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Analysis of Radius of Curvature errors in Recycling Cavity and their Compensation

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1 Introduction

1.1 Purpose

The purpose of this document is to describe the effect of radius of curvature (ROC) errors in the recycling cavities (RCs) in Advanced LIGO. Specifically, we evaluate the effect of mode matching losses due to these ROC errors. The errors considered here are due to fabrication/tolerance of the optics. Here we assume that the signal loss is equivalent to the mode mismatch between the arm and the recycling cavity.

1.2 Scope

For finalizing the COC design, we need to put some requirements on the flatness (un-flatness) of the various components in the RC. Here we consider optics from ITM to the beam splitter. The rest of the recycling cavity optical components are not included in this analysis as they are included in the IOO design document. So the scope is limited to beam splitter (BS), compensation plate (CP), and the test mass (TM) AR side. Note that this document is written with intent to put some realistic limits on the ROC errors. The exact analysis of the losses and their frequency dependence is outside the scope of this document. An FFT analysis would be carried out in future to better describe these losses. One more limitation of the analysis is the excluding of diffraction effects in the recycling cavity.

1.3 Definitions

1.4 Acronyms

BS: Beam Splitter

ROC: Radius of curvature

RC: Recycling Cavity

PRC: Power recycling Cavity

SRC: Signal Recycling Cavity

ITM: Input Test Mass

1.4.1 LIGO Documents

1. Muzammil A. Arain, "Effect of BS wedge on mode-matching in Advanced LIGO," LIGO technical note, available at http://www.ligo.caltech.edu/docs/E/E080170-00.pdf .

1.4.2 Non-LIGO Documents

2 General description

Advanced LIGO will have dual recycled configuration with both PRC and SRC. Also, the target for the PRC round trip loss is 1000 ppm while that for SRC is 2000 ppm. This requires that the mode resonating in PRC and SRC should have a very high degree of coupling with the arm cavity modes. Fabrication errors and tolerances can decrease the coupling of the RCs to the arm cavities. We need to calculate the mode-matching losses due to these errors and put some limits on these errors. Specially important is to analyze the effect of BS un-flatness as it changes the PRC and SRC differently.

In this document, we will analyze the effect of any manufacturing ROC errors on BS surfaces in the PRC and the SRC. We will also investigate the effect of BS wedge angle. Apart from the BS, errors in the CP flatness, folding mirrors in the folded interferometer (H2), and AR side of the ITM also affects the mode in the RC. But since these surfaces are close by, therefore, we can treat them as one combined ROC error. Any deviation due to fabrication error at the HR side affects both the AC mode and the RC mode. This is more complex in nature because of the necessity of considering two SRC formed by the X and the Y arm.

3 Calculation of RC modes

We are using a simple modal model where the RC mode is determined by propagating the beam coming out from the ITMx to the PRM (or SRM) and then back to ITM. Then using the standard ABCD matrices and equating the two complex q values we determine the Eigen mode of the recycling cavity. This method neglects the effect of SRM detuning. In view of 180 degree round-trip Gouy phase difference between PRC and SRC, the Bullseye mode has a relative 360 (or 0) degree phase shift between PRC and SRC. Therefore, from ROC error perspective the behavior of the two recycling cavities is same because the build-up of Bullseye mode is same in the two cavities. We would use the following cavity for modeling.

$SR_3 \rightarrow BS \rightarrow ITMx \rightarrow BS \rightarrow SR_3 \rightarrow SR_2$

Note that this is the configuration where the BS unflatness affects the most. Since, we are not using full IFO configuration, this cavity is representative of both common mode and differential mode losses. After caluclating the resonating RC mode, we evaluate the over-lap integral of this mode with the arm cavity mode. The over-lap integral gives the decrease in the mode matching between these two modes. We present the power losses. This model has been tested before with the FFT model developed by Hiro and the results were in good agreement.

4 Geometry and ABCD Matrix of BS

The geometry of the BS is shown in Fig. 1 as described by Hiro Yamamoto in his presentation. Based upon this, we can calculate the ABCD matrices of BS when the beam passes through different directions.

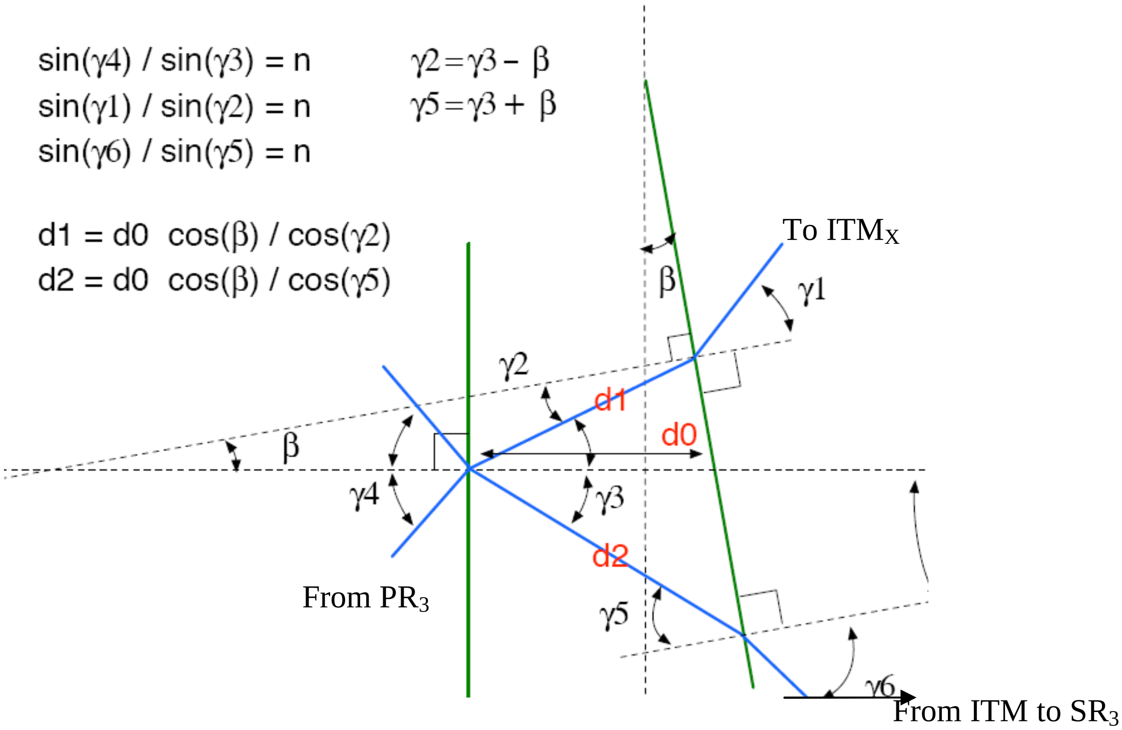

Fig. 1: Geometry of the BS wedge angle.

4.1 Wedge Angle Effect in PRC

Evaluation of wedge angle effect alone has been carried out in Ref. 1. However, Ref. 1 document was prepared when the wedge angle of the BS was 0.9 degree. Here we present the results for smaller wedge angle in Fig. 2.

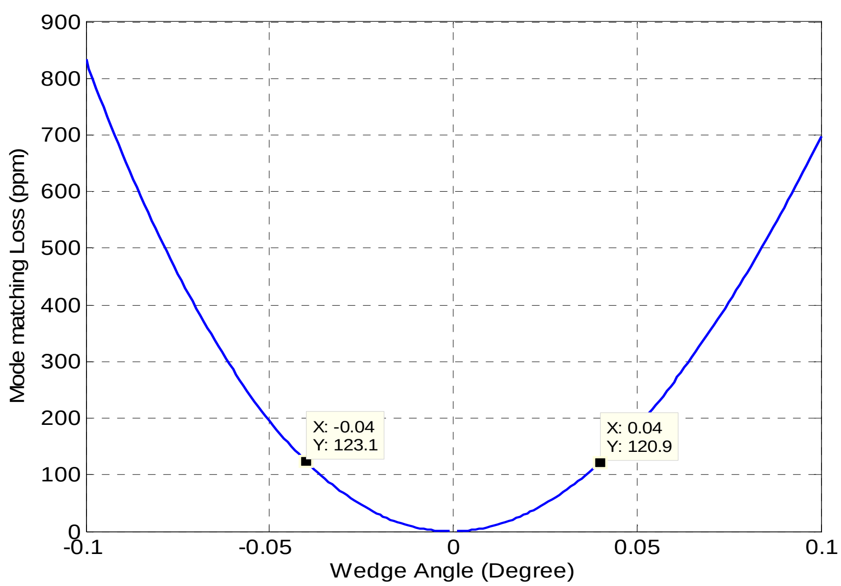

Fig. 2: Mode matching loss due to BS Wedge angle in SRC

This shows that the mode-matching loss would be around 120 ppm. However, when we consider other ROC errors, this becomes a contribution to the ROC errors.

4.2 BS Geometry for Differential Errors

The two surfaces of the BS participate differently in the differential error if the two surfaces are not flat. In general, the BS could either look like a ( ) or could be like ((or)). The ( ) is the worst configuration because it introduces more losses. First we consider the effect of these configurations for the SRC. Fig. 3 shows the effect of a BS configuration on PRC and SRC. Fig. 3a and 3b analyze the situation for the ((case for SRC and PRC respectively. A similar analysis is carried out for the ( ) BS configuration in Fig. 3c and 3d. Fig. 3a and 3b show that the relative effect of BS configuration is same for PRC and SRC when BS is ((and that this configuration requires a compensation of 2/R at CPx. A similar comparison of Fig. 3c and 3d show that ( ) BS configuration requires a compensation of 2*n/R at CPx where n is the refractive index of the material. This can be further generalized by assuming that the two ROC at the two sides of BS have different values. After some algebra, it can be concluded that a BS acts as a differential ROC where the error contribution due to the two ROCs is:

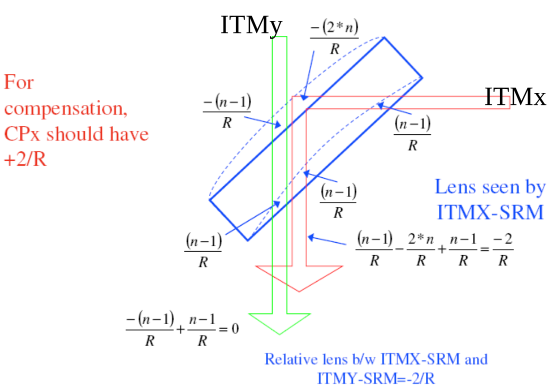

Fig. 3a: SRC and BS ((Configuration

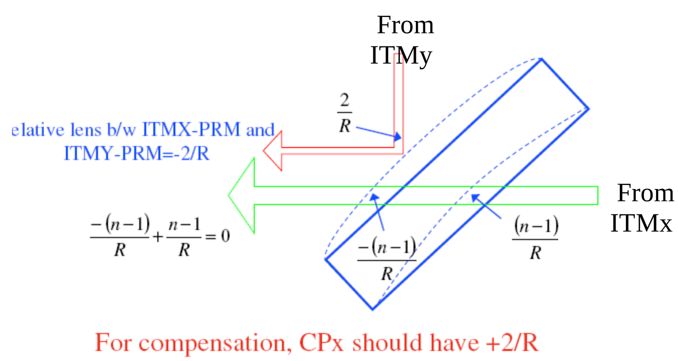

Fig. 3b: PRC and BS ((Configuration

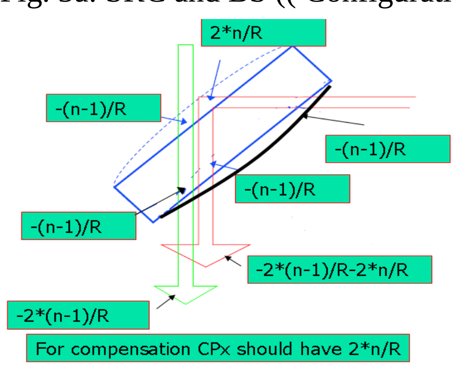

Fig. 3c: SRC and BS ( ) Configuration

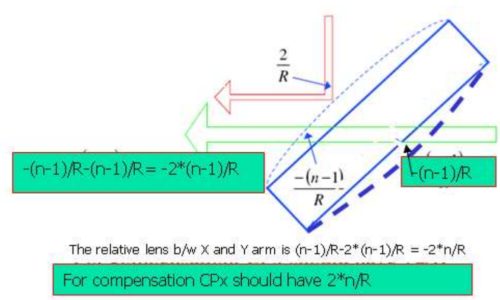

Fig. 3d: PRC and BS ( ) Configuration

Fig. 3: BS ROC error contribution in various configurations for PRC and SRC.

$$\frac{1}{R_{eq}} = \frac{(n+1)}{R_2} - \frac{(n-1)}{R_1} \qquad (1)$$

where *Req.* is the equivalent differential ROC error introduced due to a HR side ROC = R_2 and AR side ROC = R_1. Eq. 1 shows that the worst case ROC (a smaller numerical ROC) happens when R1 and R2 have opposite signs, i.e, ( ) is the worst case. If the two ROCs are equal in magnitude but opposite in sign, the worst case value of ROC is R/*n* where as the minimum value of ROC is R if the two ROC on the HR side and AR side have equal magnitude and sign. Note that here a '((' BS configuration is assumed to have a same ROC sign while '( )' BS configuration constitute a situation where the sign of the ROCs of the two sides are opposite. Thus, effectively we can model BS ROC error as a differential ROC error in the arms.

5 Losses due to Differential ROC error in the Michelson Arms

Here we consider errors due to differential ROC errors in the Michelson arm. Here specifically, we consider SRC x-arm only formed by

SRM-SR3-BS-CPx-ITMx and then back from ITMx-CPx-BS-SR3-SRM

Since we are modeling only one arm, we assume that the second arm is perfectly mode matched and the mode matching is 1. Also we are assuming same arm cavity mode in the X-arm and the Y-arm, therefore, the only mode matching decrease mechanism is due to the over-lap integral decrease between the SRCx cavity mode and the arm cavity mode. Here we introduce a ROC error in the SRC. Note that we consider ROC errors due to CP, ITM AR side, BS HR and BS AR sides. The exact location of the ROC error does not matter since these surfaces are in the far field of the same mode. So we can think of these ROC errors as the combined ROC tolerance on all these surfaces. The combined ROC error can be calculated by adding all the ROC errors inversely according to the following relationship:

$$\frac{1}{R_{combined}} = \frac{1}{R_{BS\ HR}} + \frac{1}{R_{BS\ AR}} + \frac{1}{R_{CP}} + \frac{1}{R_{FM}} + \frac{1}{R_{ITM\ AR}} + \ldots \quad (2)$$

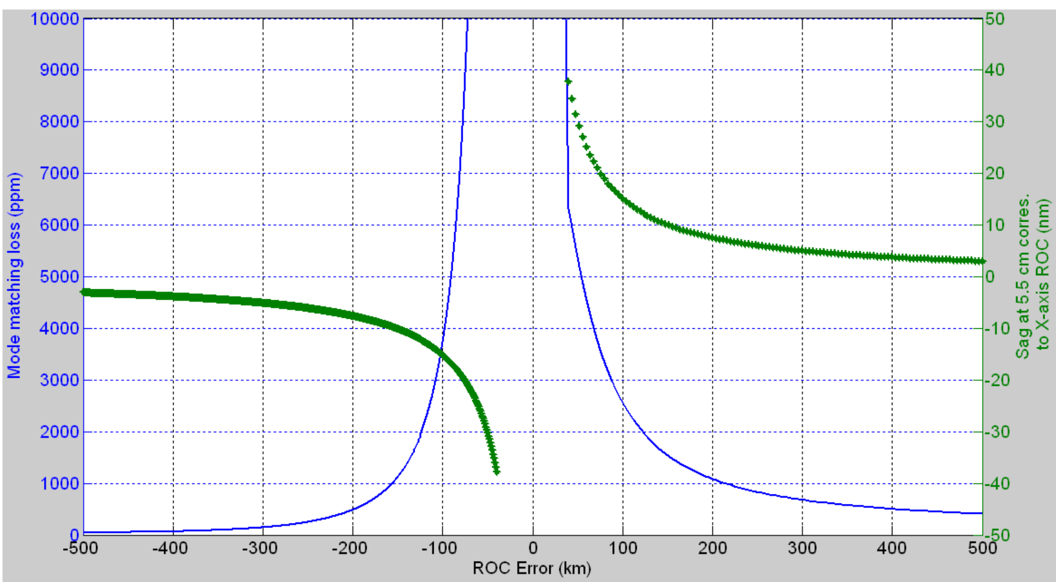

Fig. 4: Mode matching loss due to combined ROC error as defined in Eq. 2 plotted in blue against left y-axis. The corresponding sag (for a 5.5 cm beam) values are plotted in green on right y-axis.

Note that the ITM HR side ROC errors have not been considered here as this changes the AC mode also.

Figure 4 shows the expected losses due to a combined differential ROC ($R_{combined}$)error in the SRC. As mentioned earlier, the PRC is expected to have same behavior. Here the x-axis is ROC values in km. Plotted in blue on left y-axis are the expected mode matching losses. Note that larger ROC means lesser error. On the right y-axis is plotted the corresponding sag due to such a ROC error as plotted on X-axis. Therefore, when we have 0 ROC, this means that the differential sag error is infinite. Alternatively, 0 sag error means infinite ROC or no ROC error. The sag has been calculated for 5.5 cm beam size.

Although it looks like the requirements on these ROC errors is pretty stringent. We should keep in mind that different ROC errors can also have opposite signs thus different errors can add or subtract. The non-symmetrical behavior with respect to the positive and negative ROC values is due to the change in Gouy phase of the RC and is well understood. This shows that we would fare better of the net differential ROC error is concave. Another noticeable feature is that these ROC errors should have very minimal effect if the value is lower than 500 km.

6 Differential ROC error compensation using CP

The combined ROC error discussed in the above section can be compensated by operating on the CP. For example, if the differential error is 100 km ROC, then we need to apply -100 km ROC at the CP. Since the CP is in the far filed of the mode, therefore, any error can be compensated by a high degree of accuracy. The BS wedge angle can also be interperated as a differential error. So a combined compensation on the respective CP can correct all these errors in the ROC due to fabrication.

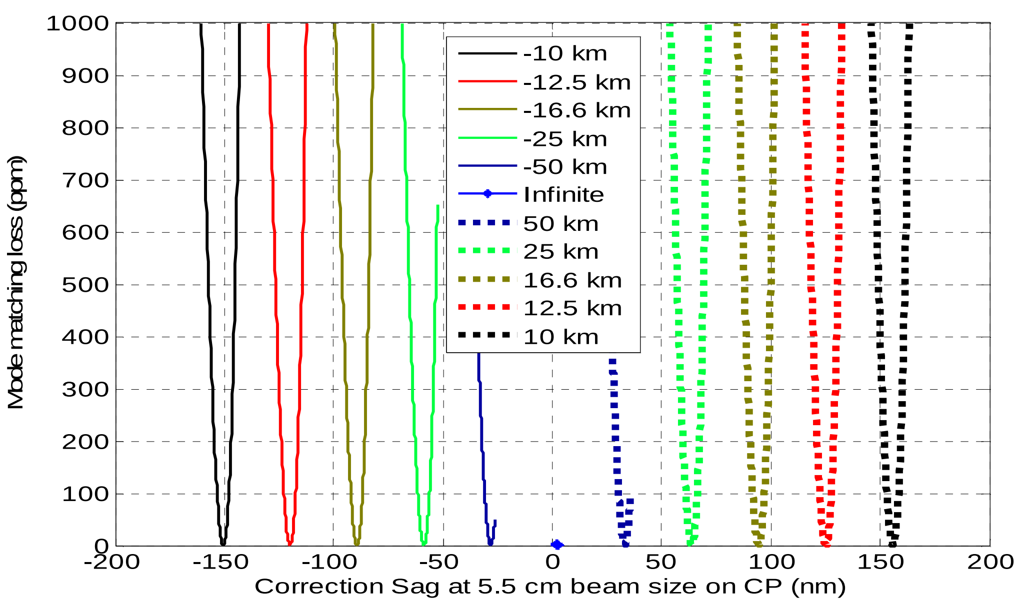

Fig. 5: Compensation of combined differential ROC error in the Michelson arm by operating on CP.

Fig. 5 shows that we can compensate any residual differential ROC errors in the Michelson cavity by operating on CP. Here on x-axis, we have taken sag applied to the compensation plate via TCS as a variable. So, for any given ROC error, there is always a compensation that can be applied at the CP and the loss can be driven to very small values. Here in the figure, different curves correspond to a particular value of ROC error. For example, the black curve shows the behavior for a 10 km ROC error. Since the ROC is positive, we would have to apply negative compensation at the CP. Here in this curve the minima ai around -150 nm. Therefore, if TCS supplies -150 nm sag change, the loss due to the differential ROC can be minimized. Fig. 6 shows the same data in another way. Here instead of ROC values, the X-axis is the differential error in terms of sag for 5.5 cm beam size. The left y-axis is the corresponding minima obtained by choosing a particular value of the TCS compensation that drives the mode matching error to a minimum. This particular value of the compensation in terms of sag in plotted in green on the right y-axis.

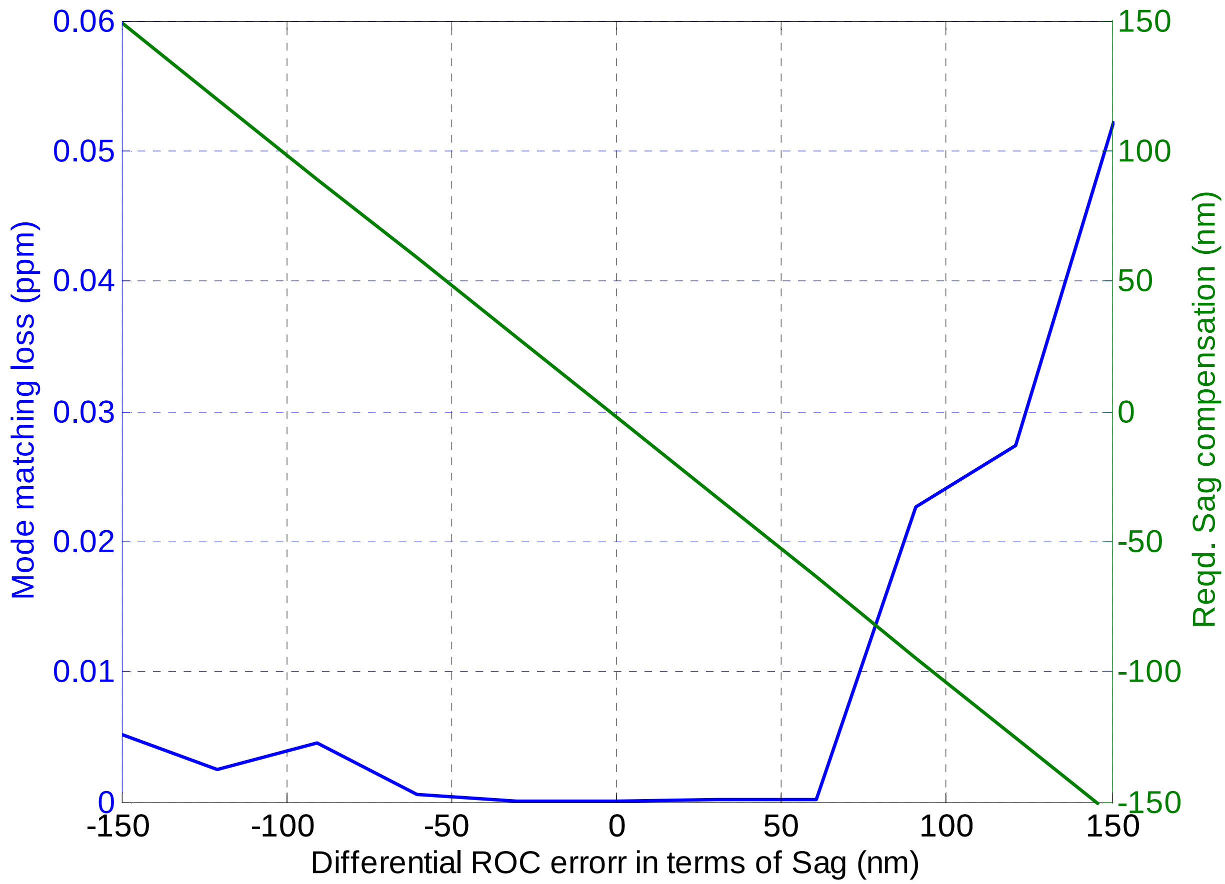

Fig. 6: Error compensation using CP for combined ROC differential errors in Michelson cavity. The x-axis values are the sag corresponding to a ROC error value in the cavity with 5.5 cm beam size. Plotted on left y-axis is the residual mode matching loss if we apply compensation at the CP according to the green curve plotted against right y-axis.

As mentioned earlier, we can compensate both PRC and SRC at the same time if we apply proper compensation on the CP. Note that the differential errors gives us the flexibility of choosing which CP we want to operate on. If there is a relative +100 km ROC error between X arm and Y arm, we can either apply -100 km compensation on CPx or else we can apply +100 km compensation on CPy. This however, may change the common mode of the RC. But for static error correction, we can use SR_2 in the SRC and PR_2 in the PRC.

6.1 Using SR_2 (PR_2) for the Common Mode Errors

In the above section we have considered differential ROC in the region from BS to ITM AR ROC. However, these errors could also be common mode. In that case, it is very easy to correct for these ROC for both PRC and SRC by moving SR_2/PR_2. Fig. 7 shows that how we can use SR_2 to correct common mode errors in SRC. This figure is equivalent of Fig. 6 assuming that the compensation mechanism is SR_2 movement instead of CP. Fig. 7 shows that by moving SR_2 by less than ±10 cm, we can drive these errors to less than one ppm.

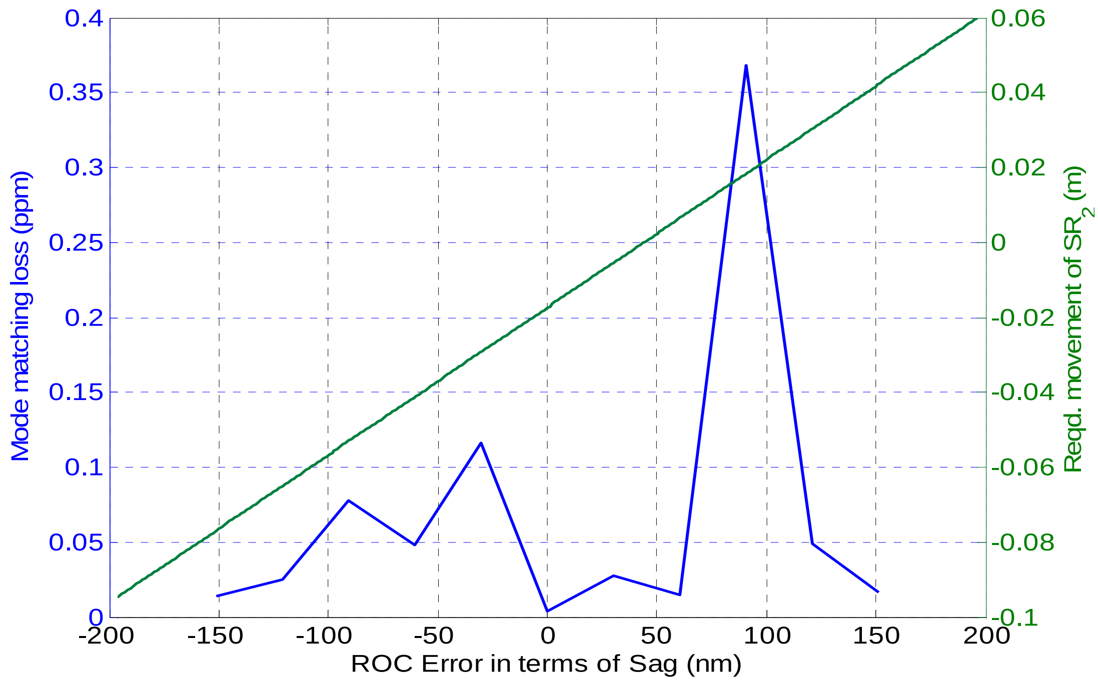

Fig. 7; Equivalent of Fig. 6 assuming that the errors are common mode and the compensation mechanism is motion of SR_2.

7 Differential ROC error in ITM and its Compensation

Another aspect of the ROC errors is the tolerance of ITM ROC. If the two ITMs have different ROC, the mode in the two arms would be different. Apart from that, the modes resonating in the two sections of SRC will also be different. Therefore, we will see some mode mismatch due to one of the ITMs being off from their designed value.

However as mentioned earlier, we can compensate these by operating on the respective CP. Fig. 7 shows how we can operate on one of the CP and be able to correct the mode mismatch. Here the x-axis is the change in ITM ROC from its designed value while the blue curve plotted on the left y-axis is the corresponding minimum mode mismatch that we would get after operating on the CP where the required compensation in terms of sag is plotted in green on the right y-axis. Comparing it with Fig. 6 we realize that ITM ROC error can not be compensated exactly as we can for the case of ROC errors in BS, CPs, and ITM HR side etc. The reason is that when ITM ROC changes, the mode in the arm changes that changes both the beam size and the beam ROC. Using only one knob can not correct both of them simultaneously. We pick the value of compensation that gives the maximum mode matching. However for a ±10 m ROC error at the ITM, the loss is less than 150 ppm.

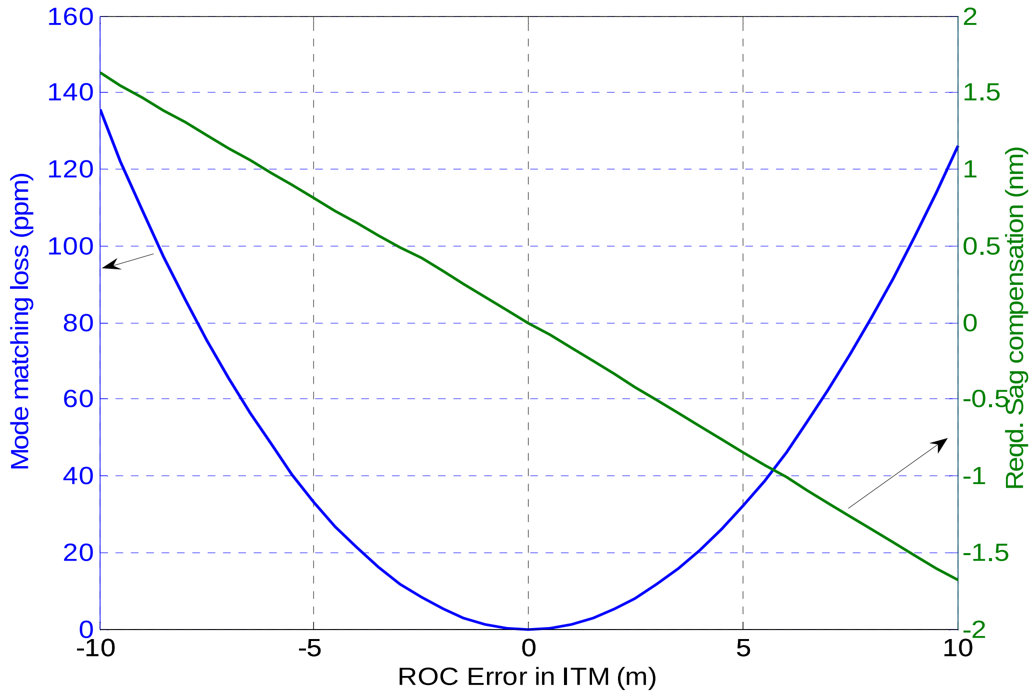

Fig. 7: Mode-matching error as a result of ITM ROC error. The green curve plotted on right y-axis shows the required compensation in terms of sag at the CP while the blue curve shows the residual mode matching loss after optimal compensation has been applied.

8 Summary

As a summary, we can say that:

1. We can compensate differential ROC errors either due to ITM ROC errors or due to BS/DP/FM un-flatness by operating on one CP while compensating both for PRC and SRC.
2. The common mode errors in SRC can be compensated by SR2 movement.
3. The common mode errors in PRC can be compensated by PR2 movement.
4. The value of losses due to these errors could be even 1000s of ppm but they can be driven to a few ppms by operating via TCS.

5. Another degree of freedom that we have for compensating these errors using CP is the ability to chose which CP we want to operate on; namely either X or Y arm compensation. So if X arm requires central heating, Y arm would require annulus or vice versa.
6. The required compensation at CPs is well within the range of TCS for reasonable ROC errors.