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Beam geometry in advLIGO arm with tilted mirror

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1. Introduction

This is a note to summarize the resonating beam geometry in the advanced LIGO arm with mirrors tilted. Because of the nearly concentric design of the advLIGO arm cavity, the tilt of the beam axis resonating in the arm is 13.5 times larger than the tilt angle of the mirror. Because of this characteristics, the dependence of various physical quantities on the mirror tilt angle is much stronger than the naive expectation.

Although the divergence angle of the arm is 29 μrad, the power in the arm drops by 40% with an ETM tilt of 1.5 μrad. The modal model calculation using up to TEM10 mode for the alignment signal will work up to this angle, i.e., higher order contribution is less than 10%, but an explicit correction of the mode coupling may be necessary to calculate the absolute value of the signal.

2. Geometry

The parameters used for the arm are as follows.

Arm Length = 3994.75m

ITM ROC = 1971m

ETM ROC = 2191m.

The following figure summarizes the geometrical relation.

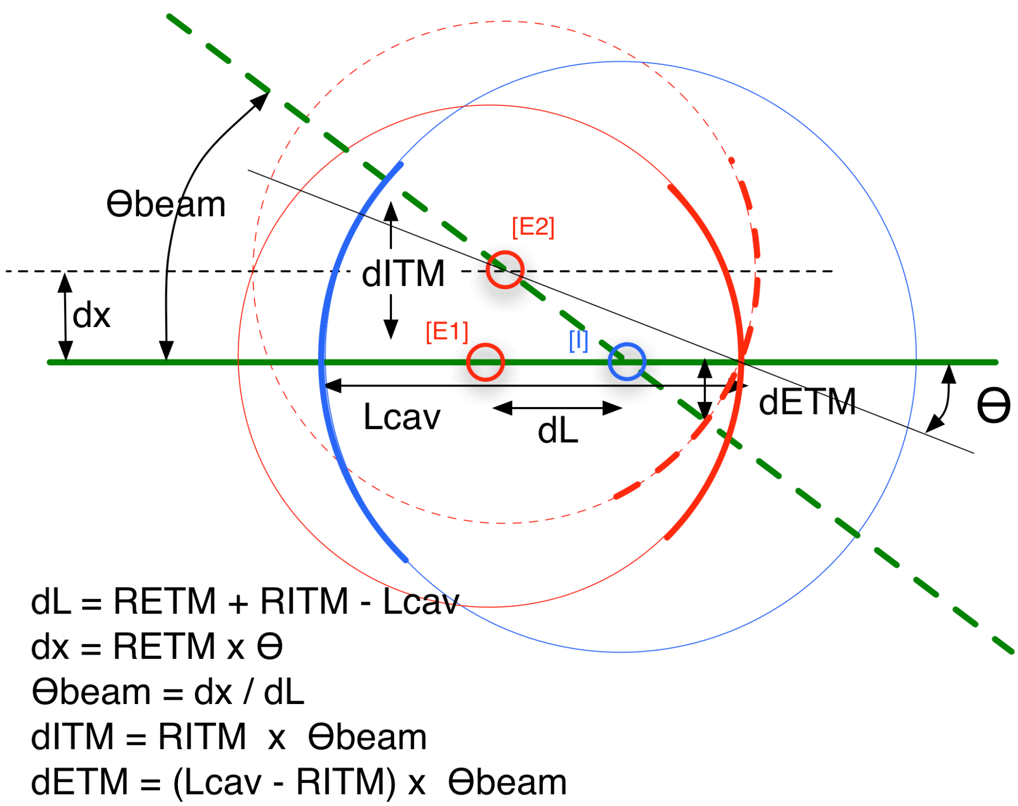

Figure 1 Geometry of mirror tilts and resonating cavity beam

When there is no misalignment of test masses, the cavity is formed by untilted ITM (solid blue) and untilted ETM (solid red), and the resonating beam axis is along the solid green line, which is the incident beam direction.

When ETM is tilted, the center of the circle of ETM moves from [E1] to [E2]. The resonating cavity beam axis of the new cavity formed by untilted ITM (solid blue) and tilted ETM (dashed red) becomes the line shown by dashed green line. The angle of the new resonating beam axis, θ_{beam}, with respect to the input beam axis is given by the ETM tilt angle, θ_{ETM}, by the following equation.

$$\theta_{beam} = \frac{R_{ETM}}{R_{ETM} + R_{ITM} - L_{cav}} \theta_{ETM} = 13.5 \cdot \theta_{ETM} \qquad (1)$$

The resonating field with the tilted ETM is defined by the same FP parameters (two ROCs and cavity length) as the original case, and the only difference is the resonating beam axis.

The power mode coupling of the incoming TEM00 mode to this new cavity is

$$\begin{aligned} P(00 \to 00, \theta) &= \exp(-(\theta_{beam} / \Theta)^2) \\ \Theta &= \frac{\lambda}{\pi w_{node}} \sim \frac{1.064\mu}{\pi \cdot 1.17cm} = 29\mu rad \end{aligned} \qquad (2)$$

When the first equation is used to express this power coupling using the ETM tilt angle, instead of the beam angle, the coupling becomes

$$\begin{aligned} P(00 \to 00, \theta) &= \exp(-(\theta_{ETM} / \Theta_{ETM})^2) \\ \Theta_{ETM} &= (\frac{\lambda}{\pi w_{node}}) / (\frac{R_{ETM}}{R_{ETM} + R_{ITM} - L_{cav}}) \sim 2.1\mu rad \end{aligned} \qquad (3)$$

The beam center on ITM is given by the following formula.

$$\begin{aligned} dITM &= R_{ITM} \cdot \theta_{beam} \\ &= \frac{R_{ITM} R_{ETM}}{R_{ETM} + R_{ITM} - L_{cav}} \theta_{ETM} \\ &= 2.7cm \cdot \theta_{ETM} (\mu rad) \end{aligned} \qquad (4)$$

When ETM is tilted by 2 μ radian, the beam moves by 1 beam size on ITM.

This is a typical case when the beam axis is tilted. Another case is the beam axis shift. When two mirrors are rotated to the opposite direction, the beam axis shifts with respect to the incident beam axis, rather than changing the angle.

The mode coupling of when the beam axis is shifted by dx is

$$P(00 \to 00, dx) = \exp(-(dx / w_0)^2) \qquad (5)$$

When $dx = R_{ETM}\, \theta_{ETM}$ is used, this becomes

$$P(00 \to 00, \theta) = \exp(-(\theta_{ETM} / \Theta_{ETM})^2)$$
$$\Theta_{ETM} = w_0 / R_{ETM} = 5.3 \mu rad \qquad (6)$$

A rigorous formula relating tilt angles of ITM and ETM is complicated, and these two are just typical cases to relate the mirror tilt angles and the mode coupling. For both cases, the normalization angle to determine the angle dependence is not the divergence angle, 27 μ radian, but a much smaller value.

3. FFT

In order to validate the above simplified arguments, the FFT simulation was done using the same cavity parameters. The FFT calculation is done completely independent of the modal model technique. The input beam is set to matched with the untilted cavity mode. For each tilt angle, the cavity length was adjusted to lock the FP arm using the nominal error signal.

The following plot shows the results. All results are consistent with the simplified argument in the previous section.

Fig.2-(1) is the power distribution on ETM and ITM along x axis when ETM is rotated by 2.5 μ radian around the y axis. The axes are chosen so that two beam shapes can be compared. The center of the beam is shifted by 7~8cm.

Fig.2-(2) is the mode coupling of the incoming field to the resonating field as a function of the ETM tilt angle. Blue line is equation (2) calculated in the previous section. With ETM tilt of 1.5 μ radian, the cavity power drops to 60% of the maximal power.

Fig.2-(3) is the beam position on ITM, compared with equation (4).

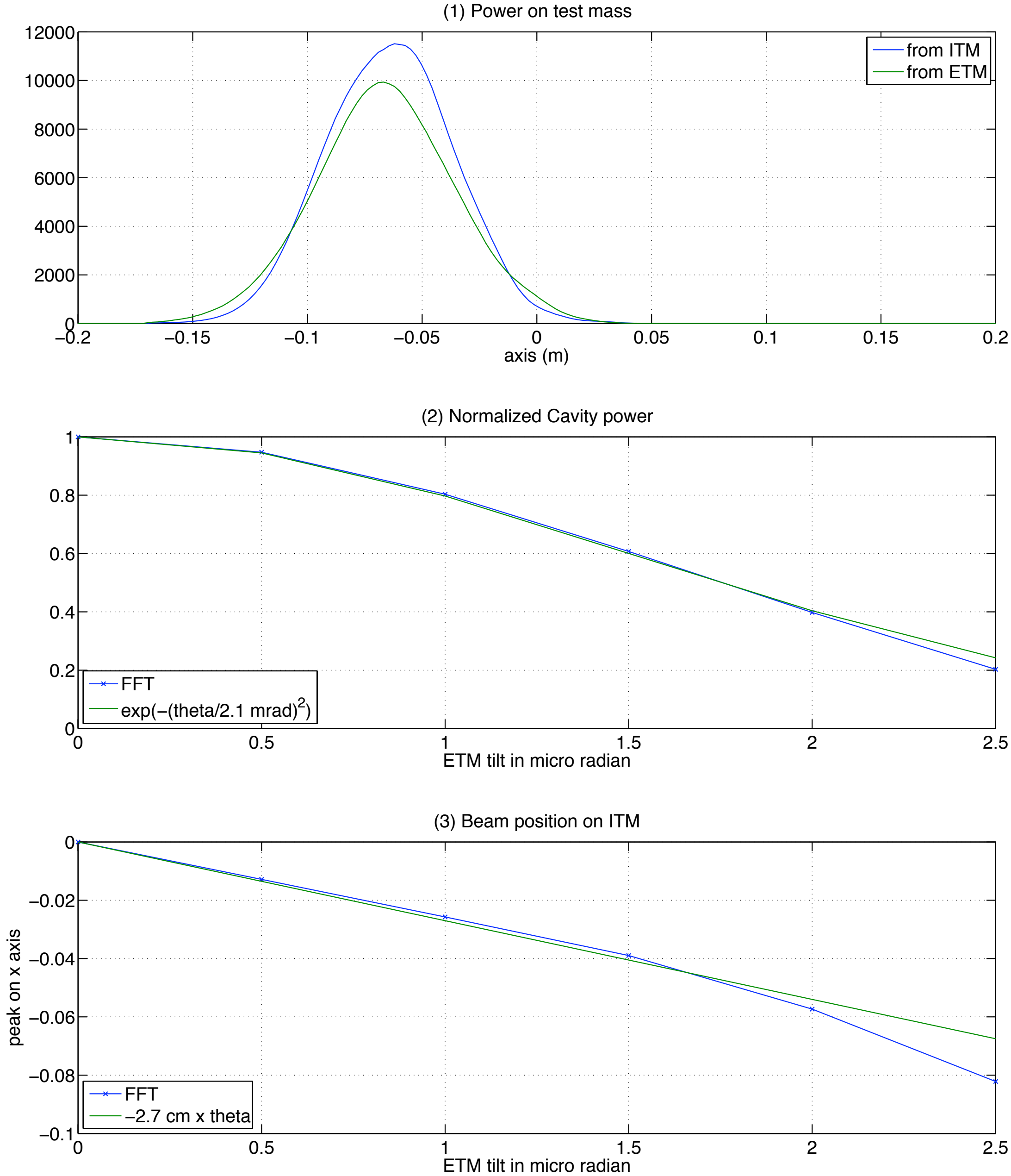

Figure 2 FFT simulation of a FP cavity with tilted ETM

4. Modal analysis of the FFT result

The FFT simulation is done without using the modal model expansion. In the previous section, it was shown that the FFT result matches with the naive modal model picture that the field resonating in the cavity is a TEM00 mode with a tilted beam axis.

Fig.3 shows the contents of the mode contents of the resonating field, i.e., the power of TEMmn mode when the resonating field is mode expanded using the original mode base defined by the incoming beam.

Fig.3-(1) shows the fractional power of 4 modes, TEM00, TEM10, TEM20 and TEM30, i.e., the power of each mode normalized by the power of the resonating field as a function of the ETM tilt angle. The shape of the TEM00 mode is the same as Eq.(3).

The power fraction of these modes are expected to be

$$P(00) = Exp(-\theta^2)$$
$$P(10) = P(00) \cdot \theta^2$$
$$P(20) = P(00) \cdot \theta^4 / 2$$
$$P(30) = P(00) \cdot \theta^6 / 6$$

Fig.3-(2) shows the power fraction of these modes as a function of various powers of the normalized angle. All modes show nice linearity as a function of θ^n, n = 2, 4 and 6 for (10),(20) and (30) mode respectively.

From this mode analysis, one can do a simple modal model analysis using the lowest orders, i.e., TEM00 and TEM01, up to ETM tilt of $1.5\,\mu rad$, but the power resonating in the cavity should be explicity corrected to predict the absolute value of the signal.

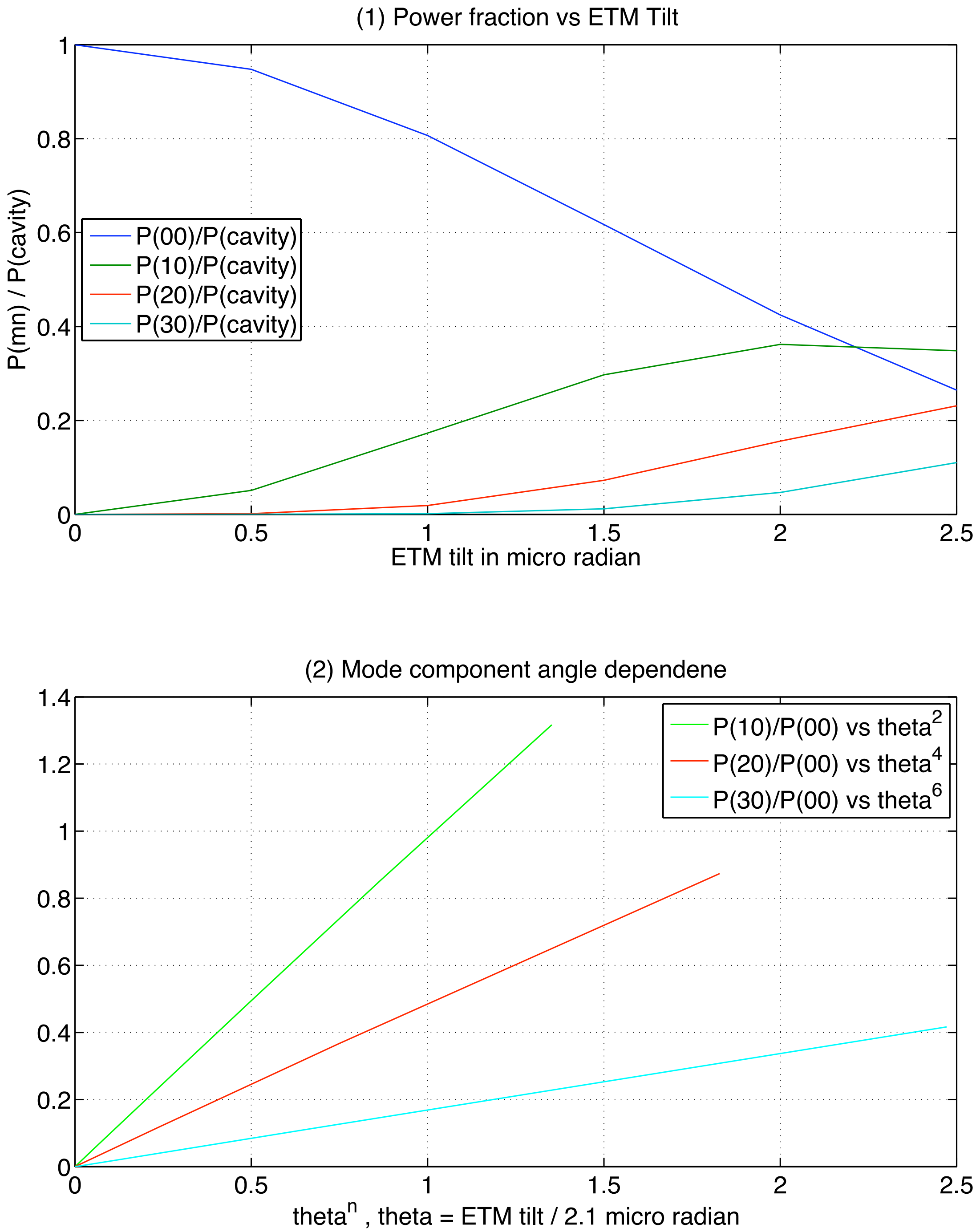

Figure 3 Mode components as a function of ETM angle