LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY

LIGO Laboratory / LIGO Scientific Collaboration

LIGO-T080268-00-I *ADVANCED LIGO* 10/08/08

Optical Layout and Parameters for the Recycling Cavities

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1 Introduction

1.1 Purpose and Scope

This document describes the optical parameters of the recycling cavities for Advanced LIGO. The lengths between various optical elements, ROC values of the mirrors, and their tolerances are listed. The various cavity parameters like Finesse, linewidths, and transversal mode spacing are calculated. Included also are the higher order modes offsets from resonances.

1.2 Definitions

Finesse: Measure of the selectiveness/build-up of the cavity given by $F = \frac{\pi\sqrt{r_1 r_2}}{1 - r_1 r_2}$

Free Spectral Range: FSR is given by FSR = $\frac{c}{2L}$ where c is the speed of light while L is the length of the cavity. The units we use are Hz.

Linewidth: The point at which the normalized transmission through a cavity becomes 1/e^2. This is calculated as Linewidth = $\frac{0.5 * FSR}{F}$ = Half-Width-Half-Max (HWHM).

Transversal Mode Spacing: Transversal mode spacing in the frequency difference between two Gaussian modes. For example, this is the frequency difference between TEM_{00} mode and TEM_{01}. For any higher order TEMnm mode, the difference between TEMoo and TEMnm mode is given by $\frac{(n+m)FSR * a\cos(\pm\sqrt{g})}{\pi}$ where g is the G-factor of the cavity. Note that we will use Hz as the units of transversal mode spacing.

Sagitta or Sag: For a beam with 1/e^2 beam size of w incident on a mirror if ROC R, the sag is given by $\frac{w^2}{2R}$.

1.3 Acronyms

ROC: Radius of Curvature

PRC: Power Recycling Cavity

SRC: Signal recycling Cavity

1.3.1 LIGO Documents

1. Mike Smith, recycling-cavity-length-nfold-stable_range_0.08-ITMvert_9-12-08.pdf.
2. Mike Smith, non-fold_src_coord_0.08-ITMvert_9-12-08.pdf.
3. R. Abbott, “Advanced LIGO Interferometer Sensing and Control Conceptual Design,” LIGO-T070247-00-I.

2 Optical Configuration

The optical configuration of the Advanced LIGO cavities is given in Fig. 1 where we include both recycling cavities and the arm cavities.

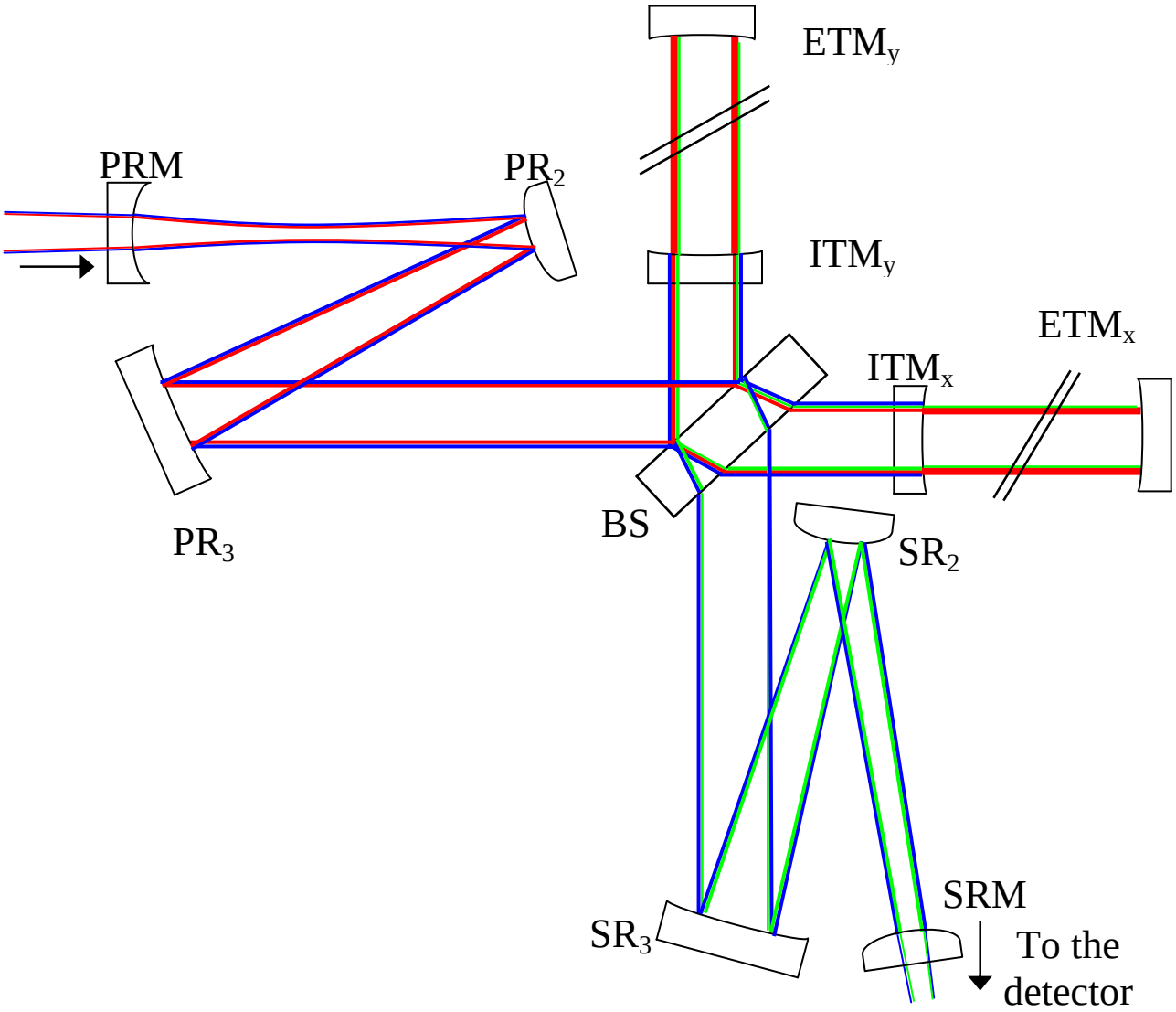


Fig. 1: Optical layout of Advanced LIGO cavities.

The various distances involved between optical elements are taken from Ref. 1 and 2. These are shown in Table 1.

Table 1: Optical Parameters and Distances in Advanced LIGO Cavities

Definition	***Unit***	***PRC***		***SRC***	
		Straight	Folded	Straight	Folded
PRM radius of curvature	m	9.0	10.43	-10.11	-71.09
Distance b/w P(S)RM and P(S)R_2	m	16.6329	15.7972	15.7064	15.9407
P(S)R_2 ROC	m	-2.26	-1.56	-3.78	-2.49
Distance b/w P(S)R_2 and P(S)R_3	m	16.1709	15.2	15.470	16.0016
P(S)R_3 ROC	m	34	31.5	34.0	34
Distance b/w P(S)R_3 and BS	m	19.511	19.4139	19.395	20.0991
BS Effective thickness	mm	0	0	131.5	132
Distance b/w BS and CP	m	4.8526	9.4652	4.8076	9.4455
Distance b/w CP and ITM	mm	5	5	5	5
ITM ROC	m	1971	1971	1971	1971
Reqd. beam waist size in arm	mm	11.8	11.8	11.8	11.8
Spot Size at ITM	cm	5.55	5.55	5.55	5.55
Beam waist location from ITM	m	1882.6	1882.6	1882.6	1882.6
ETM ROC	m	2191	2191	2191	2191
Angle of Incidence at P(S)R_2	degree	0.748	0.963	0.892	0.878
Angle of incidence at P(S)R_3	degree	0.608	1.144	0.774	0.916

The associated component specifications for the recycling cavity mirrors are given in Table 2.

Table 2: Component Parameters for Recycling Cavity Mirrors

Optics	ROC (m)		Beam Size (mm)		Sag (±μm)		Tolerance in % and m			Tol. Sag (±nm)	
	Straight	Folded	Straight	Folded	Straight	Folded	Both (%)	Straight (±m)	Folded (±m)	Straight	Folded
PRM	9.00	10.43	1.90	2.30	0.20	0.25	1.00	0.09	0.10	1.99	2.51
PR2	-2.26	-1.56	3.40	2.60	-2.56	-2.17	1.00	0.02	0.02	25.83	21.90
PR3	34.00	31.50	56.52	56.70	46.98	51.03	0.50	0.17	0.16	233.75	253.89
SRM	-10.11	-71.09	2.00	2.90	-0.20	-0.06	1.00	0.10	-0.71	2.00	-0.59
SR2	-3.78	-2.49	5.70	4.00	-4.30	-3.22	1.00	0.04	0.02	43.47	32.51
SR3	34.00	34.00	56.50	56.72	46.94	47.31	0.50	0.17	0.17	233.55	235.39

Note that the tolerances of P(S)R_3 are based upon our ability to correct any manufacturing tolerance by repositioning P(S)R_2. From layout standpoint, we can reposition P(S)R_2 by ± 10 cm requiring P(S)RM be moved by ± 20 cm. Thus we had to select 0.5% tolerance for P(S)R_3. Tolerance of P(S)R_2 and P(S)RM is loosely based upon what we can get from the manufacturers easily. Any error in ROC of P(S)R_2 and P(S)RM can also be corrected by repositioning the mirrors but the range of motion required for these mirrors is very small.

Note: Here 'Sag' is the sagitta change due to ROC while 'Tol Sag' is the change in sagitta between the nominal ROC value and when the ROC is at the end of the tolerance. For example, for PRM, 'Tol. Sag' = (Beam size)^2/(2*10.43) - (Beam size)^2/{2*(10.43+0.1)}

Note that we have used the HR side of the BS for designing the PRC while for the SRC, the beam passing through the BS AR side is chosen. Thus for the straight cavity, PRC is designed for the Y-arm while SRC is designed for the X-arm. For the folded cavity, PRC is designed for the X-arm while SRC is designed for the Y-arm. The difference in the resulting ROC for the cavity mirrors is very small and well within the proposed tolerance.

3 Derived Cavity Parameters

To derive cavity parameters we have to use some mirror transmittances and distances. These are given in Table 3 and are taken from Ref. [1-3].

Table 3: Derived Cavity Parameters

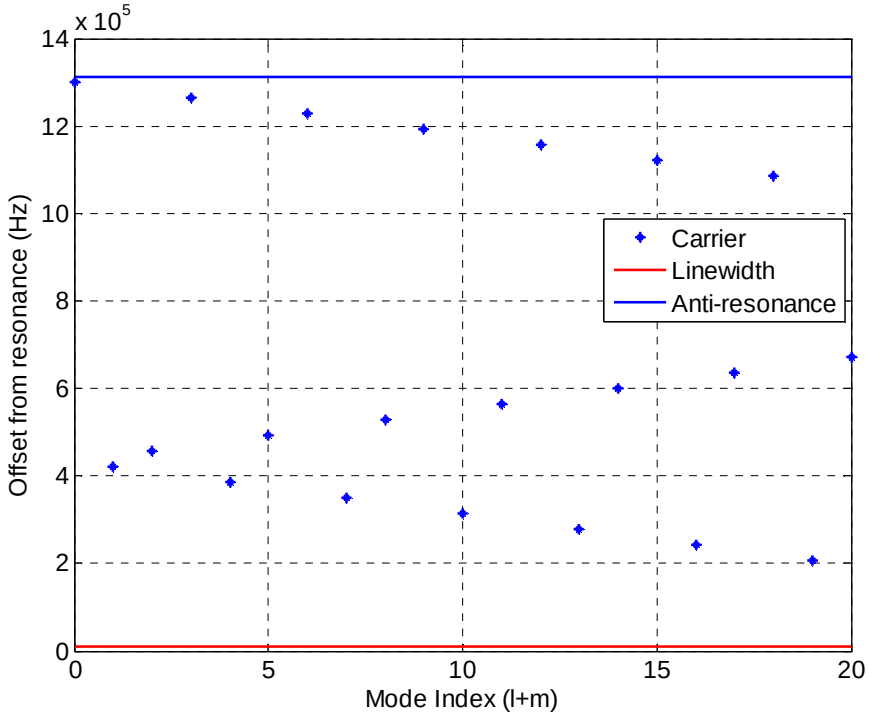
Quantity	***Unit***	***Straight IFO***	
ITM Transmittance	%	1.4	
PRM Transmittance	%	3.0	
SRM Transmittance	%	20.0	
ETM Transmittance	ppm	10	
PRC Length	M	57.676	
SRC Length	m	56.028	
Arm cavity length	m	3994.75	
Lower Mod. Frequency	MHz	9.1011	
Distance b/w P(S)R_3 and BS	MHz	45.5055	
Arm cavity Finesse		443	
Arm cavity FSR	KHz	37.52	
Arm cavity TMS	KHz	32.69	
Arm cavity Linewidth	Hz	42.37	
Arm cavity G-factor		0.8455	
		PRC	***SRC***
Carrier Recycling cavity Finesse		114	25.36
Recycling cavity FSR	MHz	2.6	2.67
Recycling cavity TMS	MHz	1.722	0.434
Carrier Recycling cavity Linewidth	KHz	11.35	52.7
RF Recycling cavity Finesse		141	26.46
RF Recycling cavity Linewidth	KHz	9.22	50.57
G-factor		0.2376	0.7617
One way Gouy Phase	Radian	2.08	0.51

Note that there is a little difference between the Finesse for the carrier and the RF sidebands in the recycling cavities. This happens because the carrier is resonant in the arms while the sidebands are not. Therefore we use effective reflectance for the over-coupled arm cavity. The amplitude reflectivity of ITM based upon 1.4% power transmittance is 0.9930 while the amplitude reflectivity of the combined ITM-ETM combination when the carrier is resonant inside the arm cavity is -0.9879. Thus the slightly higher losses for the carrier accounts for slightly reduced Finesse of the PRC and SRC for the carrier as compared to the sideband Finesse in these cavities.

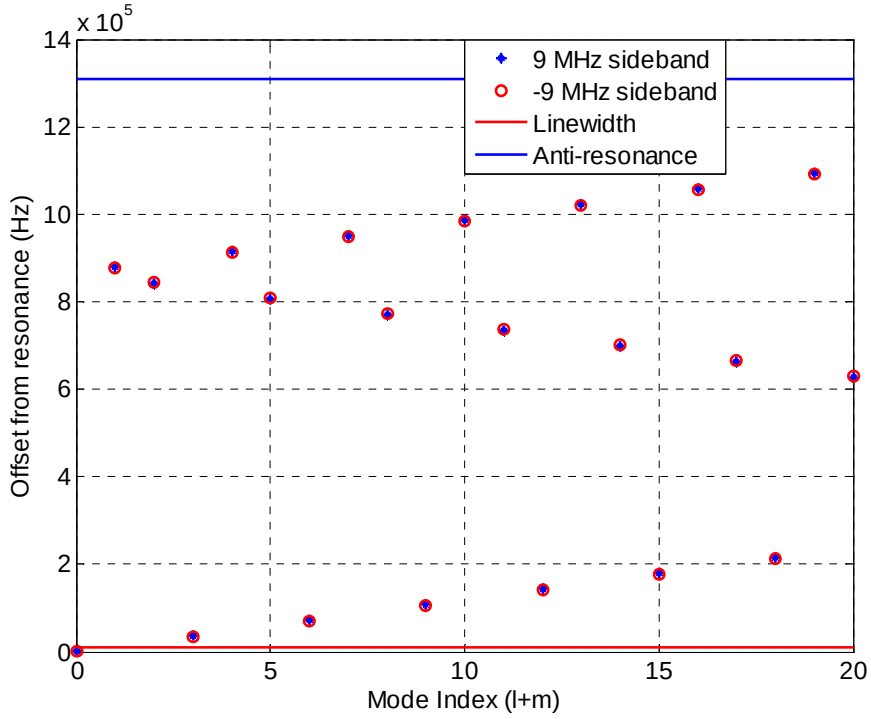
4 Higher Order Mode Resonance

Next we have used straight IFO to check the higher order mode resonance in the recycling cavities. The results are shown in the figures below.

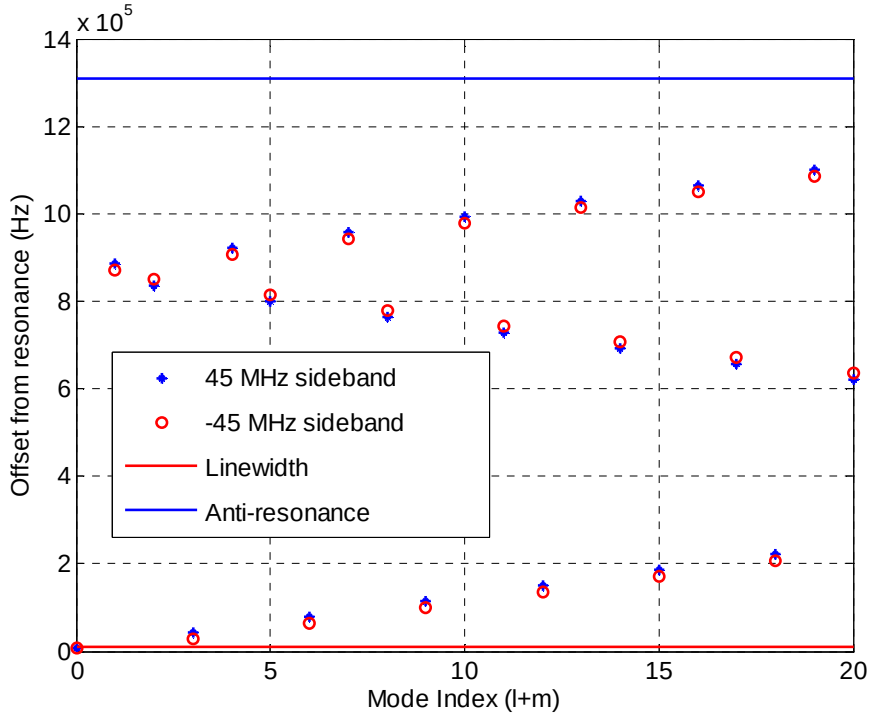
4.1 Carrier in the PRC



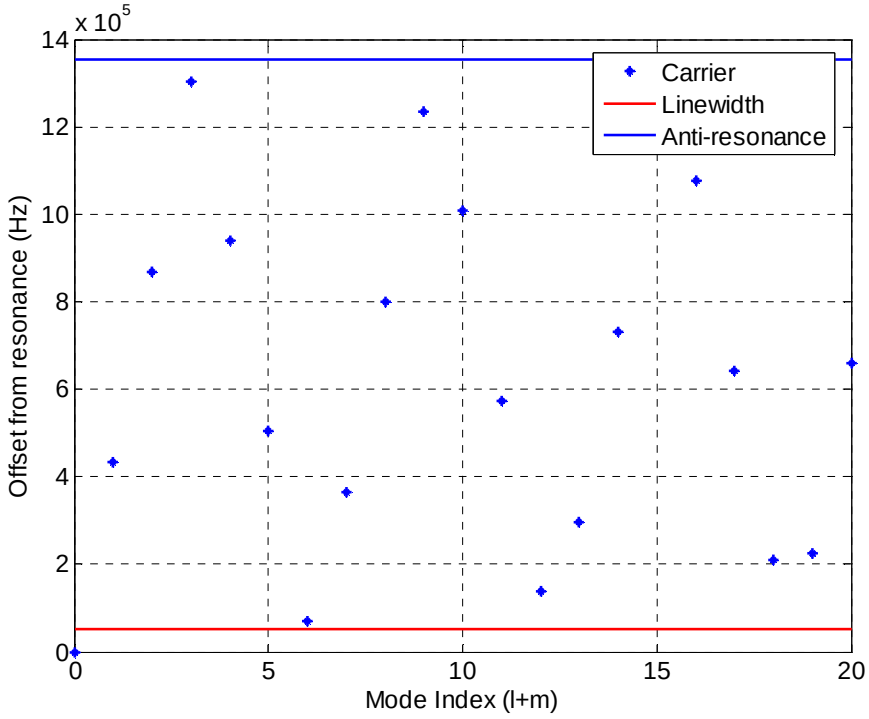
4.2 Low Frequency Sideband in the PRC



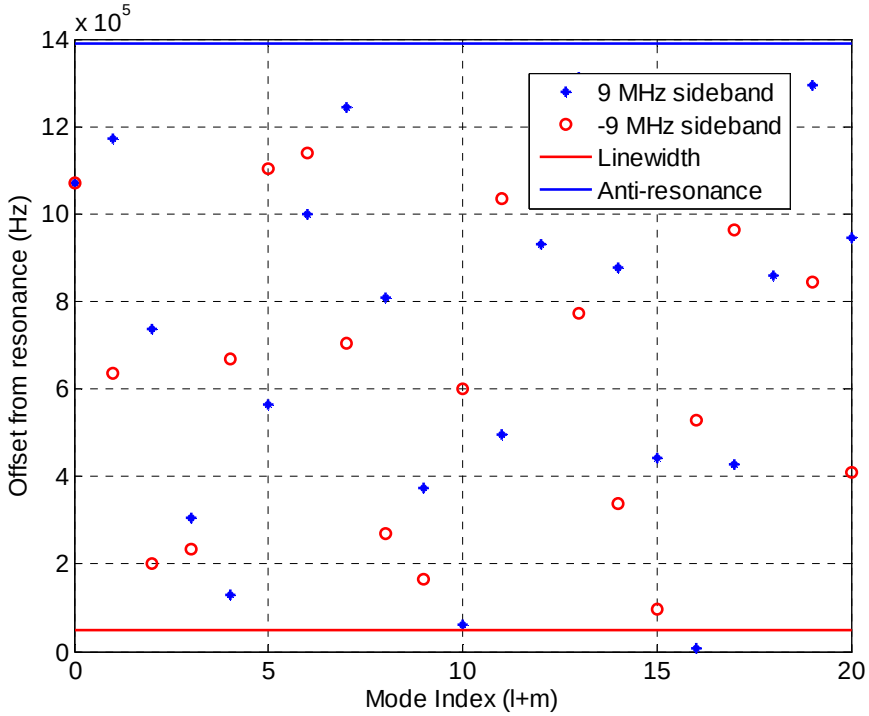
4.3 High Frequency Sideband in the PRC



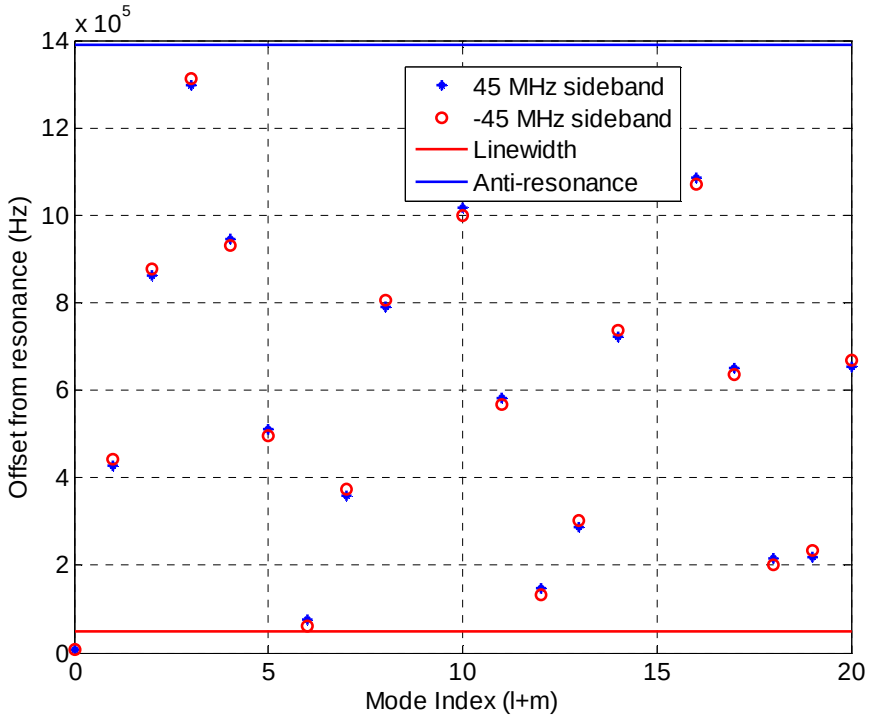
4.4 Carrier in the SRC



4.5 Low Frequency Sideband in the SRC



4.6 High Frequency Sideband in the SRC



5 Summary

We have evaluated the higher order mode resonance for the carrier and the sidebands in the PRC and SRC. None of the higher order mode with a mode index of less than 15 is within the cavity linewidths anywhere. The l+m=6 mode in the signal recycling cavity for the carrier and the 45 MHz sideband is almost at the edge of the cavity linewidth. The choice of PRC and SRC Gouy phase seems appropriate.