for an isotropic stochastic background is defined (e.g., in eqn (2.11) of Allen and Romano[1]) by

$$\langle h^*_A(f,\hat{n})\, h_{A'}(f',\hat{n}')\rangle = \delta^2(\hat{n},\hat{n}')\,\delta_{AA'}\,\delta(f-f')\,H(f) \tag{1.3}$$

The natural extension of this definition to a potentially anisotropic, unpolarized background is

$$\langle h^*_A(f,\hat{n})\, h_{A'}(f',\hat{n}')\rangle = \delta^2(\hat{n},\hat{n}')\,\delta_{AA'}\,\delta(f-f')\,H(f,\hat{n}) \tag{1.4}$$

If we further assume that the spatial distribution of the background is non-frequency-dependent[1], we can factor the background strength to get

$$\langle h^*_A(f,\hat{n})\, h_{A'}(f',\hat{n}')\rangle = \delta^2(\hat{n},\hat{n}')\,\delta_{AA'}\,\delta(f-f')\,H(f)\,\mathcal{P}(\hat{n}) \tag{1.5}$$

which is equation (2.8) of Allen and Ottewill [2]. The separation into $H(f)$ and $\mathcal{P}(\hat{n})$ is of course arbitrary, but we will get the closest correspondence to the isotropic formulas if we choose it so that

$$\iint d^2\Omega_{\hat{n}}\mathcal{P}(\hat{n}) = 4\pi \ . \tag{1.6}$$

This is the normalization chosen by Allen and Ottewill [2], in their equation (3.10). Note, however, that this is not the notation used by Ballmer[3], as can be seen from appendix C of his thesis[4], so his $H(f)$ differs from that associated with the Allen and Ottewill normalization by a factor of 4π from the division into $H(f)$ and $\mathcal{P}(\hat{n})$; additionally his (implicit) definition of $H(f,\hat{n})$ is four times that used in the generalization (1.4) of Allen and Romano. So Ballmer's $H(f)$ is 16π times that used in this note.

2 Expected Cross-Correlation in a Pair of Detectors

Let detector i at position $\vec{r}_i$ have response tensor $\overleftrightarrow{d_i}$, so that the strain it measures is

$$h_i(t) = d^{ab}_i\, h_{ab}(\vec{r}_i,t) = \sum_{A=+,\times} \int_{-\infty}^{\infty} df \iint d^2\Omega_{\hat{n}}\, h_A(f,\hat{n})\, d^{ab}_i\, e_{A\,ab}(\hat{n}) \exp\left(i2\pi f\left[t - \frac{\hat{n}\cdot\vec{r}_i}{c}\right]\right) \tag{2.1}$$

[1] This is not a good assumption in general, but we should be able to resolve a background into a sum of contributions which individually satisfy it.

Note that in addition to the explicit time dependence, there is a slow time variation hidden in the quantities $\hat{n}\cdot\vec{r}_i$ and $d_i^{ab}\, e_{A\,ab}(\hat{n}) = F_i^A(\hat{n})$. This is because $\hat{n}$ is a sky direction vector associated with a fixed right ascension and declination while $\vec{r}_i$ and $\overleftrightarrow{d_i}$ are quantities with constant components in an Earth-fixed basis, and are thus rotating with respect to the sky-fixed basis. However, if the data are analyzed in chunks which are small compared to the rotation period of one siderial day, we can neglect that time dependence in identifying

$$\widetilde{h}_i(f) \approx \sum_{A=+,\times} \iint d^2\Omega_{\hat{n}}\, h_A(f,\hat{n})\, d_i^{ab}\, e_{A\,ab}(\hat{n}) \exp\left(-i2\pi f \frac{\hat{n}\cdot\vec{r}_i}{c}\right) \tag{2.2}$$

The usual calculation tells us that

$$\begin{aligned}\langle \widetilde{h}_i^*(f)\, \widetilde{h}_j(f')\rangle &= \delta(f-f') \iint d^2\Omega_{\hat{n}} \sum_{A=+,\times} d_i^{ab} e_{A\,ab}(\hat{n})\, e_{A\,ab}(\hat{n})\, e_{A\,cd}(\hat{n})\, d_j^{cd}\, H(f,\hat{n}) e^{i2\pi f \hat{n}\cdot(\vec{r}_i-\vec{r}_j)/c} \\ &= 2\delta(f-f') \iint d^2\Omega_{\hat{n}}\, d_i^{ab}\, P^{\mathrm{TT}\hat{n}ab}{}_{cd}\, d_j^{cd}\, H(f,\hat{n}) e^{i2\pi f \hat{n}\cdot(\vec{r}_i-\vec{r}_j)/c}\end{aligned} \tag{2.3}$$

where $P^{\mathrm{TT}\hat{n}ab}{}_{cd}$ is the projector onto traceless, symmetric tensors transverse to the unit vector $\hat{n}$, which can be expanded in the standard polarization basis as

$$P^{\mathrm{TT}\hat{n}ab}{}_{cd} = \frac{1}{2} \sum_{A=+,\times} e_A^{ab}(\hat{n})\, e_{A\,cd}(\hat{n}) \tag{2.4}$$

If we recall the overlap reduction function appropriate for isotropic stochastic background searches

$$\gamma_{12}(f) = d_{1\,ab}\, d_2^{cd}\, \frac{5}{4\pi} \iint d^2\Omega_{\hat{n}}\, P^{\mathrm{TT}\hat{n}ab}{}_{cd}\, e^{i2\pi f \hat{n}\cdot(\vec{r}_2-\vec{r}_1)/c} \tag{2.5}$$

and call integrand, which depends on both frequency and sky direction,

$$\frac{d^2\gamma_{12}}{d^2\Omega}(f,\hat{n}) = \frac{5}{4\pi} \left(d_{1\,ab}\, d_2^{cd}\, P^{\mathrm{TT}\hat{n}ab}{}_{cd}\right) \left(e^{i2\pi f \hat{n}\cdot(\vec{r}_2-\vec{r}_1)/c}\right) \tag{2.6}$$

then (2.3) becomes

$$\begin{aligned}\langle \widetilde{h}_i^*(f)\, \widetilde{h}_j(f')\rangle &= \delta(f-f') \frac{8\pi}{5} \iint d^2\Omega_{\hat{n}}\, \frac{d^2\gamma_{12}}{d^2\Omega}(f,\hat{n})\, H(f,\hat{n}) \\ &= \delta(f-f') \frac{8\pi}{5} H(f) \iint d^2\Omega_{\hat{n}}\, \frac{d^2\gamma_{12}}{d^2\Omega}(f,\hat{n})\, \mathcal{P}(\hat{n})\end{aligned} \tag{2.7}$$

If we extend the definition of the overlap reduction function to one specific to a particular background $\mathcal{P}(\hat{n})$ as follows:

$$\gamma_{12}^{\mathcal{P}}(f) = \iint d^2\Omega_{\hat{n}}\, \frac{d^2\gamma_{12}}{d^2\Omega}(f,\hat{n})\, \mathcal{P}(\hat{n}) \tag{2.8}$$

then we have a generalization of the usual formula:

$$\langle \widetilde{h}_i^*(f)\, \widetilde{h}_j(f')\rangle = \delta(f-f') \frac{8\pi}{5} \gamma_{12}^{\mathcal{P}}(f) H(f) \tag{2.9}$$

This is the equivalent of equation (3.56) in Allen and Romano[1]. [See also Allen & Romano's equation (2.15).]

Note that $\gamma_{12}^{\mathcal{P}}(f)$ depends on the detectors, the spatial distribution of the source, and also on siderial time.

Note also that, subject to the normalization (1.6), a background coming from a single direction $\hat{n}_0$ is described by a distribution

$$\mathcal{P}_{\hat{n}_0}(\hat{n}) = 4\pi\delta^2(\hat{n},\hat{n}_0) \tag{2.10}$$

which corresponds to the overlap reduction function

$$\begin{aligned}\gamma_{12}^{\hat{n}_0}(f) &= 4\pi\frac{d^2\gamma_{12}}{d^2\Omega}(f,\hat{n}_0) = 5\left(d_{1\,ab}\,d_2^{cd}\,P^{\mathrm{TT}\hat{n}_0}{}^{ab}_{cd}\right)\left(e^{i2\pi f\hat{n}_0\cdot(\vec{r}_2-\vec{r}_1)/c}\right)\\ &= \frac{5}{2}\left(\sum_{A=+,\times}F_{A\,1}(\hat{n}_0)\,F_{A\,2}(\hat{n}_0)\right)\left(e^{i2\pi f\hat{n}_0\cdot(\vec{r}_2-\vec{r}_1)/c}\right)\end{aligned} \tag{2.11}$$

This is 5 times what Ballmer[3] calls $\gamma_{\hat{\Omega}}$.

3 Calculation of Overlap Reduction Function Integrand

For a given sky direction, the overlap reduction function integrand (2.6) factors as shown above into a piece depending only on detector orientation and a factor depending only on the frequency and separation. Note that in a basis co-rotating with the Earth, the components of the detector response tensors $\overleftrightarrow{d_1}$ and $\overleftrightarrow{d_2}$ and the separation vector $\vec{r}_2-\vec{r}_1$ are fixed, but the direction $\hat{n}$ associated with a particular right ascension and declination changes with siderial time.

The explicit form of the projector $P^{\mathrm{TT}\hat{n}}{}^{ab}_{cd}$, and thus of the overlap reduction function integrand $\frac{d^2\gamma_{12}}{d^2\Omega}(f,\hat{n})$ can be worked out by noting that it must be traceless and symmetric on both pairs of indices ($\{ab\}$ and $\{cd\}$); with $\hat{n}$ as the only preferred direction, there are only three independent tensors which can be created with these properties:

$$T_1{}^{ab}_{cd} = P^{\mathrm{T}}{}^{ab}_{cd} \tag{3.1a}$$

$$T_2{}^{ab}_{cd}(\hat{n}) = P^{\mathrm{T}}{}^{ab}_{ef}\hat{n}^f\hat{n}_g P^{\mathrm{T}}{}^{eg}_{cd} \tag{3.1b}$$

$$T_3{}^{ab}_{cd}(\hat{n}) = P^{\mathrm{T}}{}^{ab}_{ef}\hat{n}^e\hat{n}^f\hat{n}_g\hat{n}_h P^{\mathrm{T}}{}^{gh}_{cd} \tag{3.1c}$$

where

$$P^{\mathrm{T}}{}^{ab}_{cd} = \delta^a_{(c}\delta^b_{d)} - \frac{1}{3}\delta^{ab}\delta_{cd} \tag{3.2}$$

is the projector onto traceless symmetric tensors. We can thus write

$$P^{\mathrm{TT}\hat{n}}{}^{ab}_{cd} = \sum_{n=1}^{3}\beta_n T_n{}^{ab}_{cd}\ ; \tag{3.3}$$

to figure out the coëfficients $\{\beta_n\}$ we just need to contract each of the $\{T_{n\,cd}^{\ ab}\}$ with $P^{\mathrm{TT}\hat{n}ab}_{\ \ \ \ cd}$ and each other. The former set of contractions is straightforward, because $P^{\mathrm{TT}\hat{n}ab}_{\ \ \ \ cd}$ is a projector onto a two-dimensional subspace which is transverse to $\hat{n}$.

$$T_{1\,ab}^{\ cd} P^{\mathrm{TT}\hat{n}ab}_{\ \ \ \ cd} = P^{\mathrm{TT}\hat{n}ab}_{\ \ \ \ ab} = 2 \tag{3.4a}$$
$$T_{2\,ab}^{\ cd} P^{\mathrm{TT}\hat{n}ab}_{\ \ \ \ cd} = P^{\mathrm{TT}\hat{n}ab}_{\ \ \ \ ac} n_b n^c = 0 \tag{3.4b}$$
$$T_{3\,ab}^{\ cd} P^{\mathrm{TT}\hat{n}ab}_{\ \ \ \ cd} = P^{\mathrm{TT}\hat{n}ab}_{\ \ \ \ cd} n_a n_b n^c n^d = 0 \tag{3.4c}$$

The latter set of contractions is worked out in the appendix of [5] [equation (21)] and they give us

$$\begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} T_{1\,ab}^{\ cd} \\ T_{2\,ab}^{\ cd} \\ T_{3\,ab}^{\ cd} \end{pmatrix} P^{\mathrm{TT}\hat{n}ab}_{\ \ \ \ cd} = \overbrace{\begin{pmatrix} T_{1\,ab}^{\ cd} \\ T_{2\,ab}^{\ cd} \\ T_{3\,ab}^{\ cd} \end{pmatrix} \begin{pmatrix} T_{1\,cd}^{\ ab} & T_{2\,cd}^{\ ab} & T_{3\,cd}^{\ ab} \end{pmatrix}}^{\begin{pmatrix} 5 & 5/3 & 2/3 \\ 5/3 & 17/18 & 4/9 \\ 2/3 & 4/9 & 4/9 \end{pmatrix}} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} \tag{3.5}$$

Inverting the matrix gives

$$\begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & -1 & \frac{1}{4} \\ -1 & 4 & -\frac{5}{2} \\ \frac{1}{4} & -\frac{5}{2} & \frac{35}{8} \end{pmatrix} \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 1/2 \end{pmatrix} \tag{3.6}$$

so

$$P^{\mathrm{TT}\hat{n}ab}_{\ \ \ \ cd} = P^{\mathrm{T}ab}_{\ \ cd} - 2P^{\mathrm{T}ab}_{\ \ ef} \hat{n}^f \hat{n}_g P^{\mathrm{T}eg}_{\ \ cd} + \frac{1}{2} P^{\mathrm{T}ab}_{\ \ ef} \hat{n}^e \hat{n}^f \hat{n}_g \hat{n}_h P^{\mathrm{T}gh}_{\ \ cd} \tag{3.7}$$

which means

$$\frac{d^2\gamma_{12}}{d^2\Omega}(f,\hat{n}) = \frac{5}{4\pi} e^{i2\pi f\hat{n}\cdot(\vec{r}_2-\vec{r}_1)/c} \left[d_1^{\mathrm{T}ab} d^{\mathrm{T}}_{2\,ab} - 2d_1^{\mathrm{T}ab} \hat{n}_b \hat{n}^c d^{\mathrm{T}}_{2\,ac} + \frac{1}{2} d_1^{\mathrm{T}ab} \hat{n}_a \hat{n}_b \hat{n}^c \hat{n}^d d^{\mathrm{T}}_{2\,cd} \right] \tag{3.8}$$

where

$$d^{\mathrm{T}ab} = P^{\mathrm{T}ab}_{\ \ cd} d^{cd} = d^{ab} - \frac{1}{3}\delta^{ab} d^c_c \tag{3.9}$$

is the traceless part of the response tensor (which, for an interferometer, is just the tensor itself).

This calculation is implemented in the matapps routine `orfintegrand()` in the directory `src/utilities/detgeom/matlab`; if you pass it a $3 \times N$ matrix representing N different sky directions in Earth-fixed Cartesian coördinates, and a column vector ($M \times 1$ matrix) of M frequencies, it will return an $M \times N$ matrix containing the value of $\frac{d^2\gamma_{12}}{d^2\Omega}$ at each frequency and sky direction. The routine `getcartesiandirectionfromsource()` converts the combination of declination and minus hour angle (which is right ascension minus Greenwich Mean Siderial Time) into Earth-fixed Cartesian unit vectors.

4 Autocorrelation (Power Spectral Density) in a Given Detector

One measure of the stochastic background strength is the strain spectrum it would generate in a suitable detector. Applying (2.9) to a single detector and defining the one-sided strain power spectral density by

$$\langle h(f)^* h(f') \rangle = \frac{1}{2} S_{\text{det}}(f) \delta(f - f') \tag{4.1}$$

we have

$$S_{\text{det}}(f) = \frac{16\pi}{5} \gamma_{\text{det}}^{\mathcal{P}} H(f) \tag{4.2}$$

where

$$\gamma_{\text{det}}^{\mathcal{P}} = \iint d^2\Omega_{\hat{n}} \, \frac{d^2\gamma_{\text{det}}}{d^2\Omega}(\hat{n}) \, \mathcal{P}(\hat{n}) \tag{4.3}$$

and

$$\frac{d^2\gamma_{\text{det}}}{d^2\Omega}(\hat{n}) = \frac{5}{4\pi} d_{ab} \, d^{cd} \, P^{\text{TT}\hat{n}ab}{}_{cd} = \frac{5}{4\pi} \left[d^{\text{T}ab} d^{\text{T}}{}_{ab} - 2 d^{\text{T}ab} \hat{n}_b \hat{n}^c d^{\text{T}}{}_{ac} + \frac{1}{2} d^{\text{T}ab} \hat{n}_a \hat{n}_b \hat{n}^c \hat{n}^d d^{\text{T}}{}_{cd} \right] \tag{4.4}$$

Now, for the special case of pointlike source at sky position $\hat{n}_0$, one can consider the strain PSD in an optimally oriented detector, i.e., an interferometer whose perpendicular arms lie in the plane transverse to the propagation direction so that

$$d_{ab} \, d^{cd} \, P^{\text{TT}\hat{n}_0 ab}{}_{cd} = d_{ab} d^{ab} = \frac{1}{2} \tag{4.5}$$

and then $\gamma_{\text{det}} = \frac{5}{2}$

$$S_{\text{gw,opt}}(f) = 8\pi H(f) \tag{4.6}$$

However, this concept of an "optimally oriented detector" doesn't generalize particularly well to sources with non-trivial sky distributions.

5 Detector-Independent Measures of Background Strength

5.1 Energy Density

A property of the gravitational wave background itself, without reference to the strain in any hypothetical detector, is the energy density[1, 6]

$$\rho_{\text{gw}} = \frac{c^2}{32\pi G} \langle \dot{h}_{ab}(t, \vec{r}) \dot{h}^{ab}(t, \vec{r}) \rangle = \frac{\pi c^2}{G} \int_0^\infty f^2 \iint d^2\Omega_{\hat{n}} \, H(f, \hat{n}) = \frac{4\pi^2 c^2}{G} \int_0^\infty f^2 \, H(f) \, df \tag{5.1}$$

where we have used the Allen and Ottewill normalization (1.6) in the last step. Note that subject to this normalization the relation of $H(f)$ to energy density has the same form for both isotropic and anisotropic backgrounds.

The energy density per unit frequency interval is this

$$\frac{d\rho_{\text{gw}}}{df} = \frac{4\pi^2 c^2}{G} f^2 \, H(f) \tag{5.2}$$

and the usual definition of energy density per logarithmic frequency interval as a fraction of the critical energy density is

$$\Omega_{\text{gw}}(f) = \frac{f}{\rho_{\text{crit}}} \frac{d\rho_{\text{gw}}}{df} = \frac{32\pi^2}{3H_0^2} f^3 H(f) \tag{5.3}$$

One can also consider the energy density per hertz per steradian coming from a given direction:

$$\frac{d^3\rho_{\text{gw}}}{df \, d^2\Omega}(f, \hat{n}) = \frac{\pi c^2}{G} f^2 H(f, \hat{n}) \tag{5.4}$$

5.2 Energy Flux

An even more natural quantity, if we want to restrict attention waves coming from a given direction, is the power per square meter per hertz per steradian (which is apparently called the spectral radiance, although I would have called it flux density)

$$\frac{d^3\vec{J}_{\text{gw}}}{df \, d^2\Omega}(f, \hat{n}) = -\hat{n} \, \frac{\pi c^3}{G} f^2 \, H(f, \hat{n}) \tag{5.5}$$

For a general background this makes more sense than flux itself (for instance, the net flux through any surface from an isotropic background is zero), but for a pointlike source this diverges because of the delta function in $H(f, \hat{n})$. In that case, the useful quantity is the total power per square meter per hertz through a surface perpendicular to the direction to the pointlike source (which is apparently called spectral irradiance–although I would have called it flux–and is the thing that's measured in Janskys)

$$\frac{d\Phi_{\text{gw}}}{df} = \frac{4\pi^2 c^3}{G} f^2 \, H(f) \tag{5.6}$$

5.3 Quantitative Relationship

To see how all of these are related, consider a stochastic background of total strength $h_{100}^2 \Omega_{\text{gw}}(f) = 10^{-6}$, in two cases: (i) isotropic (ii) coming from a single direction $\hat{n}_0$ The values of these various reference quantities are shown in Table 1.

References

[1] Allen B and Romano J D 1999 *Phys Rev* **D59** 102001; e-Print: gr-qc/9710117.

	isotropic	pointlike
$H(f,\hat{n})$	$9.98 \times 10^{-50} \left(\frac{100\,\mathrm{Hz}}{f}\right)^3 \mathrm{Hz}^{-1}$	$1.25 \times 10^{-48} \left(\frac{100\,\mathrm{Hz}}{f}\right)^3 \mathrm{Hz}^{-1} \delta^2(\hat{n},\hat{n}_0)$
$H(f)$	$9.98 \times 10^{-50} \left(\frac{100\,\mathrm{Hz}}{f}\right)^3 \mathrm{Hz}^{-1}$	$9.98 \times 10^{-50} \left(\frac{100\,\mathrm{Hz}}{f}\right)^3 \mathrm{Hz}^{-1}$
$S_{\mathrm{gw,opt}}(f)$	$1.00 \times 10^{-48} \left(\frac{100\,\mathrm{Hz}}{f}\right)^3 \mathrm{Hz}^{-1}$	$2.51 \times 10^{-48} \left(\frac{100\,\mathrm{Hz}}{f}\right)^3 \mathrm{Hz}^{-1}$
$h_{100}^2 \Omega_{\mathrm{gw}}(f)$	10^{-6}	10^{-6}
$\frac{d\rho_{\mathrm{gw}}}{df}$	$5.30 \times 10^{-17} \left(\frac{100\,\mathrm{Hz}}{f}\right) \mathrm{J/m^3\,Hz^{-1}}$	$5.30 \times 10^{-17} \left(\frac{100\,\mathrm{Hz}}{f}\right) \mathrm{J/m^3\,Hz^{-1}}$
$\frac{d^3\rho_{\mathrm{gw}}}{df\,d^2\Omega}(f,\hat{n})$	$4.22 \times 10^{-18} \left(\frac{100\,\mathrm{Hz}}{f}\right) \mathrm{J\,m^3\,sr^{-1}\,Hz^{-1}}$	$5.30 \times 10^{-17} \left(\frac{100\,\mathrm{Hz}}{f}\right) \mathrm{J\,m^3\,sr^{-1}} \delta^2(\hat{n},\hat{n}_0)$
$-\hat{n}\cdot\frac{d^3\vec{J}_{\mathrm{gw}}}{df\,d^2\Omega}(f,\hat{n})$	$1.27 \times 10^{-9} \left(\frac{100\,\mathrm{Hz}}{f}\right) \mathrm{W\,m^2\,sr^{-1}\,Hz^{-1}}$	$1.59 \times 10^{-8} \left(\frac{100\,\mathrm{Hz}}{f}\right) \mathrm{W\,m^2\,sr^{-1}\,Hz^{-1}} \delta^2(\hat{n},\hat{n}_0)$
$\frac{d\Phi_{\mathrm{gw}}}{df}$	0	$1.59 \times 10^{-8} \left(\frac{100\,\mathrm{Hz}}{f}\right) \mathrm{W\,m^2\,Hz^{-1}} = 1.59 \times 10^{18}\,\mathrm{Jy}$

Table 1: Comparison of measures of SGWB strength for isotropic vs point-line backgrounds.

[2] Allen B and Ottewill A C 1997 *Phys Rev* **D56** 545; e-Print: gr-qc/9607068.

[3] Ballmer S W "A radiometer for stochastic gravitational waves"; *Class Quant Grav* **23** S179; e-Print: gr-qc/0510096.

[4] Ballmer S W, MIT Ph.D. thesis, 2006

[5] Whelan J T "Stochastic Gravitational Wave Measurements with Bar Detectors: Dependence of Response on Detector Orientation"; *Class Quant Grav* **23** 1181; e-Print: gr-qc/0509109.

[6] C.W. Misner, K.S. Thorne, and J.A. Wheeler, *Gravitation*, (Freeman, San Francisco, 1973).