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Date:	5 Jun 2008	Refer to:	T080003-04
Subject:	Thermal Noise Increase due to a Gold Coated Barrel		
To:	aligo_coc, aligo_aos, aligo_sys		
From:	Dennis Coyne, Phil Willems		

Version history:

-00	*limited release*
-01	*Initial distributed release*
-02	*Corrections to the remove the erroneous strain energy plots (see explanation in text)*
-03	*Correction for beam waist. Used $\sqrt{2}\, r_0$ instead of r_0*
-04	*Included Comsol results*

Phil Willems has proposed the addition of a gold coating on the barrel of the test masses and the compensation plates in order to reduce the emissivity and reduce the radial temperature gradients due to radiation from the barrel[1,2]. Since gold is a lossy material it may also help to passively damp higher order acoustic modes which might otherwise be excited into oscillation by higher order optical cavity modes. However, there is concern that the lossy gold coating might lead to an unacceptably large increase in Brownian thermal noise in the optical readout. The purpose of this technical note is to report on a finite element analysis used to calculate the increase in thermal noise (using Levin's approach[3]) due to the addition of a gold coating.

An axisymmetric finite element model of the Advanced LIGO test mass was developed, as depicted in Figure 1. For simplicity (and to enable an axisymmetric model) a perfect right circular cylinder was assumed; The following geometric features were not included: wedge angle, bevels, flats and bonded "ears" (used to weld the fused silica suspension ribbons). In addition, the gold layer will need a protective SiO_2 overcoat layer and may need an interstitial layer to bond it to the substrate (e.g. Ni). Neither of these layers are included. The $Si\ O_2$ protective layer should be low loss and similar to the substrate. The interstitial layer may be exceedingly thin and inconsequential.

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[1] P. Willems, "Heating of the ITM by the Compensation Plate in Advanced LIGO", LIGO-T070123-02, 3Dec2007.

[2] Note that the gold coating is not part of the baseline AOS or COC design as yet. This is just an memo on the feasibility of this proposed coating from the thermal noise perspective.

[3] Y. Levin, "Internal thermal noise in the LIGO test masses: A direct approach", Phy Rev D, v57, n2, 15Jan98, p659-663.

The elements approximating the fused silica material of the test mass substrate are comprised of parabolic quadrilateral axisymmetric solid elements. The High Reflectance (HR) and barrel coatings are comprised of axisymmtric shell elements of appropriate thickness. The mesh was refined to insure convergence. The parameters and property values used in the analysis are listed in Table 1. Similar models and analysis of thermal noise were performed and reported previously[4]. The agreement with equations from Liu & Thorne's analysis[5] (to 3 decimal places) plus the reasonableness of the stress contours gives one confidence in the results. However, it seems that the I-Deas finite element code[6] does <u>not</u> use the proper calculation when displaying strain energy[7]. As a consequence I have removed all of the strain energy plots from this version. I also think that a further check of the strain energy integrals (sums) is warranted.[8]

In lieu of strain energy plots, the stress contours are depicted in Figures 2 through 5. The non-zero stresses are σ_r, σ_θ, σ_z and σ_{rz} where r is radial, z is along the symmetry axis (cylinder axis) and θ is the angle about the z axis. The strain energy density is

$$\zeta = \{\sigma\}^T \{\varepsilon\}$$

since the stress is linearly related to the strain by the Young's modulus,

$$\{\sigma\} = [E] \{\varepsilon\}$$

The strain energy is proportional to the sums of squares of the stresses

$$\zeta \sim (\sigma_r)^2 + (\sigma_\theta)^2 + (\sigma_z)^2 + (\sigma_{rz})^2$$

The strain integrals (summations of total element strain) for the substrate, HR coating and barrel coating domains are listed in Table 2. The displacement noise (and strain), at 100 Hz, due to the Brownian motion are also compared in Table 2. The total strain noise (at 100 Hz) compares well with the Bench 6.2 calculation (3e-24 1/√Hz). The gold barrel coating only increases the strain noise ASD by 1%. This is due to the fact that the Brownian thermal noise is dominated by the HR coating (3e-24 HR, 0.3e-24 substrate, 0.5e-24 gold). Even if new HR coating formulations result in significantly lower loss factor, the HR coating is still likely to dominate over the gold barrel coating contribution.

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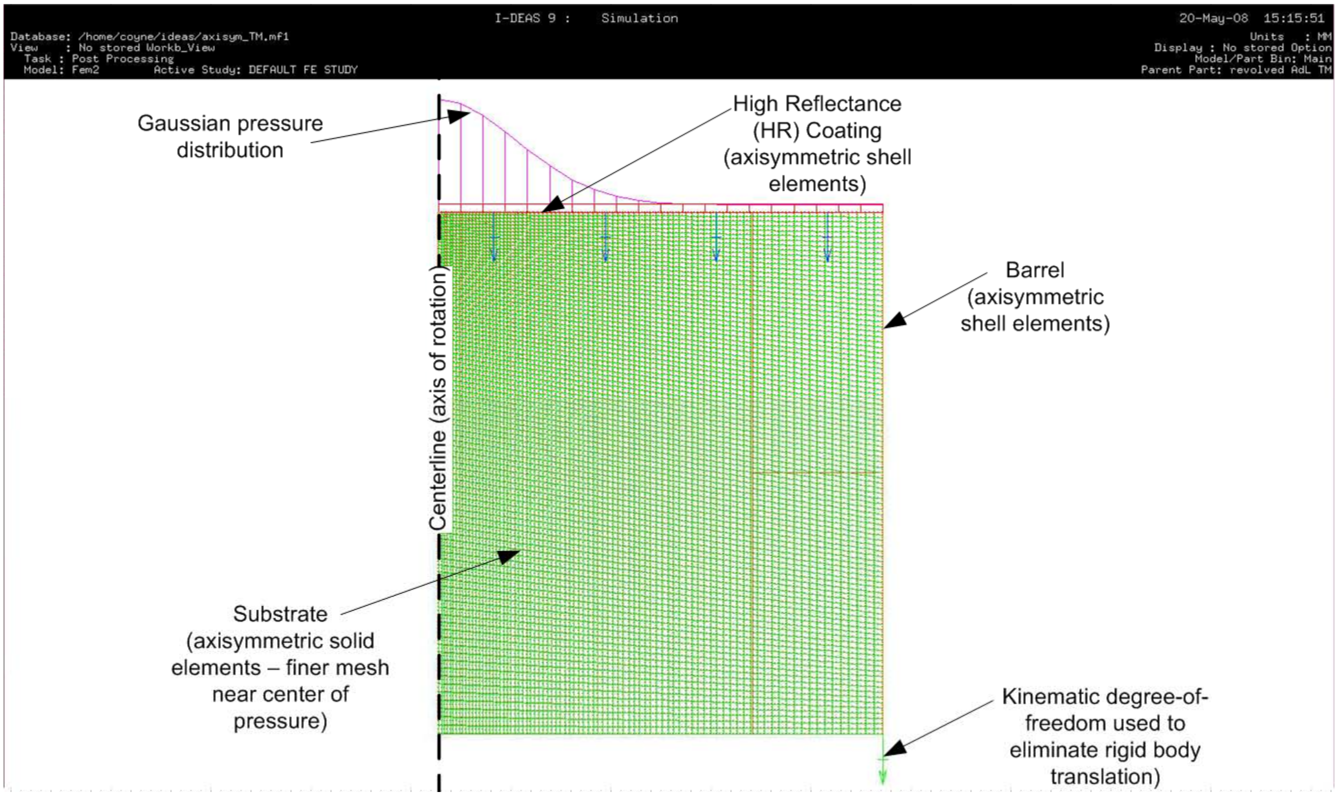
[4] D. Coyne, Thermal Noise Calculation with Inhomogeneous Loss using the Finite Element Method: Application to Test Mass Optics with Coating Loss, Attachments and Composite Assemblies, LIGO-T020070-01, 11 Jul 2002.

[5] Y. Liu and K. Thorne, "Thermoelastic noise and homogeneous thermal noise in finite sized gravitational-wave test masses", Physical Review D, vol. 62, 20 Nov 2000.

[6] I-Deas version 9, Structural Dynamics Research Corp.

[7] I think it was Andri Gretarsson who first questioned why the strain energy was so low at the center of the beam in LIGO-T020070. Riccardo DeSalvo noticed the same oddity in the -01 version of this memo.

[8] A check can be performed with Liu & Thorne's formulation (I've started this, but don't have time now to complete it), or with another finite element code.



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  Model: Fem2         Active Study: DEFAULT FE STUDY                   Parent Part: revolved AdL TM
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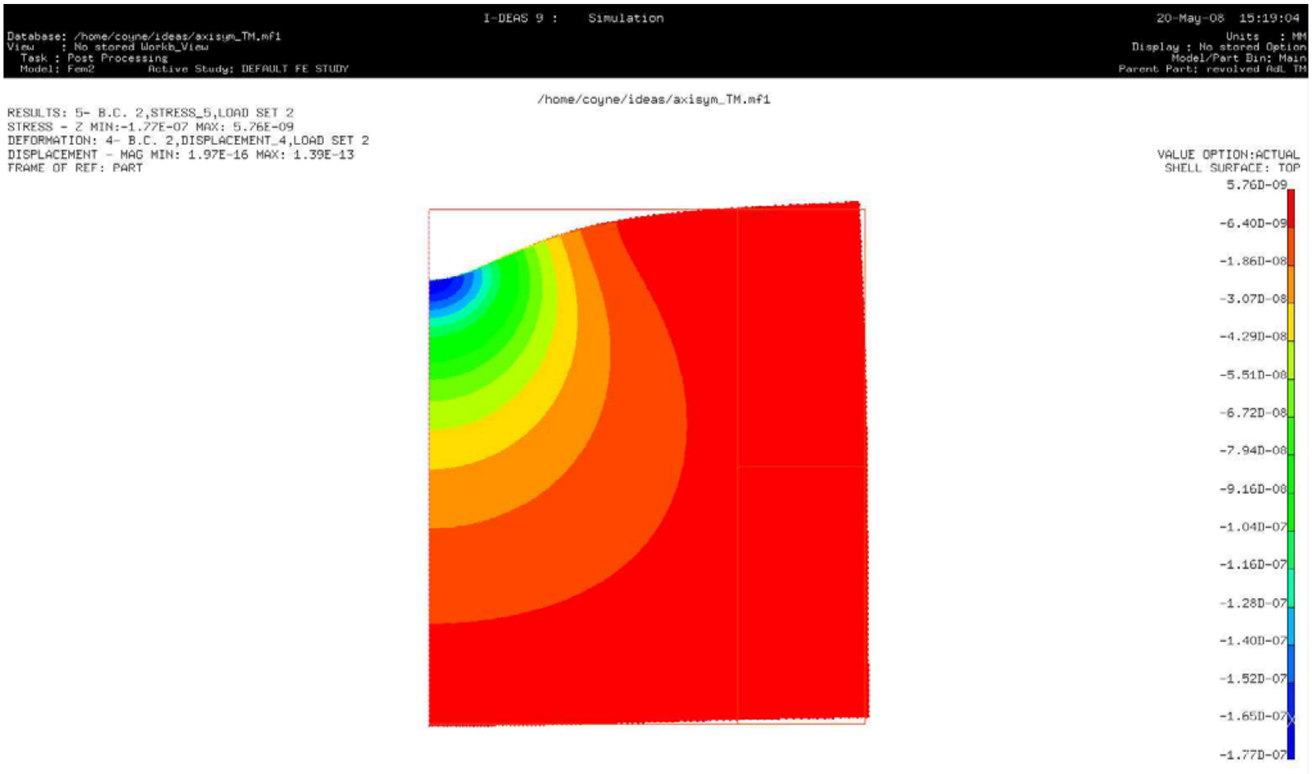
Figure 1: finite element model



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  Model: Fem2         Active Study: DEFAULT FE STUDY                   Parent Part: revolved AdL TM

                              /home/coyne/ideas/axisym_TM.mf1
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STRESS - Z MIN:-1.77E-07 MAX: 5.76E-09
DEFORMATION: 4- B.C. 2,DISPLACEMENT_4,LOAD SET 2
DISPLACEMENT - MAG MIN: 1.97E-16 MAX: 1.39E-13
FRAME OF REF: PART
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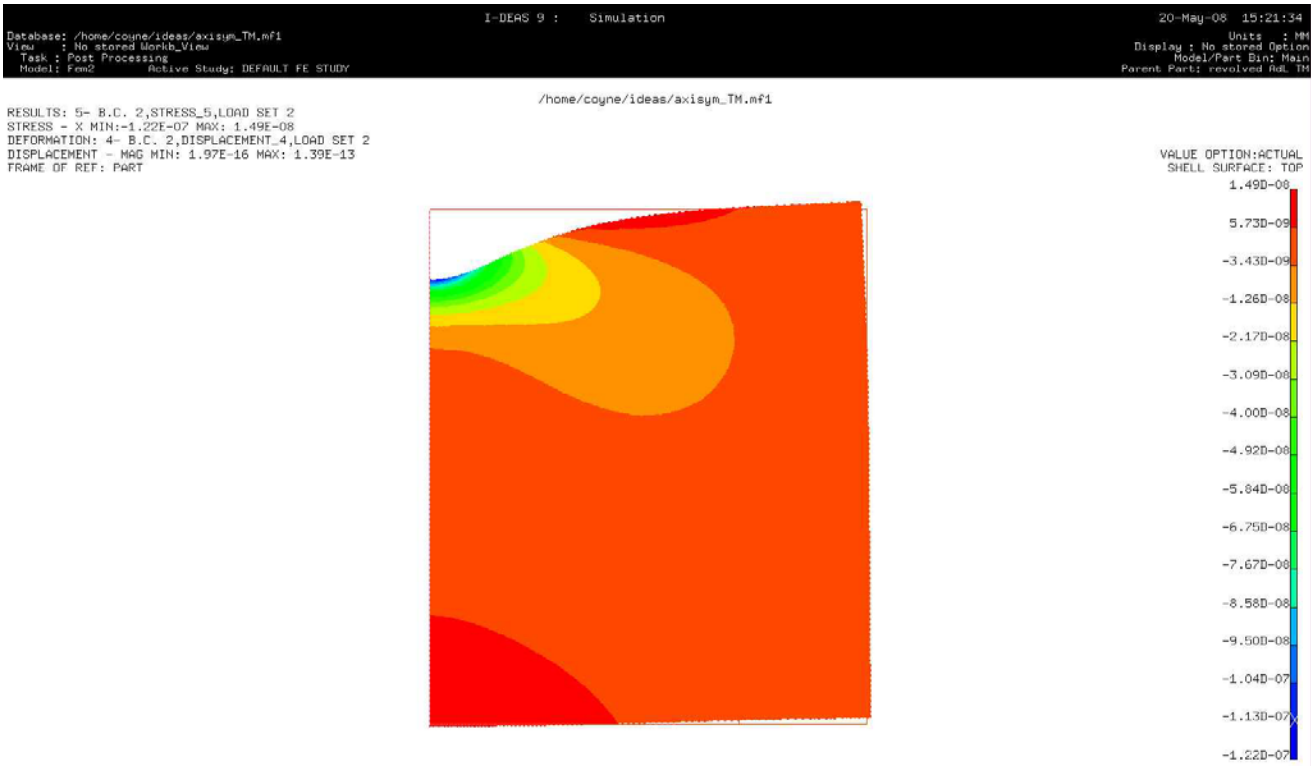
Figure 2: Stress σ_z Contours for a 1 microN Gaussian force (linear contour scale, 0 to 100%)



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  Model: Fem2        Active Study: DEFAULT FE STUDY      Parent Part: revolved AdL TM

                         /home/coyne/ideas/axisym_TM.mf1
RESULTS: 5- B.C. 2,STRESS_5,LOAD SET 2
STRESS - X MIN:-1.22E-07 MAX: 1.49E-08
DEFORMATION: 4- B.C. 2,DISPLACEMENT_4,LOAD SET 2
DISPLACEMENT - MAG MIN: 1.97E-16 MAX: 1.39E-13
FRAME OF REF: PART
                                                               VALUE OPTION:ACTUAL
                                                               SHELL SURFACE: TOP
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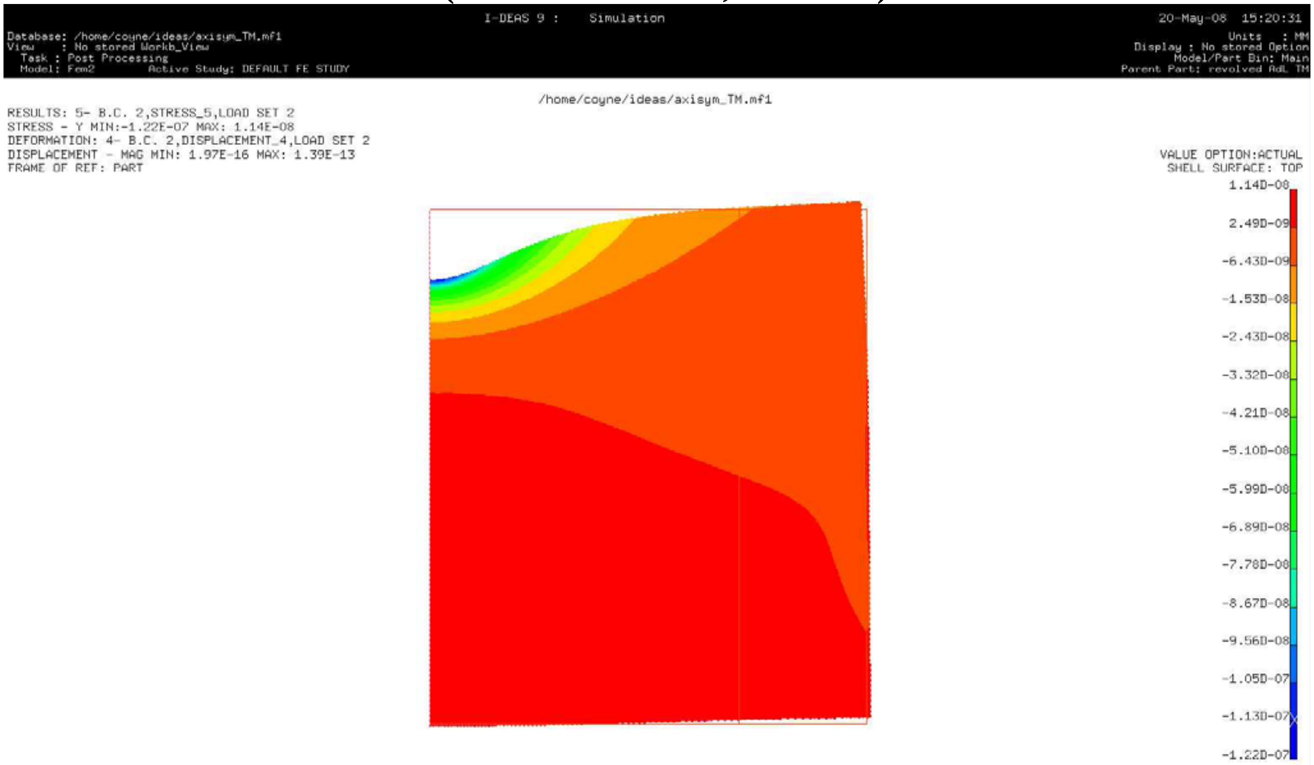
Figure 3: Stress σ_r Contours for a 1 microN Gaussian force (linear contour scale, 0 to 100%)



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View    : No stored Workb_View                           Display : No stored Option
  Task : Post Processing                                      Model/Part Bin: Main
  Model: Fem2        Active Study: DEFAULT FE STUDY      Parent Part: revolved AdL TM

                         /home/coyne/ideas/axisym_TM.mf1
RESULTS: 5- B.C. 2,STRESS_5,LOAD SET 2
STRESS - Y MIN:-1.22E-07 MAX: 1.14E-08
DEFORMATION: 4- B.C. 2,DISPLACEMENT_4,LOAD SET 2
DISPLACEMENT - MAG MIN: 1.97E-16 MAX: 1.39E-13
FRAME OF REF: PART
                                                               VALUE OPTION:ACTUAL
                                                               SHELL SURFACE: TOP
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Figure 4: Stress σ_θ Contours for a 1 microN Gaussian force (linear contour scale, 0 to 100%)

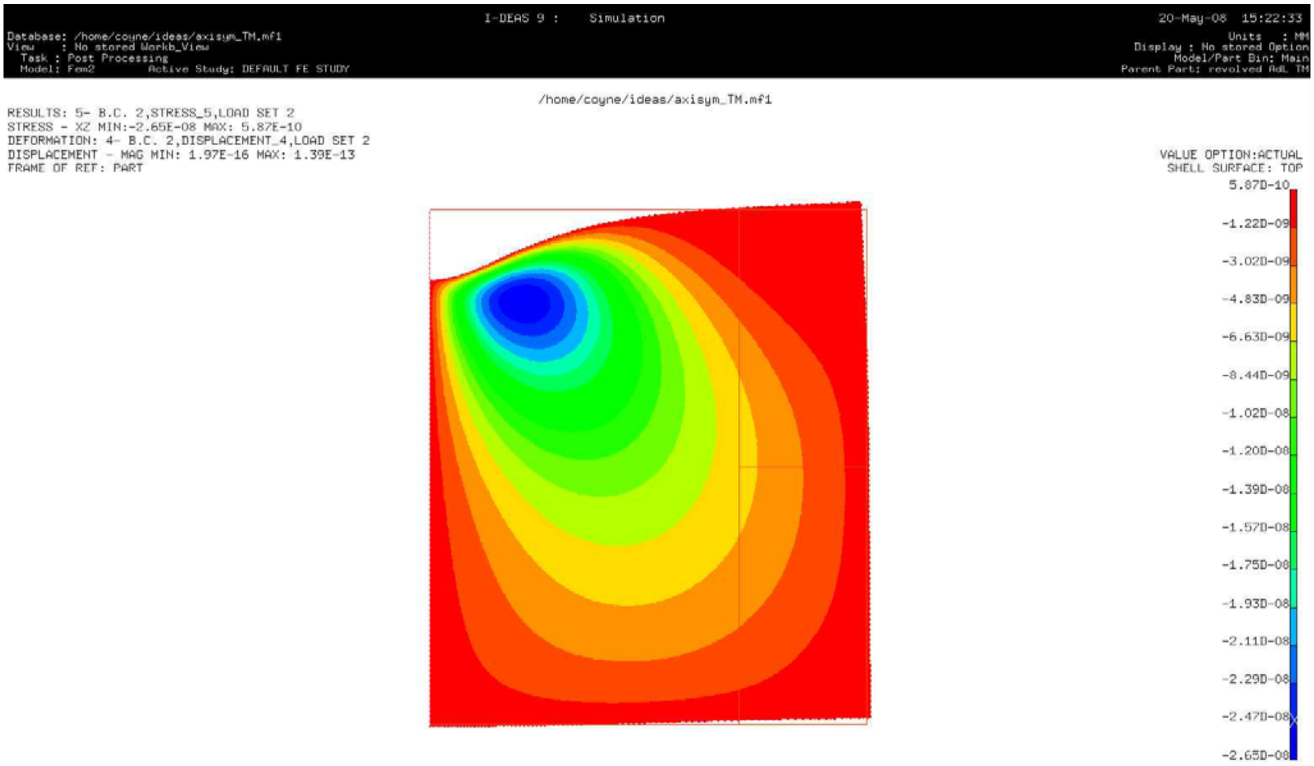

Figure 5: Stress σ_{rz} Contours for a 1 microN Gaussian force (linear contour scale, 0 to 100%)

Table 1: Parameters

Symbol	Parameter	Value	Units	Source
a	TM radius	0.170	m	M050397-02
H	TM thickness	0.200	m	Ibid
r_0	Gaussian beam radius	0.060	m	Bench 6.2
E_{FS}	TM elastic modulus	71.8	GPa	Fused Silica; Bench 6.2 uses 72.7GPa
ν_{FS}	TM Poisson's ratio	0.16	-	Bench 6.2 uses 0.167
φ_{FS}	TM bulk loss factor	2.6×10^{-10} at 100 Hz	-	Fused Silica: from Bench 6.2: $\varphi = c_2 f^n$, $c_2 = 7.6\text{e-}12$ n = 0.77, f = frequency (Hz)
t_{HR}	High Reflectance dielectric coating thickness	10	μm	~40 layers @ $\lambda/4$
E_{HR}	HR coating elastic modulus	106.5	GPa	Ta_2O_5/SiO_2: T970176-00 Bench 6.2 uses ~94 GPa
ν_{HR}	HR coating Poission's ratio	0.21	-	Ta_2O_5/SiO_2: T970176-00 Bench 6.2 uses 0.2038
φ_{HR}	HR coating loss factor	3.38×10^{-4}	-	Eqn[23], G. Harry et. al., CQG, 19(2002), with effective coating elastic modulus, Y'=94 GPa, substrate modulus, Y=72.7 GPa, $\varphi_\perp$ = 1.36e-4 and $\varphi_\parallel$ = 1.79e-4
t_{Au}	gold barrel coating thickness	100	nm	Phil Willems estimate 11-dec-2007
E_{Au}	gold barrel coating elastic modulus	80	GPa	H. Anderson (ed.), AIP Physics Vade Mecum, 1981
ν_{Au}	gold barrel coating Poission's ratio	0.42	-	Ibid
φ_{Au}	gold barrel coating loss factor	9×10^{-3}	-	recent unpublished measurement by Andri Gretarsson

Table 2: Strain Energy and Displacement Noise Results

Symbol	Parameter	Value	Units
U_{FS}	Total substrate strain energy normalized by the force amplitude squared	4.33e-11	m^2/J
U_{HR}	Total HR strain energy normalized by the force amplitude squared	5.35e-15	m^2/J
U_{Au}	Total Au barrel coating strain energy normalized by the force amplitude squared	2.40e-18	m^2/J
$S_{x,FS}$(f=100Hz)	Displacement noise PSD contribution from the substrate	5.82e-43	m^2/Hz
$S_{x,HR}$(f=100Hz)	Displacement noise PSD contribution from the HR coating	9.21e-41	m^2/Hz
$S_{x,Au}$(f=100Hz)	Displacement noise PSD contribution from the Au barrel coating	1.10e-42	m^2/Hz
$\sqrt{S_{x,}}$(f=100Hz)	Displacement noise ASD: Total	9.68e-21	m/√Hz
	Displacement noise ASD: without Au barrel coating	9.63e-21	m/√Hz
	Displacement noise ASD: substrate only	7.63e-22	m/√Hz
	Displacement noise ASD: without HR coating	1.30e-21	m/√Hz
h(f=100Hz)	Strain noise ASD: Total	4.84e-24	1/√Hz
	Strain noise ASD: without Au barrel coating	4.81e-24	1/√Hz
	Strain noise ASD: substrate only	3.81e-25	1/√Hz
	Strain noise ASD: without HR coating	6.49e-25	1/√Hz

A similar model to the one just described was implemented in Comsol. Again, the mirror model was a 2-D axisymmetric right circular cylinder without bevels, flats, bonded attachments, or wedges. In contrast to the I-DEAS model, the Comsol model did not include shell elements to individually model the barrel coating. Rather, the mirror was treated as having the Young's modulus, density, and Poisson's ratio of fused silica everywhere, including within the barrel coating. Gold has similar Young's modulus as fused silica (78 GPa vs. 72 GPa), but rather different Poisson's ratio (.44 vs. .17). We do not expect the Poisson's ratio of a thin surface layer to play a significant role in the bulk deformation of a mirror, but it will determine the expansion of the surface layer normal to the surface. This effect was not considered. Gold is also much more dense than fused silica, but the calculations described here are well in the quasistatic regime, and the materials' inertia is irrelevant.

We used Levin's approach. The pressure on the HR surface had a Gaussian distribution with 6 cm spot size and 1 N net pressure. A countervailing force of 1 N was distributed evenly throughout the body of the mirror to ensure no net acceleration of the mass within the model. The strain energy density within the mirror and its deformed shape are plotted in Figure 6. The strain energy density at the gold coating is plotted in Figure 7.

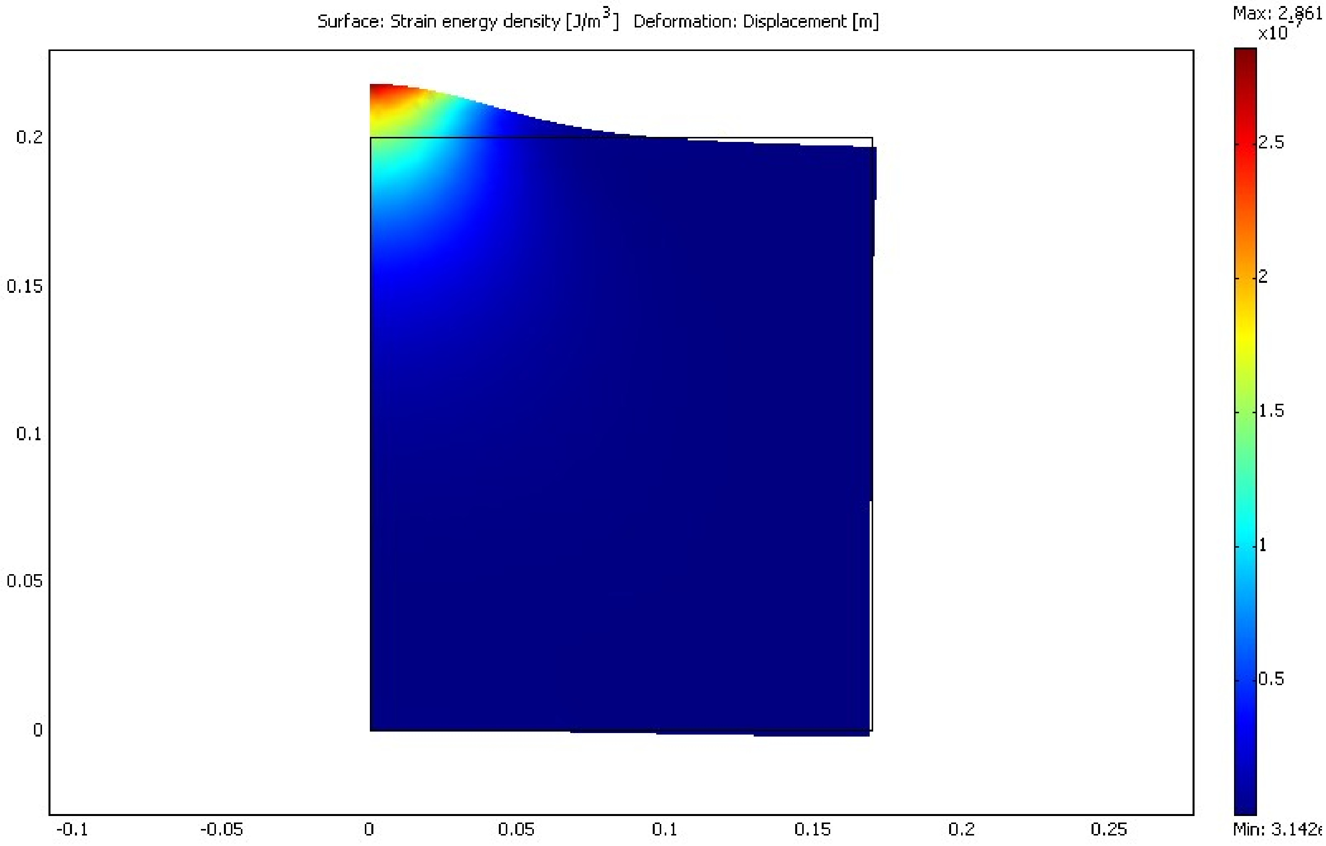

Figure 6: Strain energy density in COMSOL model

The net strain energy in the optic is found by Comsol to be 4.24e-11 J, and is in good agreement with the analytical estimate of Liu and Thorne (4.29e-11 J) and with the I-DEAS model above (4.33e-11 J, accounting for the applied force normalization). The integrated strain energy density over the barrel surface is 1.85e-17 J/µm, where we assume that the strain energy density does not change significantly within a few microns of the surface. Given the thickness of the surface layer of 0.1µm, the total barrel strain energy is 1.85e-18 J, in fair agreement with the I-DEAS model. The 23% difference may be due to the less sophisticated meshing in the COMSOL model, or to the simplifying assumption that gold has the same Poisson's ratio as fused silica in the COMSOL model. However, given the modest increase in thermal noise of the test mass noted above, the discrepancy should not be a significant source of concern.

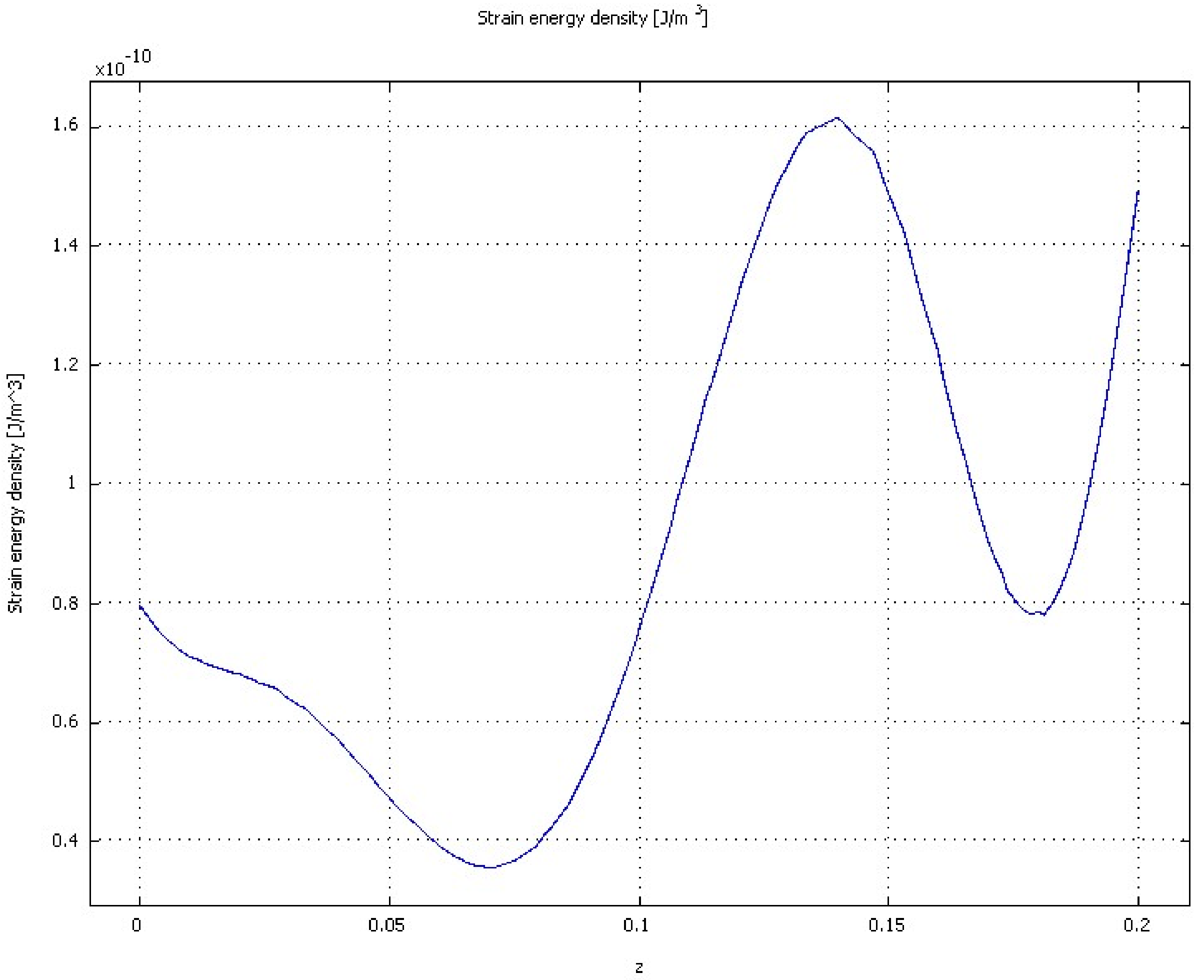

Figure 7: Strain energy density a barrel surface in the COMSOL model.