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MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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Newtonian Noise Simulation and Suppression for Gravitational-Wave Interferometers

Keenan Pepper
Mentors: Rana Adhikari and Phil Willems

California Institute of Technology
LIGO Project, MS 18-34
Pasadena, CA 91125
Phone (626) 395-2129
Fax (626) 304-9834
E-mail: info@ligo.caltech.edu

Massachusetts Institute of Technology
LIGO Project, Room NW17-161
Cambridge, MA 02139
Phone (617) 253-4824
Fax (617) 253-7014
E-mail: info@ligo.mit.edu

LIGO Hanford Observatory
Route 10, Mile Marker 2
Richland, WA 99352
Phone (509) 372-8106
Fax (509) 372-8137
E-mail: info@ligo.caltech.edu

LIGO Livingston Observatory
19100 LIGO Lane
Livingston, LA 70754
Phone (225) 686-3100
Fax (225) 686-7189
E-mail: info@ligo.caltech.edu

`http://www.ligo.caltech.edu/`

Abstract

The next generation of gravitational-wave interferometers will have mechanical isolation systems so effective that seismic noise will be negligible at 10 Hz and above. In this frequency region (which is important for both massive black hole mergers and the gravitational stochastic background), the dominant noise source will be Newtonian gravity noise caused by fluctuations in the distribution of matter around the test masses. This gravitational force cannot be shielded, even in principle, but its effect can be estimated from independent measurements and either compensated for in the online system or subtracted from the output data. In the present work, the vibrations of the vacuum chamber and support columns were modeled with finite element analysis software to estimate their contribution to the Newtonian noise. It appears that this contribution will be at least a factor of ten smaller than that caused by vibrations of the soil and the concrete foundation slab, so initial implementations of Newtonian noise cancellation should ignore the chamber and focus on the ground. To evaluate the feasibility of optimal filtering algorithms for this application, multiple-input single-output (MISO) filters were developed with the goal of implementing active cancellation of seismic noise at the 40-meter prototype lab as a proof of concept.

Contents

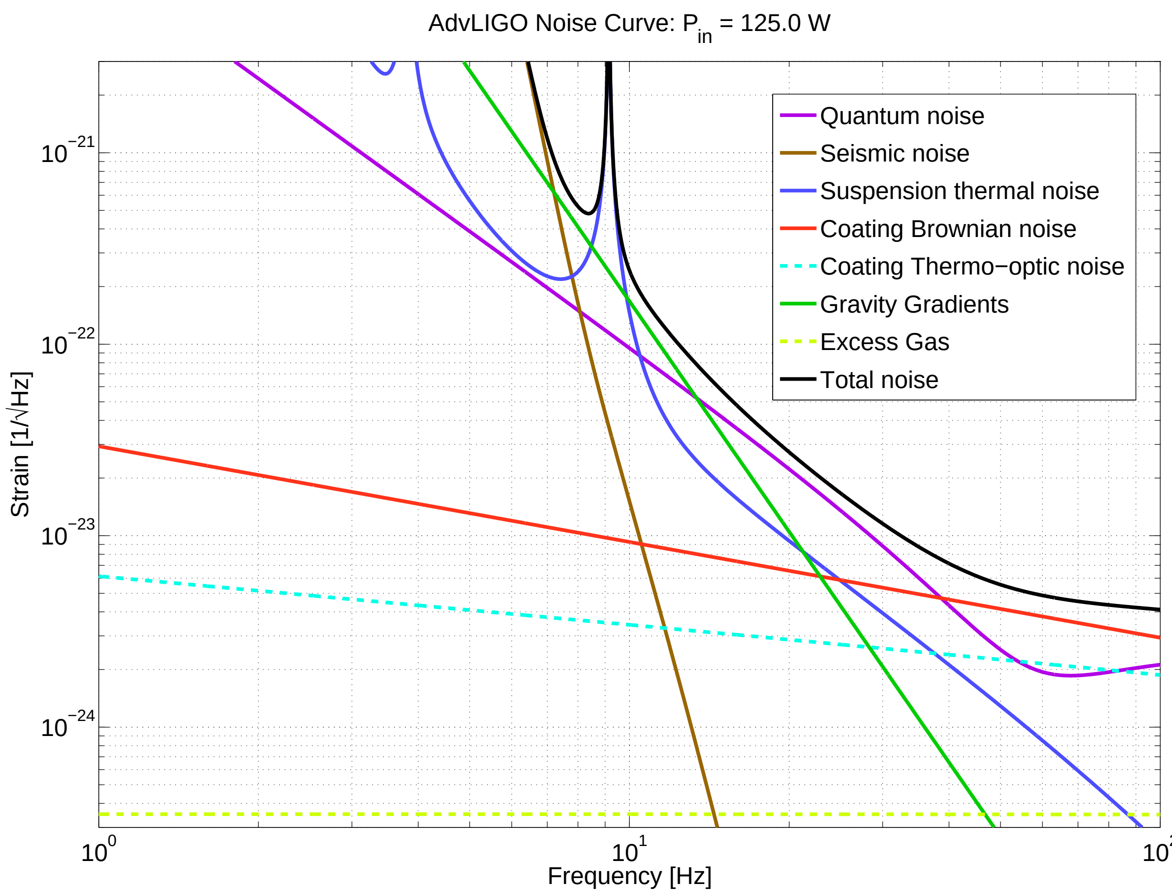

Figure 1: Expected contributions of all fundamental and some technical noise sources to the Advanced LIGO noise budget. The gravity gradient noise curve is based on a recent re-evaluation by Brian Lantz.[2]

1 Background and motivation

The ubiquitous mechanical vibrations of the outside world affect the motion of the LIGO test masses in two ways. The first, mechanical coupling, has been dealt with by placing the test masses in ultra-high vacuum, suspended as pendula from complex seismic isolation platforms.[1] For Advanced LIGO, this is expected to reduce the seismic noise to an insignificant fraction of the total noise at frequencies above 10 Hz, and to a completely negligible amount above 15 Hz (see Figure 1).

The second way in which vibration is transferred to the test masses is by variations in the local gravitational field caused by the fluctuating mass distribution in their vicinity. This is known as "Newtonian gravity noise" (because it is well approximated by Newton's familiar law) or "gravity gradient noise". Unlike seismic noise, Newtonian noise cannot be shielded, so it has been called a "short circuit" around seismic isolation systems,[1] and a fundamental "noise floor" that limits the sensitivity of any terrestrial detector.[3] The purpose of this project is to investigate the severity of this limit and explore the possibility that it can be overcome with active noise cancellation.

Newtonian noise is expected to dominate seismic noise at frequencies above 7 or 8 Hz. This is the lower limit of the frequency band in which Advanced LIGO has a reasonable chance of observing gravitational wave signals, and many astrophysical sources, such as pulsars and massive black hole mergers, are thought lie in this region. Therefore, it is desirable to extend the range of sensitivity downward as much as possible.

1.1 Outline

The first part of this paper describes several software models of the vacuum chamber and support piers, the concrete equipment foundation, and the ground beneath the test masses. These were intended to evaluate the relative contributions of these sources of Newtonian noise and characterize which modes of vibration are most important.

The second part describes a proposed method of using measurements of the ground vibration (and possibly other measurements), independent from the main interferometer, to approximate the Newtonian noise accurately enough to subtract out a large fraction of it. This approach was suggested in [4], and the present work extends that idea with a concrete implementation of one simple method, which has proven effective in cancelling seismic noise at the 40-meter prototype lab. Several possible improvements on this basic method are then discussed.

2 Simulations

2.1 Simple worst-case model of the BSC

The BSCs ("Basic Symmetric Chambers") are cylindrical steel structures, about 1.5 m in radius and 4 m in height, that enclose the vacuum around the test masses. The BSC is worthy of study because it is the closest object to the test mass that weighs more than a tonne, and since gravitational force falls off as the inverse square of the distance, nearby massive objects such as the BSC are possibly significant sources of Newtonian noise.

Even a simple cylinder has many different modes of vibration, so instead of modeling them all or choosing some arbitrarily, it was decided to create a worst-case model of a (possibly unphysical) mode of vibration that maximizes the Newtonian noise at the test mass for a given vibration amplitude. The BSC was modeled as a collection of point masses arranged in a hollow cylinder with end caps. Clearly, the Newtonian noise is maximized if each point mass moves in the direction that maximizes the gravitational field at the test mass in the same direction (along the beam axis). What is this direction?

It is convenient to work in a coordinate system in which the test mass is at the origin, and the beam axis lies along the x axis. The gravitational field vector at the origin due to a particle at $\mathbf{p} = (x, y, z)$ is proportional to

$$\frac{\hat{\mathbf{p}}}{|\mathbf{p}|^2} = \frac{\mathbf{p}}{|\mathbf{p}|^3},$$

so the x component is proportional to

$$\frac{x}{(x^2 + y^2 + z^2)^{3/2}}.$$

The gradient of this function is proportional to

$$(-2x^2 + y^2 + z^2)\hat{\imath} - 3xy\hat{\jmath} - 3xz\hat{k},$$

so if the point mass moves in that direction, the rate of increase of the component of gravitational field along the beam axis is maximized.

If every point mass in the cylinder moves the same amount along its own gradient vector, the cylinder deforms as shown in Figure 2. Intuitively, the masses on the right bunch together and move closer to the test mass to attract it, and the masses on the left spread apart and move farther away to minimize their attraction. This is not a physically reasonable mode of vibration (although it is similar to a "folding" mode), but a hypothetical worst-case scenario. No mode of vibration can cause more Newtonian noise if the maximum displacement is the same.

The ratio of the gravitational field change at the test mass to the average displacement of the points of the vibrating object has dimensions of acceleration / length, or s^{-2}, and this paper refers to it as a "gravity coefficient". Assuming the BSC has a total mass of 8 tonnes (probably an overestimate), the gravity coefficient for this worst case is 1.0×10^{-7} s^{-2} (meaning that if the points all move by one micron, then the test mass is accelerated by 10^{-7} microns per second squared).

The amount of Newtonian noise obviously depends on the amplitude of vibration (in fact, it is proportional to it in the low-amplitude limit), but to get an idea of what this gravity coefficient means, Figure 3 shows what the Newtonian noise spectrum of the BSC would look like if vibration at every frequency had as much of an effect as the worst-case mode.

The strain was calculated from real accelerometer data from the Hanford site. Many different spectra were compared and the noisiest one was chosen. The accelerometer was mounted at the top of one of the support piers, which has the largest motion of anywhere on the pier, according to [5]. The magnitude of the x, y, z vector recorded by the accelerometer was used as the displacement in the worst-case direction. Still, the noise is a factor of 10 below the Advanced LIGO benchmark noise curve even at the 18 Hz spike (which is not caused by a folding mode, so its effect is definitely overestimated). Thus the vibration of the BSC gives at most a minor contribution to the total Newtonian noise spectrum, and any proposal to suppress Newtonian noise should begin with the vibration of the ground. A paper by

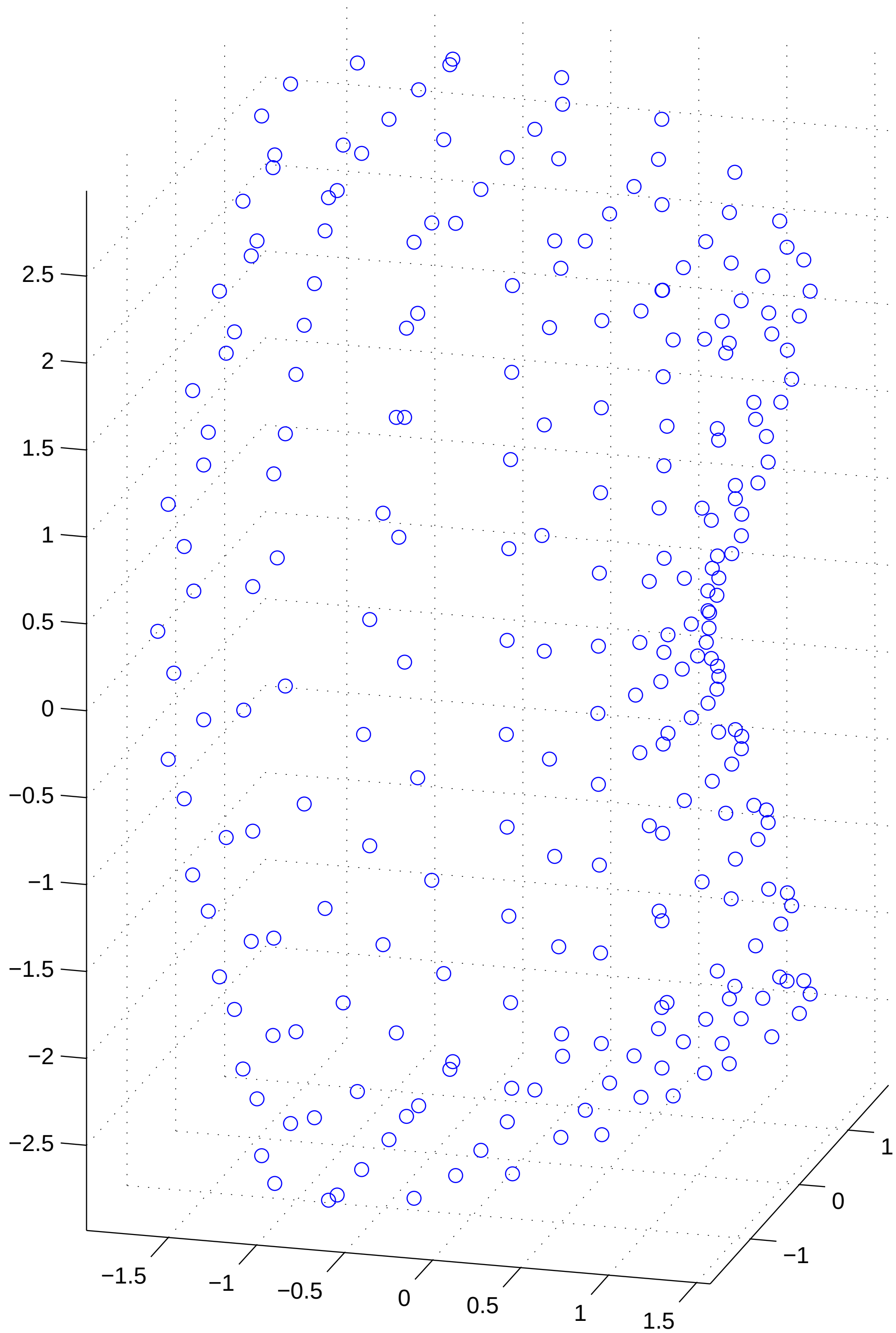

Figure 2: Worst-case distortion of the cylinder. Each point mass moves the same distance in the direction that maximizes its contribution to the x component of the gravitational field at the origin, so they become more dense on the right and more sparse on the left. The gravity coefficient is 1.0×10^{-7} s^{-2}. All coordinates are in meters.

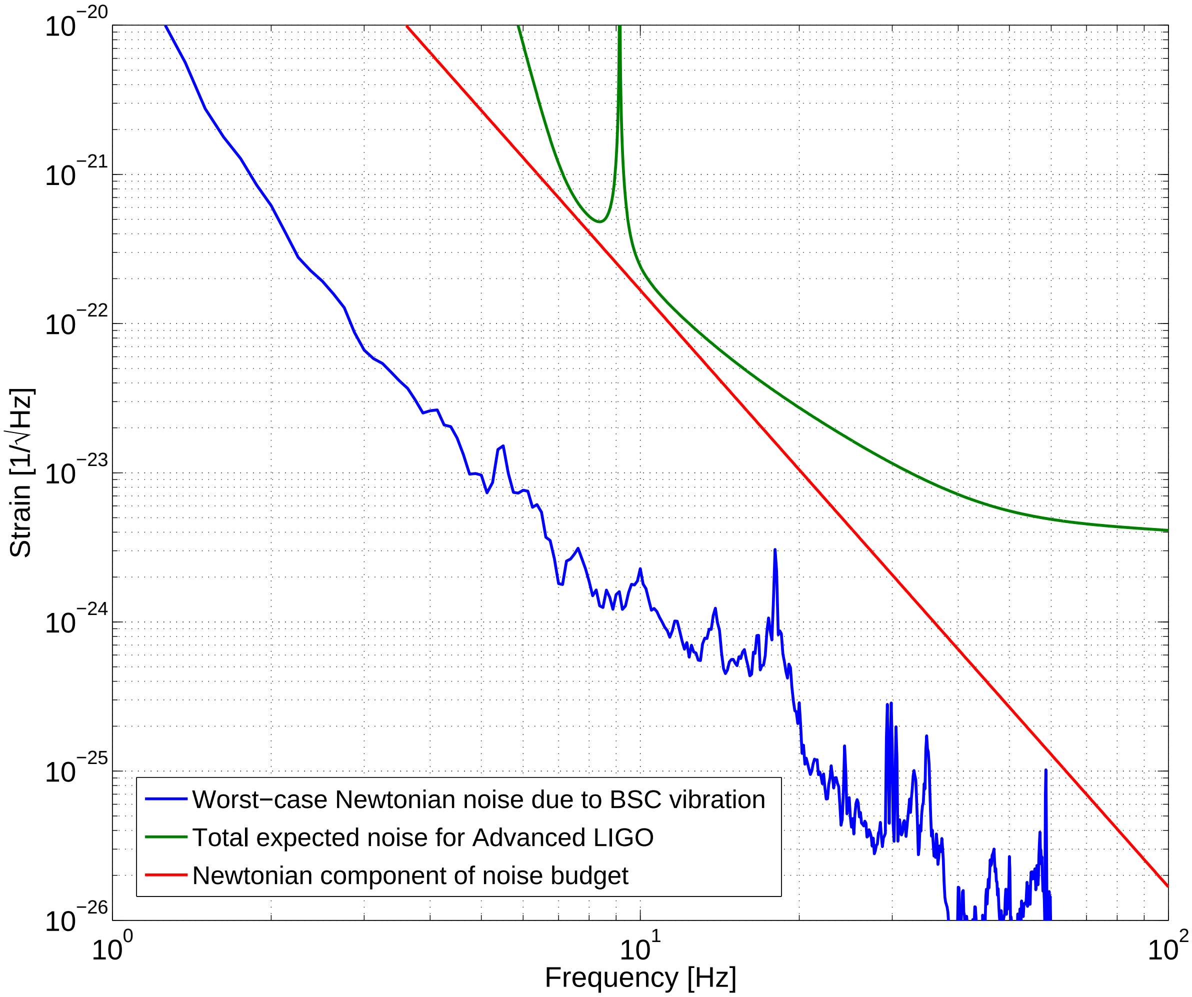

Figure 3: Comparison of the worst-case estimate of the Newtonian noise caused by BSC vibration to the total estimated Newtonian noise.

the VIRGO collaboration reached a similar conclusion for their interferometer's much larger suspension towers ("the effect of the direct coupling with the interferometer structures is negligible").[6]

2.2 Finite-element modeling with COMSOL

To create more physically reasonable models, the finite element analysis program COMSOL Multiphysics (formerly FEMLAB) was used. COMSOL is a versatile piece of software with many different application modes, but only the structural mechanics mode was used extensively for this project. The models were in three-dimensional space with no required symmetry, and they were solved in eigenfrequency mode (rather than static mode or transient mode).

COMSOL has the capability of integrating an arbitrary function over any collection of subdomains of a model. Gravitational field is not one of the built-in functions (because very few other COMSOL users have a use for it), but it is not difficult to derive an expression that can be integrated over the object to obtain the gravitational field change, and hence the gravity coefficient (see Section 5.1).

The first model is a slightly more sophisticated model of the BSC and support piers, intended to verify the worst-case model and determine how close physically reasonable vibrations modes can come to the worst case. It turns out that in many physical modes, there is some symmetry or cancellation effect that makes the gravity coefficient much smaller than the worst case. Even in modes such as the swinging mode shown in Figure 4 that have no such symmetry or cancellation, the gravity coefficient is significantly smaller than the worst-case one. Also, the modes are quite sparse: only two are predicted below 20 Hz, and only seven below 100 Hz. This supports the conclusion of the previous section that the BSC itself is not a major source of Newtonian noise.

The second model comprises a large homogeneous mass of soil with periodic boundary conditions in the horizontal directions, and the concrete foundation slab on top. This produces a large number of closely spaced modes (dozens of modes in each 1 Hz interval), and the gravity coefficients are in general much larger. The gravity coefficients for the soil are usually factors of 3-4 higher than those for the slab itself, but the shapes of the vibration modes indicate that, for the relevant frequencies, the slab follows the motion of the ground beneath it fairly closely (rather than sliding across it or moving in some other direction).

3 Active noise cancellation

3.1 Principles

The process of using a secondary sensor to estimate the noise corrupting the signal from a primary sensor, and then subtracting this estimate from the primary signal, has been extensively studied and is now well known.[7] It goes by the names of "noise cancellation", "active noise control", and "antinoise". The situation in which there is more than one secondary sensor available is almost as well known.[8] What makes this problem non-trivial is that although the noise measured by the secondary sensor is *correlated* with the noise to be removed from the primary sensor, it is not in general *equal*, so simple subtraction is usually ineffective. Instead, a filter must be created to estimate the noise as accurately as possible from the secondary sensor measurement.

In this case, the primary sensor is the gravitational-wave interferometer, and the secondary sensors are accelerometers fixed to the surrounding structures. The goal is to create a multiple-input single-output (MISO) filter that combines the data from all the accelerometers to produce an optimal estimate of the motion of the test mass.

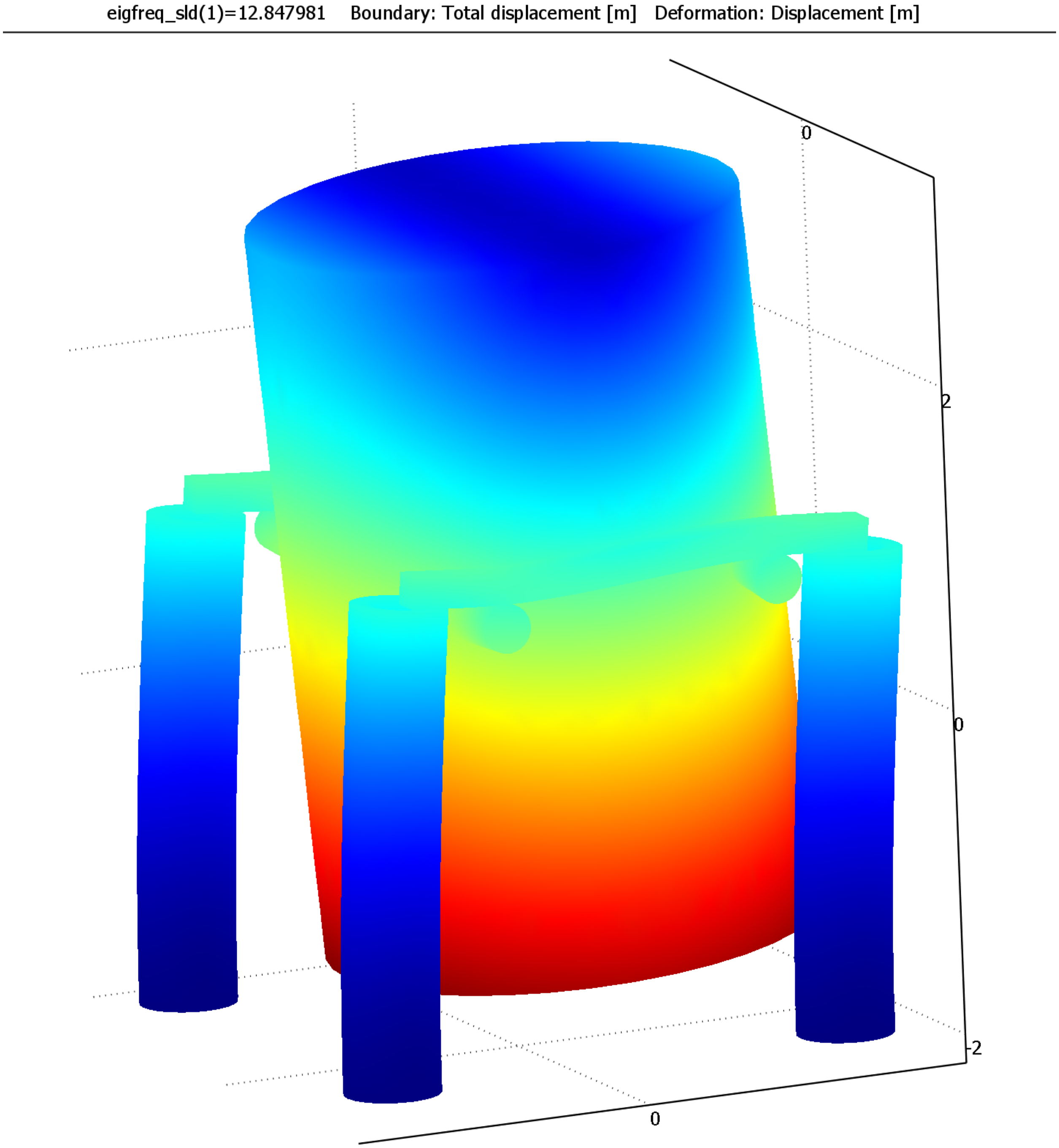

Figure 4: COMSOL predicts this as the first vibrational mode of the BSC-pier assembly, at about 13 Hz. The bottom of the cylinder swings back and forth, and the piers sway along with it. The gravity coefficient for this mode is 1.5×10^{-8} s^{-2} (compare to 1.0×10^{-7} s^{-2} for the worst case model and 6.0×10^{-7} s^{-2} for the slab/ground mode shown in Figure 5). All coordinates are in meters. The color represents the amount of displacement: nodes are blue and antinodes are red.

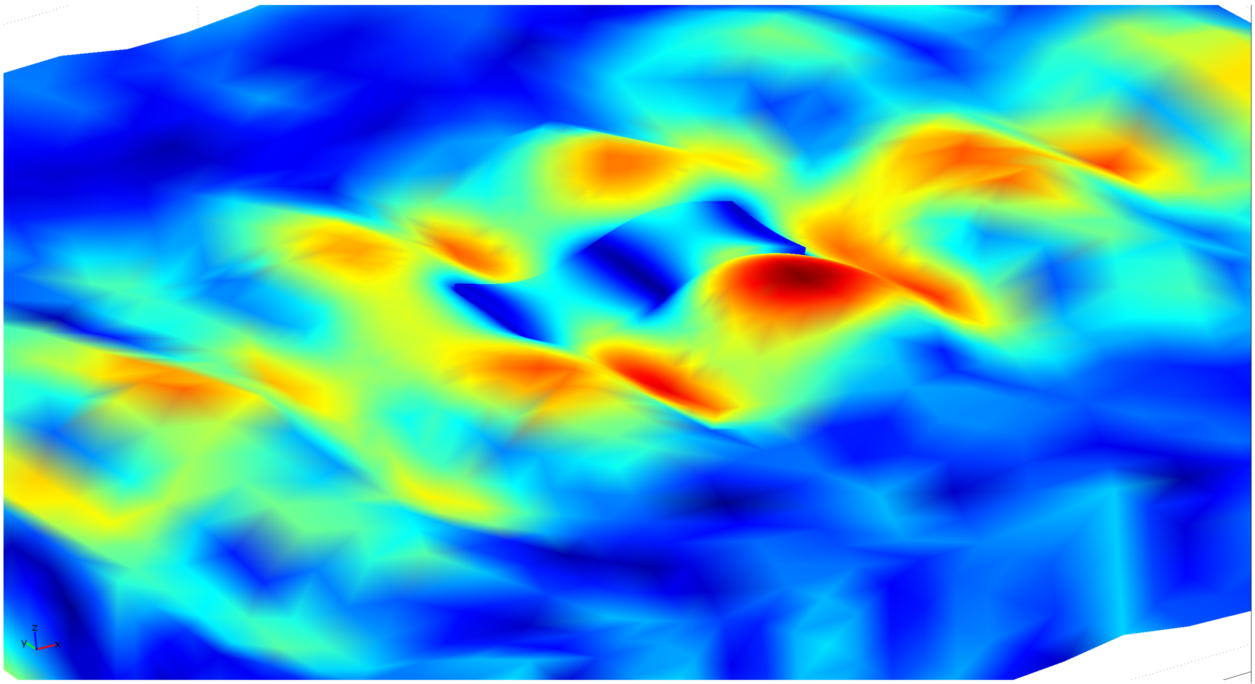

Figure 5: A vibrational mode of the end station slab and underlying soil with an especially high gravity coefficient (6.0×10^{-7} s^{-2}), predicted by COMSOL at 11 Hz. Most of the image shows the soil; the slab is the small rectangular object in the center that has been distorted into an S shape. It is about 20 meters long and 30 inches (76 cm) thick. The soil was modeled as a square block 100 meters on a side and 10 meters thick, with periodic boundary conditions on the vertical faces.

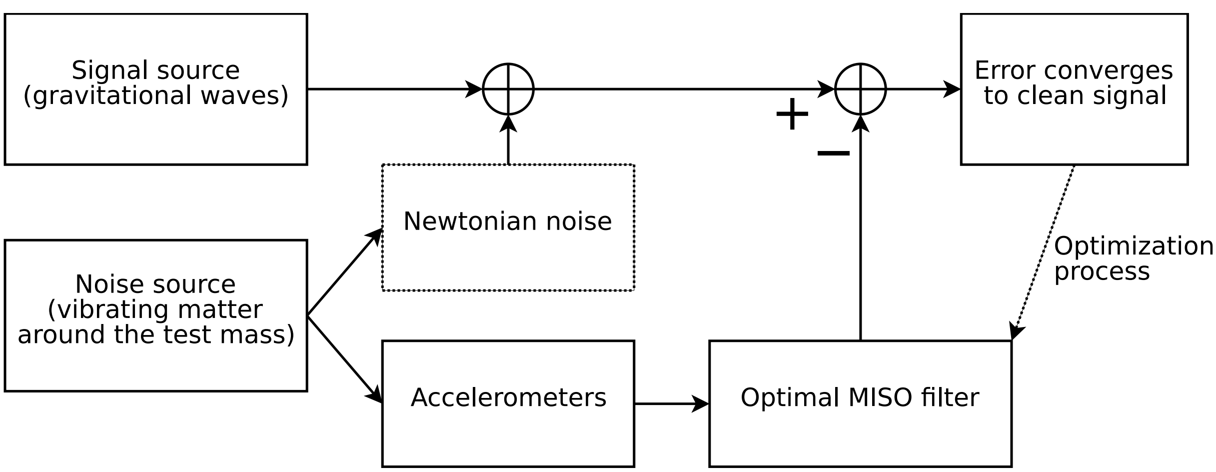

Figure 6: Block diagram showing the proposed active noise cancellation scheme for Newtonian noise in Advanced LIGO.

There are two distinct approaches to creating such a filter, depending on whether the noise is modeled as a statistically stationary process (one whose parameters are constant in time), or a non-stationary process (one that changes properties as its statistical parameters change). For a stationary process, the optimal filter is constant in time, so it need only be optimized once and then used without change after that (at least as long as the interferometer and accelerometers are in the same configuration). For a non-stationary process, the optimal filter changes as the statistical parameters change, so it must be continually adapted as more data becomes available.

For the first case of a static filter, the optimal filter satisfies the Wiener-Hopf equations (see Section 5.2), and is known as a Wiener filter. The simplest kind of Wiener filter to implement is a causal FIR (Finite Impulse Response) Wiener filter, which is simply a vector of real numbers that represent an impulse response, and which are convolved with the input signal in the time domain to obtain the optimal estimate of the noise. Causal and non-causal IIR (Infinite Impulse Response) Wiener filters also exist, but the non-causal filter is impossible to implement in real time, and the causal filter is more mathematically complicated and difficult to design.[7]

In contrast to static Wiener filters, there are literally dozens of adaptive filter algorithms.[9] This variety is a result of the difficulty of designing an algorithm that on the one hand adapts quickly to sudden changes in input parameters, but on the other hand converges reliably to the optimal filter when the input is temporarily stationary. Adaptive IIR filters, in particular, are prone to slow convergence and instability, and an effective algorithm can be "a tightly guarded trade secret" for the company that developed it.[10]

It is important to note that neither of these methods requires any *a priori* knowledge of the physical transfer function relating the input to the desired filter output. They are completely model-independent. To illustrate this point, the impulse responses in Figures 9, 10, and 11 do not come from any model of the suspensions; they are derived solely from the input and output data via the filter optimization process.

3.2 Application to seismic noise at the 40-meter lab

To evaluate the applicability of Wiener filters and adaptive filters to active noise cancellation in gravitational-wave interferometers, it was decided to implement seismic noise cancellation at the 40-meter prototype lab. The programming environment MATLAB, which is in wide use in the LIGO community, has built-in functions to create FIR Wiener filters and several kinds of adaptive filters, but they are all single-input single-output (SISO) filters. The signal from a single one-axis accelerometer is not enough to predict either Newtonian noise or seismic noise accurately enough to subtract it effectively, so these built-in functions were insufficient. Furthermore, although a SIMO filter with n outputs is equivalent to n independent SISO filters,[8] a MISO filter is not equivalent to any collection of independent SISO filters, so the built-in SISO algorithms could not be used as building blocks to create MISO filters. Instead, a function to create MISO FIR Wiener filters was written from scratch,

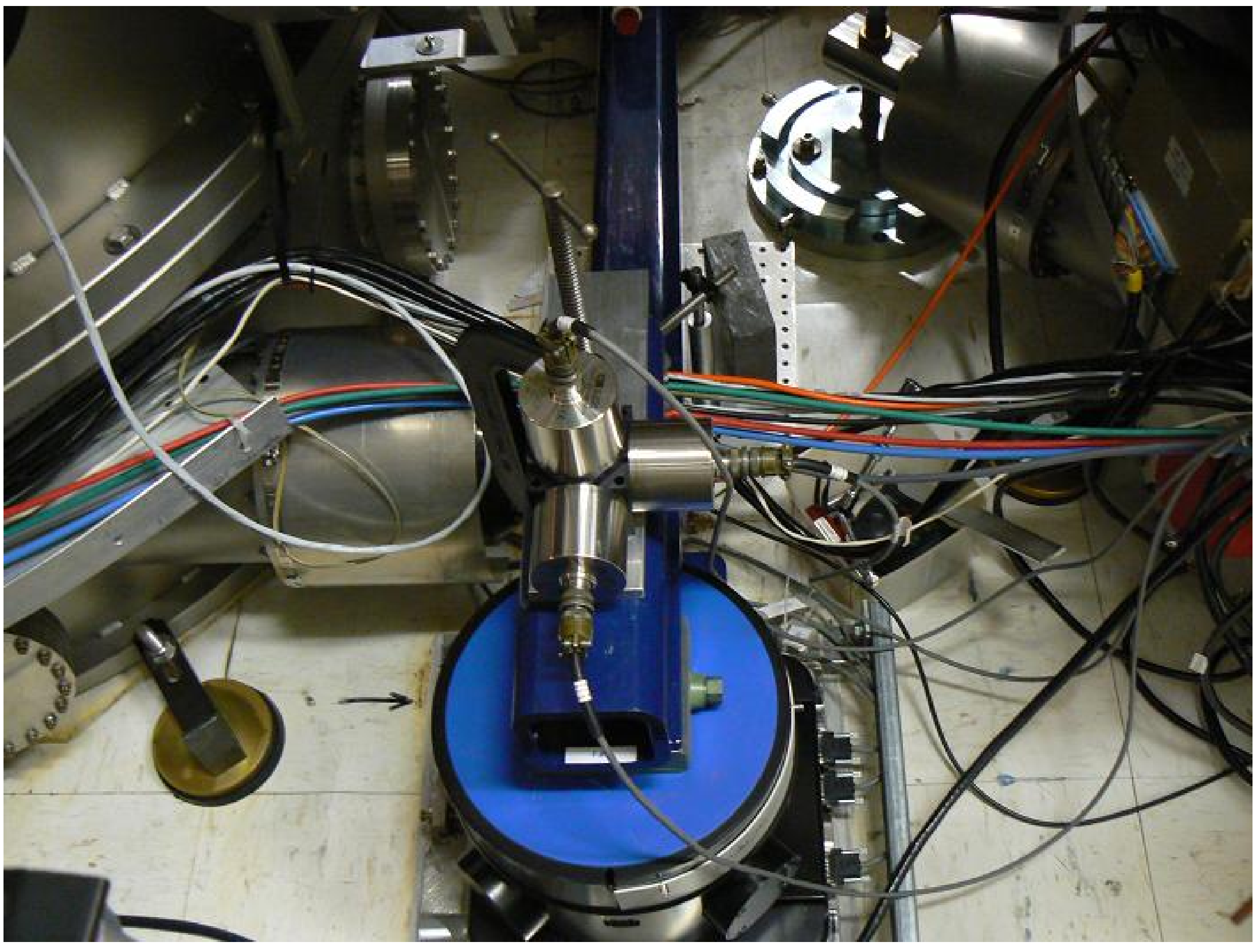

Figure 7: A photo of the accelerometer setup. The three steel cylinders are the accelerometers.

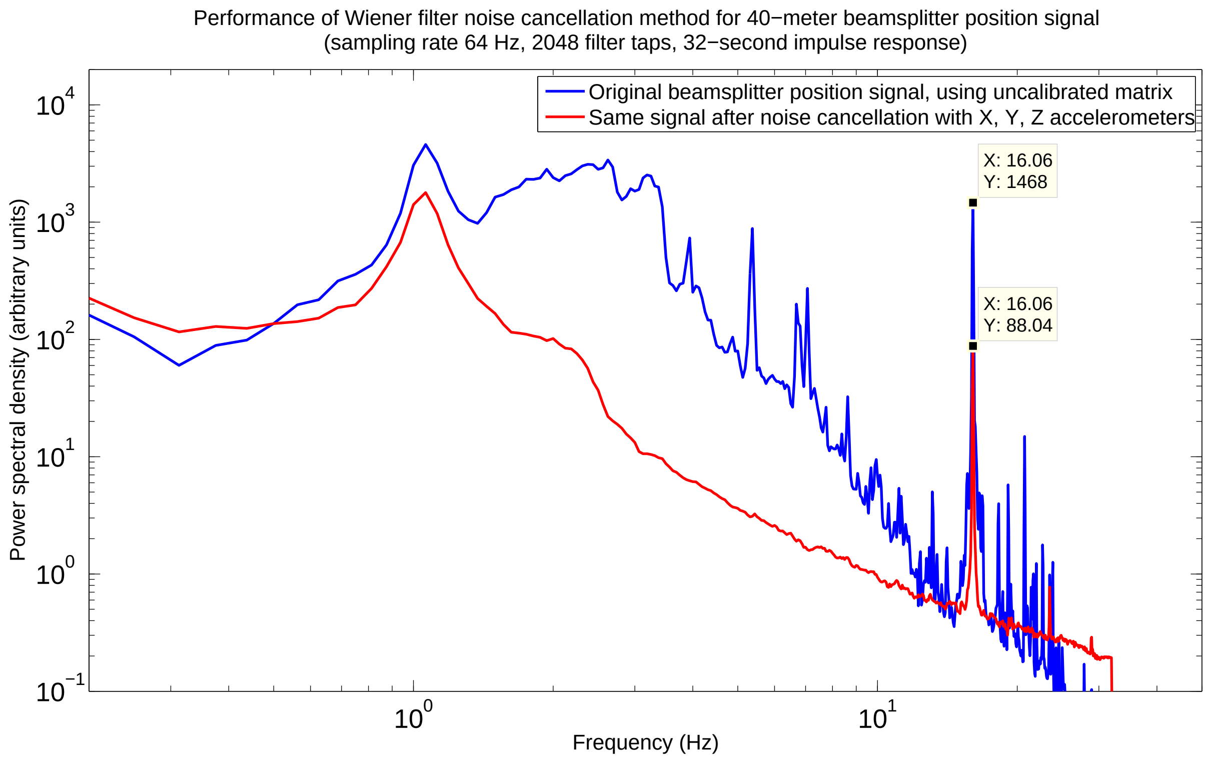

Figure 8: Effectiveness of a MISO FIR Wiener filter for seismic noise reduction at the 40-meter lab. The upper (blue in color) trace is the motion of the optic before noise cancellation and the lower (red) trace is the same signal after noise cancellation. The peak at 1 Hz is the pendulum mode (caused by the optic swinging back and forth as the suspension wires flex at their attachment points) and the spike at 16 Hz is the bounce mode (caused by the optic bouncing up and down as the wires stretch and relax).

based on the algorithm in [8] (see Section 5.3. The filters it produces agree with those produced by the built-in function in the single-input case, which provides a valuable check on its correctness.

This function was then applied to representative data from three accelerometers mounted mutually perpendicularly onto a support of the beam splitter chamber (known as the x, y, and z accelerometers), and the position signal from the optical sensors on the beam splitter inside. The result of subtracting the filter output from the position signal is shown in Figure 8. To eliminate the possibility of overfitting, the filter was optimized using one set of data and then applied to a distinct set of data to evaluate its performance. The noise reduction is clear. The noise at the 1 Hz pendulum mode is reduced by a factor of 2.5 and that at the 16 Hz bounce mode by a factor of 16, but the filter shows its best performance at the intermediate frequencies from 1.5–10 Hz, where the physical impulse response of the suspension is short. The noise at these frequencies is reduced by factors of up to 100.

Below 0.5 Hz and above 16 Hz, the noise is actually increased. This is because there is not enough coherence at those frequencies for effective noise cancellation, and although the noise approximation filter could remove those frequencies (resulting in no change to the

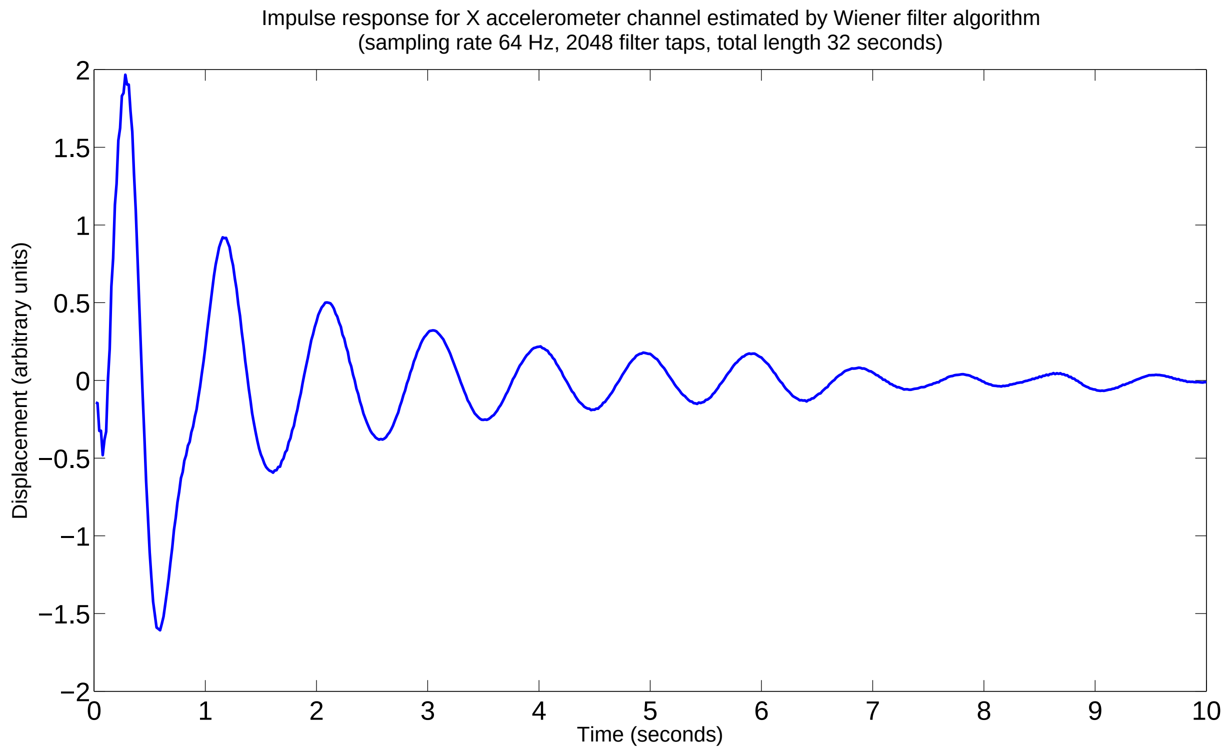

Figure 9: Response of the beam splitter position to an impulse in the X direction, as estimated by the MISO FIR Wiener filter. There are some transients at the beginning and then the appearance is dominated by the 1 Hz pendulum mode. The 16 Hz bounce mode is invisible because its amplitude is so much smaller.

original noise spectrum), it is designed to optimize the RMS power of the whole spectrum, so frequencies where there is much noise are improved at the expense of frequencies where there is little noise. In practice, this is easily dealt with by identifying the frequency band in which the noise cancellation is effective, and then filtering the estimate of the noise with a simple band-pass filter to avoid worsening the noise outside that band.

The actual coefficients of the filter, which form the estimated impulse responses, are shown in Figures 9, 10, and 11. The forms of these impulse responses seem physically reasonable, which provides another verification that the filter is doing what it should.

3.3 Possible improvements

Although already effective at reducing seismic noise, the noise cancellation method presented here is still inefficient in several important ways. First of all, the FIR Wiener filter algorithm requires solving a linear equation involving a large matrix of a special form known as "symmetric block Toeplitz". In the current implementation, this equation is solved using the generic MATLAB division operator ('/'), which does not take advantage of the special form of the matrix. This is a serious limiting factor in the computation of the filter co-

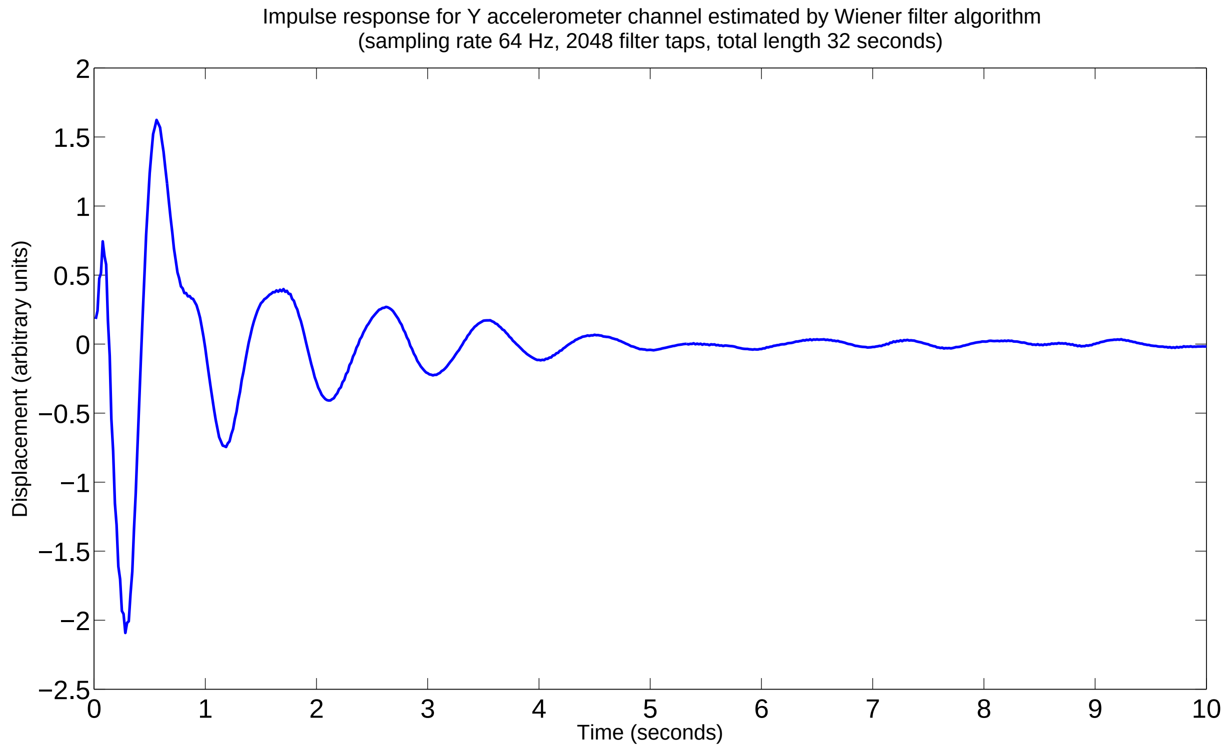

Figure 10: The response to an impulse in the Y direction is similar to that in the X direction because the beam splitter is suspended diagonally.

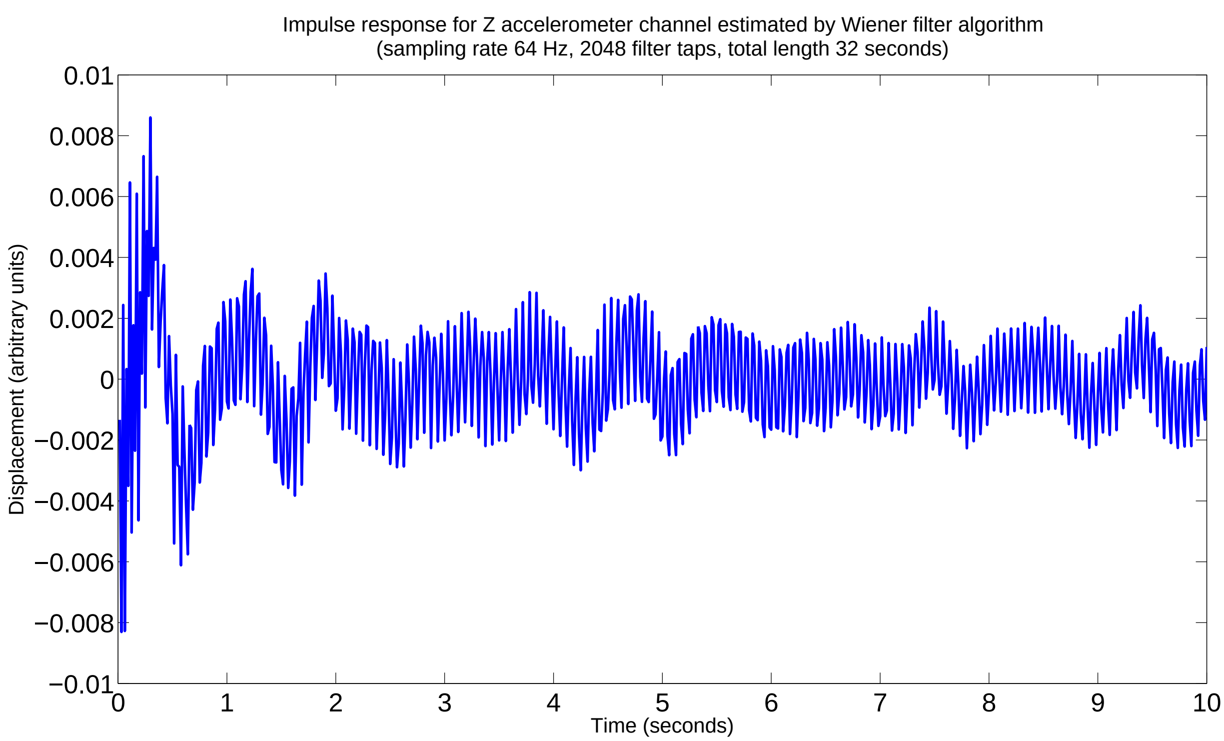

Figure 11: The response to a vertical impulse is much smaller because the pendulum mode is excited less. At this scale, the bounce mode is visible.

efficients. For example, the length-2048 Wiener filter shown here required more than one gigabyte of memory to compute. Computations involving much larger numbers n of input channels would be intractable with this naive method, because the size of the matrix grows as n^2. If an efficient algorithm designed specifically for solving Toeplitz matrices, such as Levinson-Durbin recursion,[7] were used instead, it would drastically reduce these computing requirements and make it easy to compute long filters with many input channels.

Second of all, no matter how long an FIR filter is, there will always be a "tail" of the impulse response that is not modeled, which limits the performance of the filter. It would be a good idea to implement a causal IIR Wiener filter and compare its performance to that of the FIR filter described here.

Lastly, seismic noise is not a perfectly stationary process. Therefore the Wiener filters created from different data, for example at different times of day, should be compared, and it should be determined if they are different enough that an adaptive filter is necessary. An interesting intermediate alternative is a filter that acts like a static Wiener filter most of the time, but periodically updates its coefficients so that it can never stray very far from optimality.

4 Summary of results

Worst-case models and realistic finite-element simulations demonstrate that vibrations of the vacuum chamber and support structures make only a minor contribution to Newtonian noise. The vibrations of the concrete foundation slab and underlying soil are more significant.

A proof-of-concept noise cancellation method incorporating an optimal filter was successfully implemented at the 40-meter prototype lab. Although applied to ordinary seismic noise rather than Newtonian noise, there is no reason to suspect that the same method would not work in both cases, because it does not rely on shielding or passive isolation.

Ideas for future research include injecting realistic noise spectra into the finite-element model and using virtual accelerometers to evaluate different strategies (number and placement of accelerometers), and comparing the performance of different noise estimation filters such as IIR Wiener filters and adaptive filters.

5 Methods

5.1 Derivation of Newtonian noise formula

The gravitational field at the origin due to a point mass m at position $\mathbf{p} = (x, y, z)$ is

$$\frac{Gm\hat{\mathbf{p}}}{|\mathbf{p}|^2} = \frac{Gm\mathbf{p}}{|\mathbf{p}|^3},$$

so the component along the x axis is

$$\frac{Gmx}{(x^2 + y^2 + z^2)^{3/2}}.$$

The gradient of this expression is

$$\frac{(-2x^2 + y^2 + z^2)\,\hat{\imath} - 3xy\hat{\jmath} - 3xz\hat{k}}{(x^2 + y^2 + z^2)^{5/2}},$$

so if the point mass moves by a small vector (u,v,w), the x component of the gravitational field at the origin changes by the scalar product of (u,v,w) with the gradient:

$$\frac{(-2x^2 + y^2 + z^2)\,u - 3xyv - 3xzw}{(x^2 + y^2 + z^2)^{5/2}}$$

This is the expression that should be integrated over the object and divided by its average displacement to obtain the gravity coefficient.

5.2 Theory of the MISO FIR Wiener filter

The optimal filter satisfies the Wiener-Hopf equation,

$$\mathbf{R}\mathbf{w} = \mathbf{p},$$

where $\mathbf{R}$ is the autocorrelation matrix of the input, $\mathbf{w}$ is the impulse response of the optimal filter, and $\mathbf{p}$ is the cross-correlation vector between the input and the desired output. In this equation, $\mathbf{R}$ is a symmetric[1] Toeplitz matrix, which means it is of the form

$$\begin{pmatrix} r_1 & r_2 & r_3 & \cdots \\ r_2 & r_1 & r_2 & \\ r_3 & r_2 & r_1 & \\ \vdots & & & \ddots \end{pmatrix},$$

and therefore completely determined by the first row or column. Multiplication by such a matrix is equivalent to a convolution operation.

[1] Hermitian for complex signals

5.3 MATLAB implementation of MISO FIR Wiener filter

```
function [W,R,P] = miso_firwiener(N,X,y)
%MISO_FIRWIENER Optimal FIR Wiener filter for multiple inputs.
%   MISO_FIRWIENER(N,X,Y) computes the optimal FIR Wiener filter of order
%   N, given any number of (stationary) random input signals as the columns
%   of matrix X, and one output signal in column vector Y.

%   Author: Keenan Pepper
%   Last modified: 2007/08/02

%   References:
%     [1] Y. Huang, J. Benesty, and J. Chen, Acoustic MIMO Signal
%     Processing, Springer-Verlag, 2006, page 48

% Number of input channels.
M = size(X,2);

% Input covariance matrix.
R = zeros(M*(N+1),M*(N+1));
for m = 1:M
    for i = m:M
        rmi = xcorr(X(:,m),X(:,i),N);
        Rmi = toeplitz(flipud(rmi(1:N+1)),rmi(N+1:2*N+1));
        top = (m-1)*(N+1)+1;
        bottom = m*(N+1);
        left = (i-1)*(N+1)+1;
        right = i*(N+1);
        R(top:bottom,left:right) = Rmi;
        if i ~= m
            R(left:right,top:bottom) = Rmi';  % Take advantage of hermiticity.
        end
    end
end

% Cross-correlation vector.
P = zeros(1,M*(N+1));
for i = 1:M
    top = (i-1)*(N+1)+1;
    bottom = i*(N+1);
    p = xcorr(y,X(:,i),N);
    P(top:bottom) = p(N+1:2*N+1)';
end

% The following step is very inefficient because it fails to exploit the
% block Toeplitz structure of R. It's done the same way in the built-in
% function "firwiener".
W = P/R;
```

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