LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY

LIGO Laboratory / LIGO Scientific Collaboration

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Finite Element Modelling of Test Mass Flexure Due to OSEM Forces

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1 Introduction

It is well known that the calibration of the test mass displacement caused by the photon calibrator can be significantly in error if the photon calibrator beam reflects from the mirror within the area probed by the interferometer beam. This is because the indentation of the mirror under the pressure of the photon calibrator beam changes the optical path length of the reflected interferometer beam, just as the test mass recoil does.

This technical note explores the deformation of the test mass under the force applied by the OSEMs, to see whether this too significantly alters the effective displacement or phase response of the mirror.

2 The Model

I modeled the test mass as a simple right cylinder of radius 127 mm and thickness 100 mm. No wedges, bevels, or attachments were included. The force applied by an OSEM was taken as uniform over a circle of radius 2 mm. Each OSEM was located 120 mm from the center of the mirror's rear face, one each at the northeast, northwest, southeast, and southwest quadrants. I assumed that the OSEM magnets and their standoffs had no internal dynamics and no mass.

Table 1 shows the material parameters used for fused silica by the model. Note that the loss factor, which can be understood as the imaginary part of the Young's modulus in a frequency response analysis of the elastodynamics, corresponds to a quality factor of 10^5, lower than typically measured for fused silica. This would make a significant difference to the deformation amplitude very close to the frequency of an acoustic mode of the mirror, but that is never the case in this analysis.

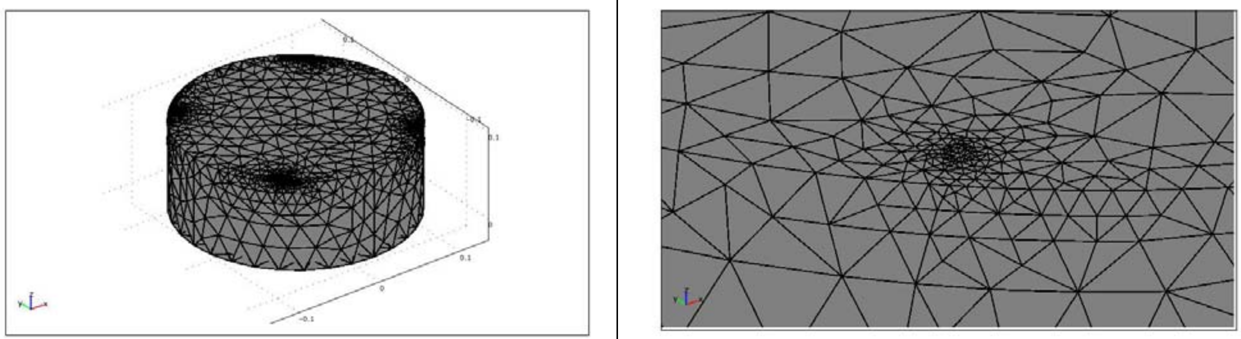
Table 1: material parameters of fused silica used by the model.

Young's modulus	73.1 GPa
Poisson's ratio	0.17
Mass density	2203 kg/m^3
Mechanical loss factor	0.00001

I used COMSOL 3.4 to solve the model. The mirror displacement was sampled at 2673 mesh points, distributed by taking the default 'Normal' mesh density for the bulk of the mirror, and additionally specifying that, at the circles where the OSEMs act, the maximum element size be .001 of the maximum distance in the geometry. Figure 1 shows the optic and two views of the mesh applied to it, one view of the whole mirror and another of the region near the OSEM.

Solutions of elastic problems in COMSOL often include unphysical rotations in models where there are no constraints specifically excluding them (that is, when the strain energy in the system is insensitive to rotation of the system). Therefore, four points on the HR face of the mirror were constrained in the following manner. The HR face of the mirror is centered in the x-y plane (the OSEM forces acting only in the z-direction). The two points at y=0 at the edge of the HR face were constrained not to move in the y-direction, and the two points at x=0 at the edge of the HR face were constrained not to move in the x-direction. By the symmetry of the system, these points cannot move in those directions anyway, so these constraints do not alter the physics of the system.

Figure 1: left side, meshing of the mirror; right side, closer view of meshing near the OSEM.



3 Results

The frequency response of the mirror to 1 N of force per OSEM (4 N total) was calculated at 40, 70, 100, 200, 400, 700, 1000, 2000, 4000, and 7000 Hz. The displacement of the mirror HR surface was dominated by bulk recoil of the mirror, which falls inversely with frequency, at low frequencies. The quasistatic bending of the mirror due to the concentrated distribution of the force is nearly independent of frequency below the drumhead mode of the mirror at 9327 Hz, and became relatively important at the highest frequency. For each frequency, I found the phase with respect to the driving force of the solution that exhibited the largest displacement of the center of the HR surface. In each case this frequency was 180° to well within 1°. This is to be expected, since the mass recoil is 180° out of phase with the force at all driving frequencies, and the displacement of the center of the HR face is also ≈180° out of phase with the force from DC up to within ~0.00001 of the drumhead mode frequency in relative units.[1]

The interferometer sees the position of the mirror surface as weighted by the intensity distribution of the arm cavity beam spot. To calculate this weighting, I exported the displacement of a line from the center of the HR face to the edge from the COMSOL model into MATLAB. I then integrated this displacement multiplied by a normalized Gaussian matching the intensity pattern of the arm cavity mode profile, assuming circular symmetry over the mirror surface.

As a check on the model, I also calculated the motion of the center of mass of the mirror, which is most easily obtained by taking the average of the displacement of all mesh elements within the volume of the mirror. This motion should be exactly the same as that for a rigid body:

$$\frac{4N}{(2\pi f)^2 m}.$$

[1] Note that although the butterfly mode of the test mass at ~6800 Hz is within the frequency range of the model, it cannot be excited by the OSEMs operating in phase with one another due to the symmetry of the system. For a wedged optic or imbalanced OSEMs the influence of this resonance might be visible.

In the COMSOL model, the center of mass motion is generally 0.016% lower than given by the above formula, which is a bit larger than one would like for a finite element model but very good agreement nonetheless.

Figure 2 shows the frequency response of the mirror HR surface and center of mass. Clearly, above 1 kHz, the HR surface displacement diverges drastically from the simple rigid body approximation.

Figure 2: frequency response of the HR surface weighted for the interferometer beam (blue), and of the mirror center of mass (red).

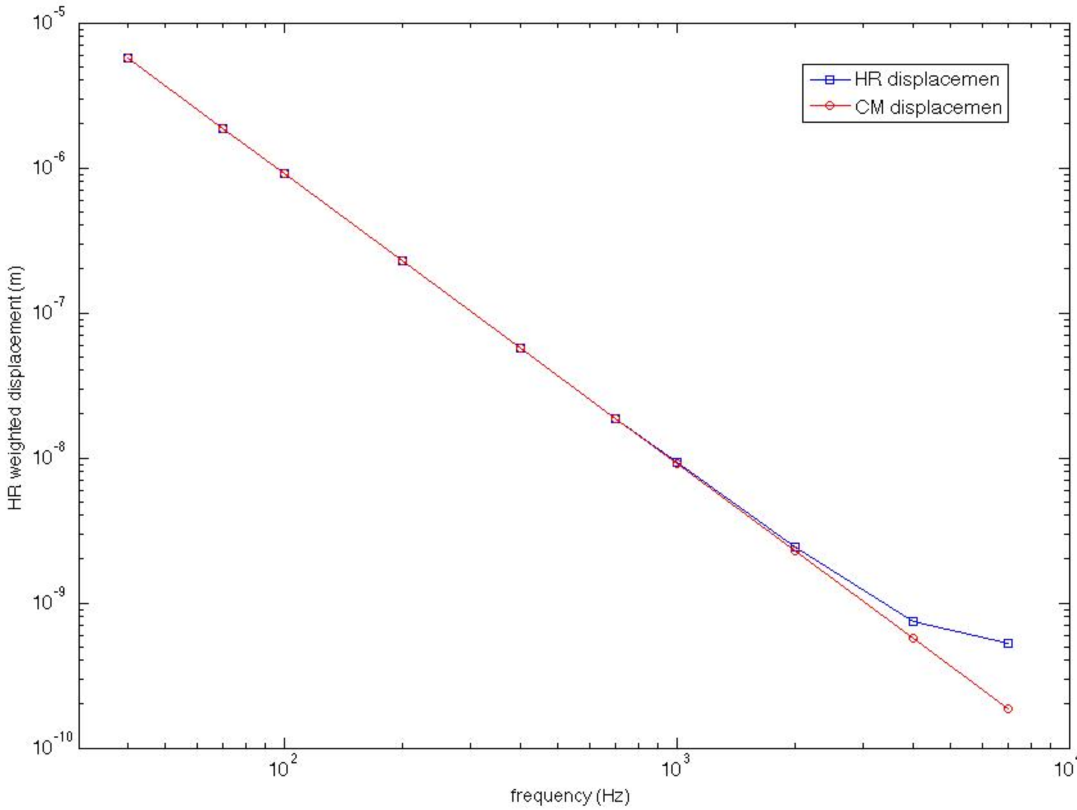


Table 2 contains the data points in Figure 2. An interpolation of the data may be useful for applying a correction to measured calibration measurements.

Table 2: displacement values vs. drive frequency

Frequency (Hz)	Weighted displacement of HR surface (m)	Displacement of center of mass (m)
40	$5.6721x10^{-6}$	$5.6721x10^{-6}$
70	$1.8522x10^{-6}$	$1.8521x10^{-6}$
100	$9.0767x10^{-7}$	$9.0753x10^{-7}$
200	$2.2702x10^{-7}$	$2.2688x10^{-7}$
400	$5.6864x10^{-8}$	$5.6720x10^{-8}$
700	$1.8665x10^{-8}$	$1.8521x10^{-8}$
1000	$9.2203x10^{-9}$	$9.0753x10^{-9}$
2000	$2.4195x10^{-9}$	$2.2688x10^{-9}$
4000	$7.4536x10^{-10}$	$5.6720x10^{-10}$
7000	$5.2957x10^{-10}$	$1.8521x10^{-10}$