LIGO Laboratory / LIGO Scientific Collaboration

LIGO-T080337-00-D *LIGO* Nov. 28 2008

Minimum Brownian Noise Dichroic Dielectric Mirror Coatings for AdLIGO

Maria Principe, Innocenzo M. Pinto, Vincenzo Pierro
TWG, University of Sannio at Benevento, ITA

Riccardo DeSalvo
CALTECH, LIGO Lab

Distribution of this document:
LIGO Science Collaboration

This is an internal working note
of the LIGO Project.

California Institute of Technology
LIGO Project – MS 18-34
1200 E. California Blvd.
Pasadena, CA 91125
Phone (626) 395-2129
Fax (626) 304-9834
E-mail: info@ligo.caltech.edu

Massachusetts Institute of Technology
LIGO Project – NW17-161
175 Albany St
Cambridge, MA 02139
Phone (617) 253-4824
Fax (617) 253-7014
E-mail: info@ligo.mit.edu

LIGO Hanford Observatory
P.O. Box 1970
Mail Stop S9-02
Richland WA 99352
Phone 509-372-8106
Fax 509-372-8137

LIGO Livingston Observatory
P.O. Box 940
Livingston, LA 70754
Phone 225-686-3100
Fax 225-686-7189

http://www.ligo.caltech.edu/

1. Motivation and Requirements

The latest AdLIGO design requirements include double-wavelength operation for the cavity mirrors (both ETM and ITM) with further allowance for sufficient reflectance on a third wavelength (used for Hartmann sensors) and for optical levers at wavelengths as yet to be chosen among 670, 946, 980, 1319, 1550 nm. The requirements on the second wavelength (532nm, second harmonic of the main beam) are stringent, being related to the new (fast) alignment locking system. The nominal specs from the AdLIGO wiki are given below [0]:

Surface 1	ITM	ETM
Transmission at 1064 nm	0.014	< 6 ppm
Transmission at 532	max 0.01	0.01 to 0.15 pref 0.05
Reflection at 670 nm	> 0.05	unsp

(Source : AdvLIGO Wiki)

Note that among solutions satisfying the requirements, some may be preferred for a better interferometer operation. For example, a 0.01 power transmittance @532nm, although satisfying all requirements, dumps a lot of light on the beam splitter. A solution with lower ETM reflectance and/or higher ITM reflectance @532 nm (and less cavity transmittance) would be preferable.
The std. quarter wavelength (QWL) designs going closest to the prescribed transmittance at 1064nm (consisting of 20 and 9 doublets, respectively for the ETM and ITM coatings) cannot cope with all the above requirements, as shown below.

1064	10008.4	[ppm]	1064	4.17714	[ppm]
532	0.127124		532	0.202163	
670	0.0853936		670	0.109082	
946	0.827508		946	0.304031	
1319	0.379098		1319	0.0965333	
1550	0.215577		1550	0.0658356	

Table 0 – Quarter wavelength designs closest to prescribed transmittance at 1064nm. ETM (right) and ITM (left).

2. Reference design

The simplest ETM coating design [1] which is ideally (in a sense to be specified below) capable of fulfilling the dichroic AdLIGO requirements is shown below:

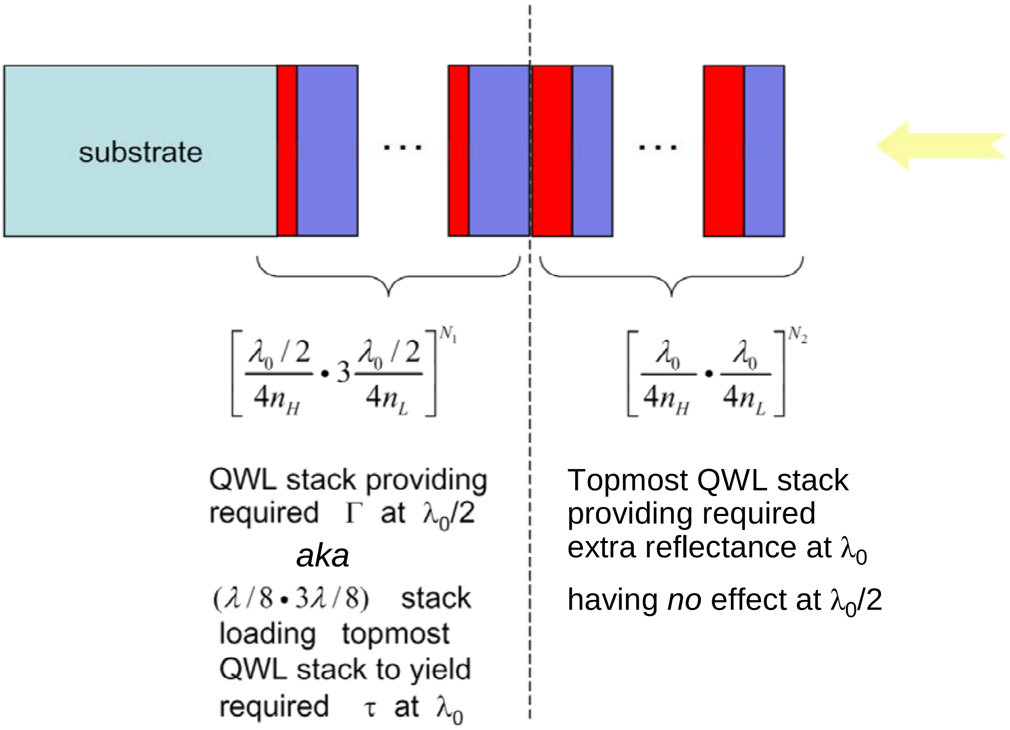


Figure 0 – Reference dichroic design

At $\lambda=\lambda_0/2=532$nm the topmost stack consists of N_2 half-wavelength doublets, which give no contribution to reflectance. The bottom stack, on the other hand, consists of N_1 quarter-wavelength doublets (the low-index layers are topped with irrelevant half-wavelength layers), and N_1 is chosen in such a way as to provide the required reflectance at 532nm. On the other hand, at $\lambda=\lambda_0=1064$nm, the bottom stack consists of N_1 (1/8)wavelength-(3/8)wavelength doublets, which load the topmost stack consisting of N_2 quarter-wavelength doublets, and N_2 can be freely chosen to fulfill the transmittance requirement at 1064nm.

For the ITM the 532nm high reflectivity constraint is the most stringent, and a pure $(\lambda_0/8)$-$(3\lambda_0/8)$ stacked doublet design (behaving as QWL at $\lambda_0/2$) can meet the nominal requirements both at 1064nm and at 532nm.

The above reasoning, however, is only approximate as it ignores material dispersion (dependence of refractive indexes on wavelength).

If material dispersion is included, the "reference" designs can meet the requirements only approximately.
Tables 1 and 2 below show (grey boxes) the actual reference configurations that are compatible with the nominal specs on transmission and reflection coefficients at 1064 and 532 nm. Obviously, the particular configurations minimizing the Brownian noise are those with the *minimum* number of doublets, i.e. ($N_1 = 4$ and $N_2 = 17$) for the ETM and $N = 12$ for the ITM.

N_1 \ N_2	10	11	12	13	14	15	16	17	18	19	20
1	3316,2	1635,3	806,01	397,2	195,72	96,432	47,513	23,409	11,534	5,6826	2,7998
	0,3621	0,3909	0,41563	0,43632	0,45306	0,46597	0,47517	0,48076	0,48279	0,4813	0,47626
2	2107,8	1039	512,07	252,32	124,33	61,256	30,181	14,87	7,3264	3,6097	1,7785
	0,61658	0,64006	0,65946	0,67515	0,68746	0,69667	0,70299	0,70657	0,70752	0,70584	0,70149
3	1306,5	643,93	317,31	156,35	77,035	37,955	18,701	9,2136	4,5395	2,2366	1,1019
	0,79441	0,80856	0,82004	0,82916	0,8362	0,84139	0,84486	0,84675	0,84709	0,84591	0,84316
4	800,48	394,47	194,37	95,771	47,187	23,249	11,455	5,6436	2,7806	1,37	0,67497
	0,89572	0,90326	0,90932	0,9141	0,91776	0,92043	0,9222	0,92314	0,92326	0,92258	0,92107
5	487,7	240,32	118,41	58,341	28,745	14,162	6,9777	3,4379	1,6938	0,83453	0,41117
	0,94848	0,95229	0,95534	0,95773	0,95956	0,96089	0,96176	0,96222	0,96226	0,96191	0,96113
6	296,29	145,99	71,932	35,441	17,462	8,6033	4,2388	2,0884	1,029	0,50696	0,24977
	0,97487	0,97674	0,97824	0,97942	0,98031	0,98096	0,98139	0,98161	0,98163	0,98145	0,98106
7	179,74	88,56	43,634	21,498	10,592	5,2187	2,5712	1,2668	0,62415	0,30751	0,15151
	0,98781	0,98873	0,98946	0,99003	0,99046	0,99078	0,99099	0,99109	0,9911	0,99101	0,99082
8	108,94	53,677	26,447	13,03	6,4199	3,163	1,5584	0,76781	0,37829	0,18638	0,09183
	0,99411	0,99455	0,9949	0,99518	0,99539	0,99554	0,99564	0,99569	0,9957	0,99566	0,99556
9	66,001	32,519	16,022	7,8939	3,8893	1,9162	0,94411	0,46515	0,22918	0,11291	0,055632
	0,99715	0,99737	0,99754	0,99767	0,99777	0,99785	0,9979	0,99792	0,99792	0,9979	0,99786
10	39,974	19,695	9,7038	4,781	2,3556	1,1606	0,5718	0,28172	0,1388	0,068387	0,033694
	0,99863	0,99873	0,99881	0,99888	0,99893	0,99896	0,99899	0,999	0,999	0,99899	0,99897

Table 1 – ETM reference "hybrid" SD design. In each cell: ETM power transmission coefficient @1064 nm [ppm] (1st line) and power reflection coefficient @532 nm (2nd line) for different values of N_1 and N_2

N	5	6	7	8	9	10	11	12	13	14
$\lvert\tau\rvert^2$ @1064 nm	0,32238	0,209	0,13199	0,082008	0,050439	0,030831	0,018775	0,011407	0,006921	0,004195
$\lvert\Gamma\rvert^2$ @532 nm	0,86159	0,93077	0,96602	0,98347	0,99199	0,99613	0,99813	0,9991	0,99957	0,99979

Table 2 – ITM reference (1/8-3/8) design. Power transmission coefficient @1064 nm [ppm] and power reflection coefficient @532 nm for different values of N

In the above tables (and throughout) the following refractive index values have been used (courtesy Mark Gross, CSIRO).

material \ wavelength	532	670	946	1064	1319
Silica	1.47809	1.47337	1.47044	1.46995	1.46937
Doped Tantala	2.13890	2.10980	2.09570	2.09418	2.09238

3. Optimized Design

For single wavelength operation, blind (i.e., unconstrained insofar as the coating geometry is concerned) genetic optimization suggests [2] that the minimum noise design at prescribed transmittance consists of a stack of almost identical non-QWL doublets, with physical thicknesses

$$d_{L,H} = \left(\frac{\lambda_0}{n_{L,H}}\right)\left(\frac{1}{4} \pm \xi\right). \tag{1.1}$$

Almost identical here refers to the fact that the first (bottom) and last (top) layer in the stack have slightly different thicknesses. We shall ignore these differences at the moment.

Note that according to (1.1) the low (high) index layers, which are less (more) noisy are thicker (thinner) than a (local) quarter of wavelength. The total phase-thickness of each doublet is however π, as for the QWL.

Now, in order to satisfy the *double* requirements on wavelengths 1064 nm and 532 nm, it is reasonable to use *two* design parameters, obtained by allowing the doublet layers to have different optical lengths, i.e.

$$d_L = \left(\frac{\lambda_0}{n_L}\right)\left(\frac{1}{4} + \xi_L\right),\ d_H = \left(\frac{\lambda_0}{n_H}\right)\left(\frac{1}{4} - \xi_H\right). \tag{1.2}$$

Here the total phase thickness of each doublet is *no* longer π.

It is easy to draw the domain in the (ξ_L, ξ_H)-plane where the dichroic requirements on the reflection coefficients are fulfilled. For different vaues of the number of doublets N in the coating, these domains have different shapes. We limit ourselves to the nonempty domains corresponding to the minimum number of doublets (minimum noise). The minimum number of doublets needed to have a non-empty domain is 20 for the ETM and 10 for the ITM.
The corresponding domains are shown below.

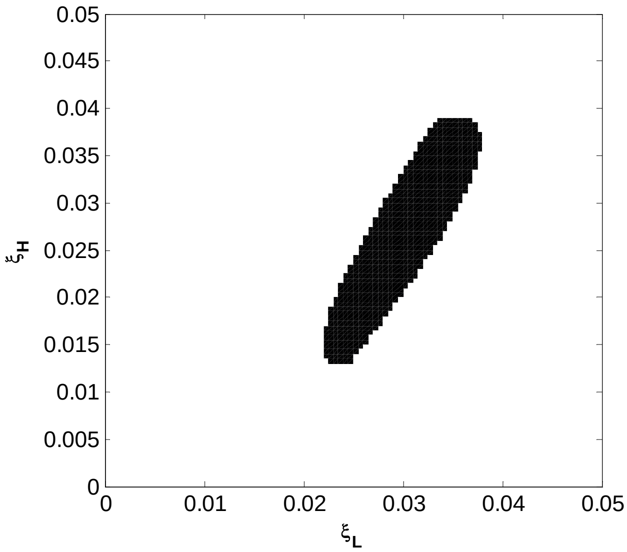


Figure 1. Region in the (ξ_L, ξ_H) plane where ETM transmittance/reflectance requirements at 1064 and 532 nm are fulfilled. N=20 doublets.

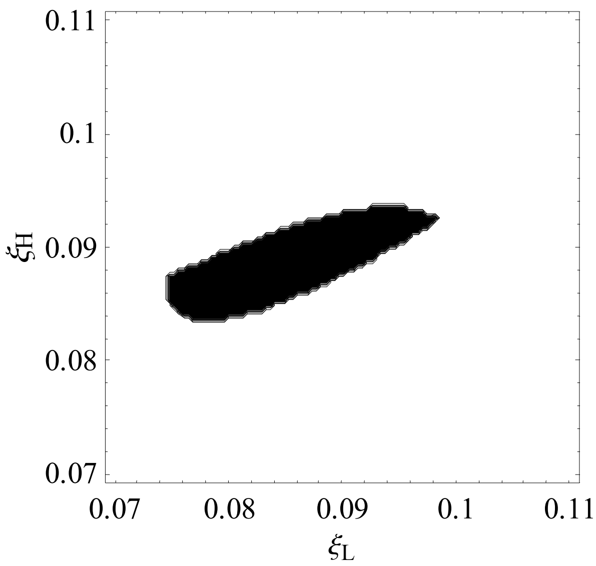


Figure 2. Region in the (ξ_L, ξ_H) plane where the ITM transmittance/reflectance requirements at 1064 and 532 nm are fulfilled. N =10 doublets.

Drawing in the same (ξ_L, ξ_H) plane the coating Brownian noise contour levels, helps visually identifying the minimum-noise (optimal) configuration(s)

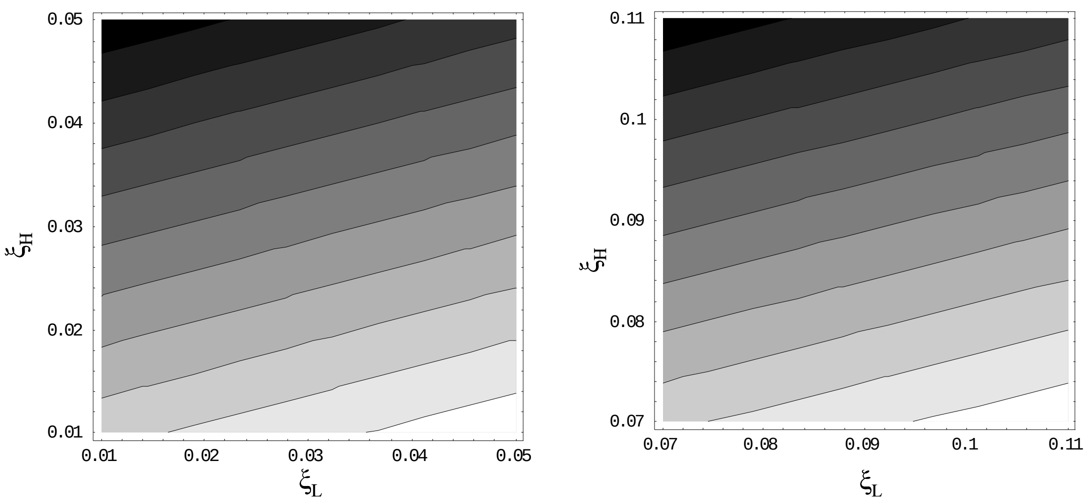


Figure 3. Brownian noise levels in a sub-domain of the (ξ_L, ξ_H) plane, N=20 doublets (left) and N=10 doublets (right).

.

The minimum noise configuration are singled out using a (robust) genetic algorithm, yielding the following optimal configuration:

	N	ξ_L	ξ_H
ETM	20	0.0344	0.0373
ITM	10	0.0885	0.0927

Table 3. Optimal dichroic ETM and ITM designs.

We mention in passing that, as seen from Figs. 1,2 and 3, *several* (ξ_L, ξ_H) configurations, corresponding to points on the upper border of the requirement-acceptance region domain yield almost equivalent (minimal) noises. A best design among these may be chosen, on the basis of cavity transmittance @532nm and on a preferred response on one or more further wavelengths.

4. Comparison Between the Reference and Optimized Design

In this section we wish to compare the two designs for the mirror ETM/ITM coatings in terms of Brownian noise, frequency response and sensitivity to layer thickness deposition errors.

a) Power reflection and Brownian noise level

In Table 4 the power transmission coefficient in ppm @1064 nm and the power reflection coefficients at several wavelengths of interest are shown for the reference coating design (R) described in Section 1 and for the proposed optimized design (O) specified in Table 3.

λ [nm]	(R)	(O)	nL	nH
1064	5.6436 [ppm]	5.9995 [ppm]	1.46995	2.094183
532	0.9231367	0.9887330	1.47809	2.138899
670	0.2405062	0.2055937	1.47337	2.109801
946	0.6279549	0.1684466	1.47044	2.095705
980	0.9982649	0.9998407	1.47027	2.0951
1319	0.1581090	0.2110564	1.46937	2.092376
1550	0.0273024	0.0251009	1.46822	2.08399

Table 4. Power transmission coefficient [ppm] @1064 nm and power reflection coefficients at several wavelengths for the ETM. Reference configuration (R) vs one satisfying the requirements and optimized (O) for thermal noise.

λ [nm]	(R)	(O)	nL	nH
1064	0.0114069	0.0139999	1.46995	2.094183
532	0.9990990	0.9930867	1.47809	2.138899
670	0.1885469	0.0734824	1.47337	2.109801
946	0.3639995	0.6370665	1.47044	2.095705
980	0.8435680	0.9606214	1.47027	2.0951
1319	0.0087962	0.1999338	1.46937	2.092376
1550	0.1006466	0.0201776	1.46822	2.08399

Table 5. Power transmission coefficient @1064 nm and power reflection coefficients at several wavelengths for ITM. . Reference configuration (R) vs one satisfying the requirements and optimized (O) for thermal noise.

The ratio between the PSD levels of the optimized vs reference design are listed below for the ETM and ITM coatings:

	PSD_{opt}/PSD_{ref}
ETM	0.918431
ITM	0.921495

Table 6. Brownian noise PSD ratio. ETM and ITM.

The ratio between the (total) PSD levels, and the corresponding event rate boost obtained by using a partially optimized (ETM only) coating design, and a fully optimized (ETM and ITM) one are listed below

	PSD_{opt}/PSD_{ref}	Event Rate Boost
ETM opt + ITM ref	0.942812	1.092
ETM opt + ITM opt	0.919347	1.134

Table 7. PSD reduction and Event Rate boost for partial and fully optimized design

The fully-optimized (ETM+ITM) design yields a +13.4% event rate boost.

b) Frequency Response

The frequency responses of the optimized and reference ETM designs are shown below

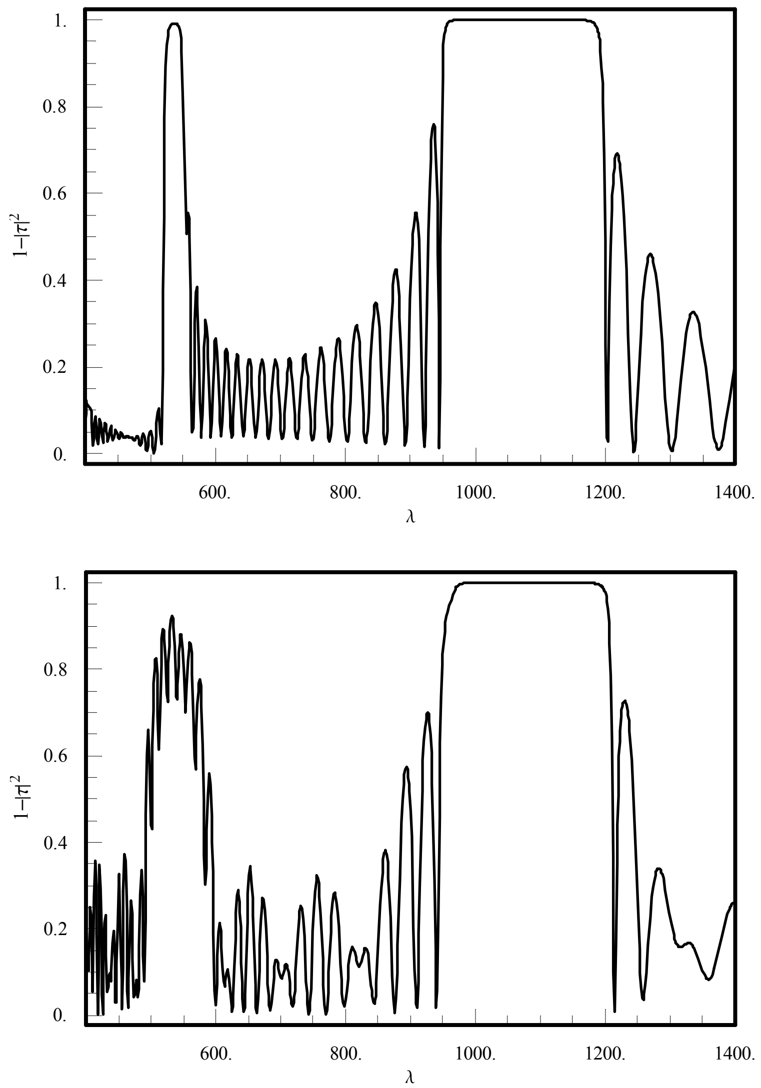


Figure 4. Power reflection coefficient vs wavelength for the optimized ETM coating design (top) and the reference coating design (bottom).

From Figure 4 we see that in the main reflection wavelength band the two spectra behave similarly, on the 2nd harmonic of interest the spectrum of the reference design is oscillating much more wildely, thus potentially opening the design to thickness error and thermal sensitivity. In particular at λ = 532 nm the optimized design, has a much flatter behavior compared to the reference one. This is further appreciated from the close-ups below:

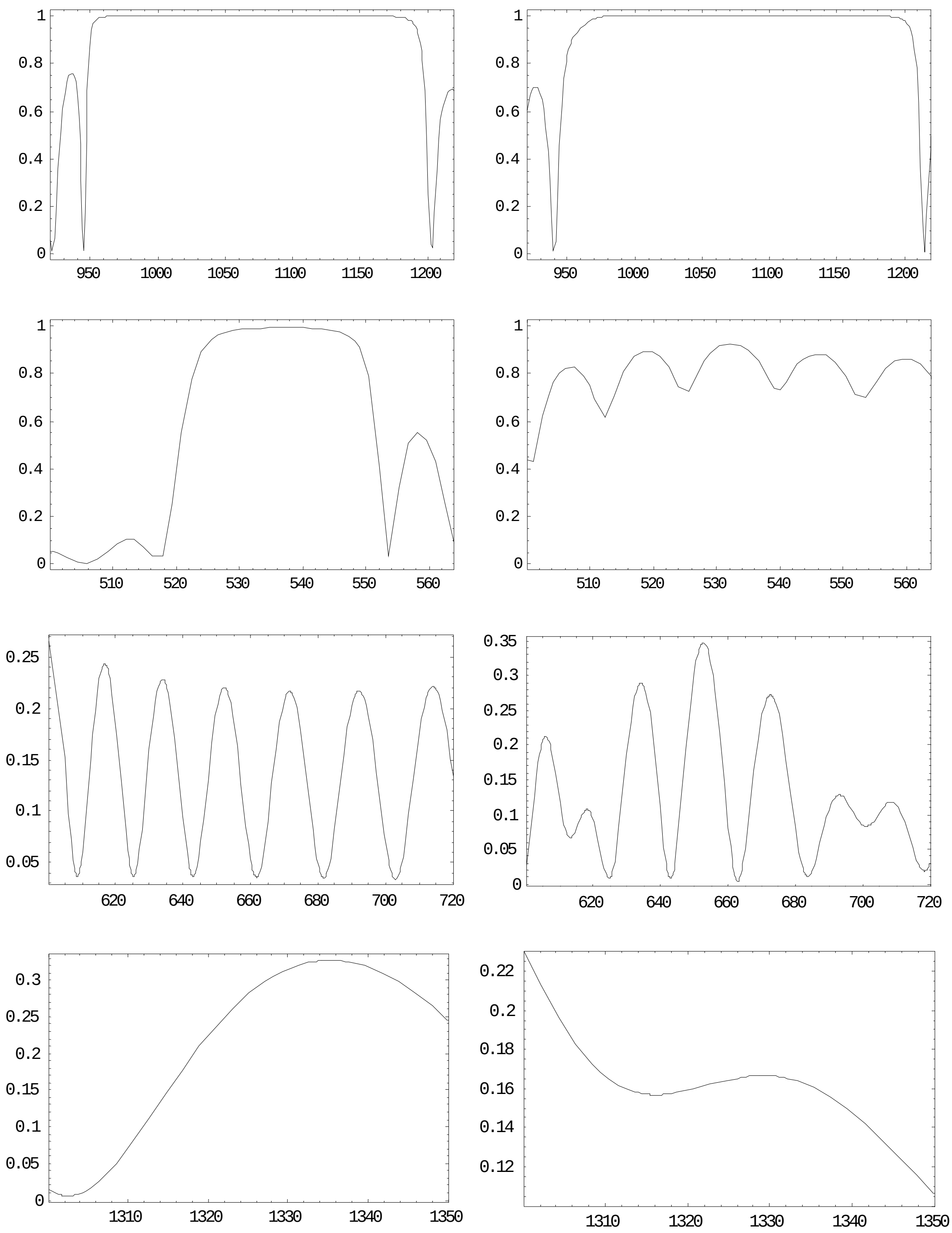
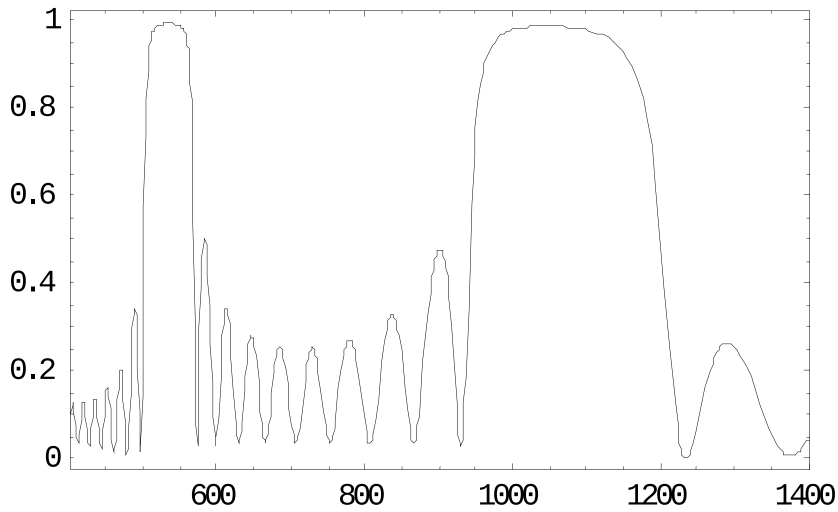


Figure 5 - **Power reflection coefficient vs wavelength closeups for the optimized ETM coating design (left) and the reference coating design (right).**

The frequency responses of the optimized and reference ITM designs are shown below



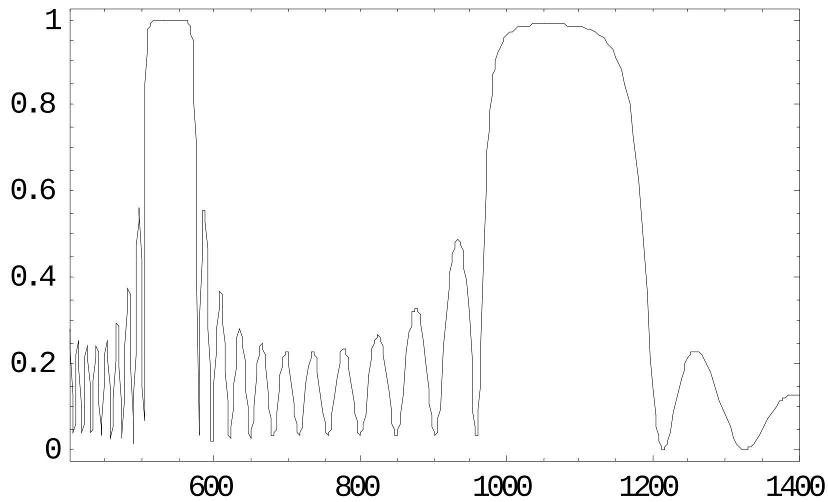


Figure 6. Power reflection coefficient vs wavelength for the optimized ETM coating design (top) and the reference coating design (bottom).

Similar to the ETM case, we show in the next page some close-ups of the spectral response of the ITM mirror, for the reference (bottom) and optimized (top) dichroic design.

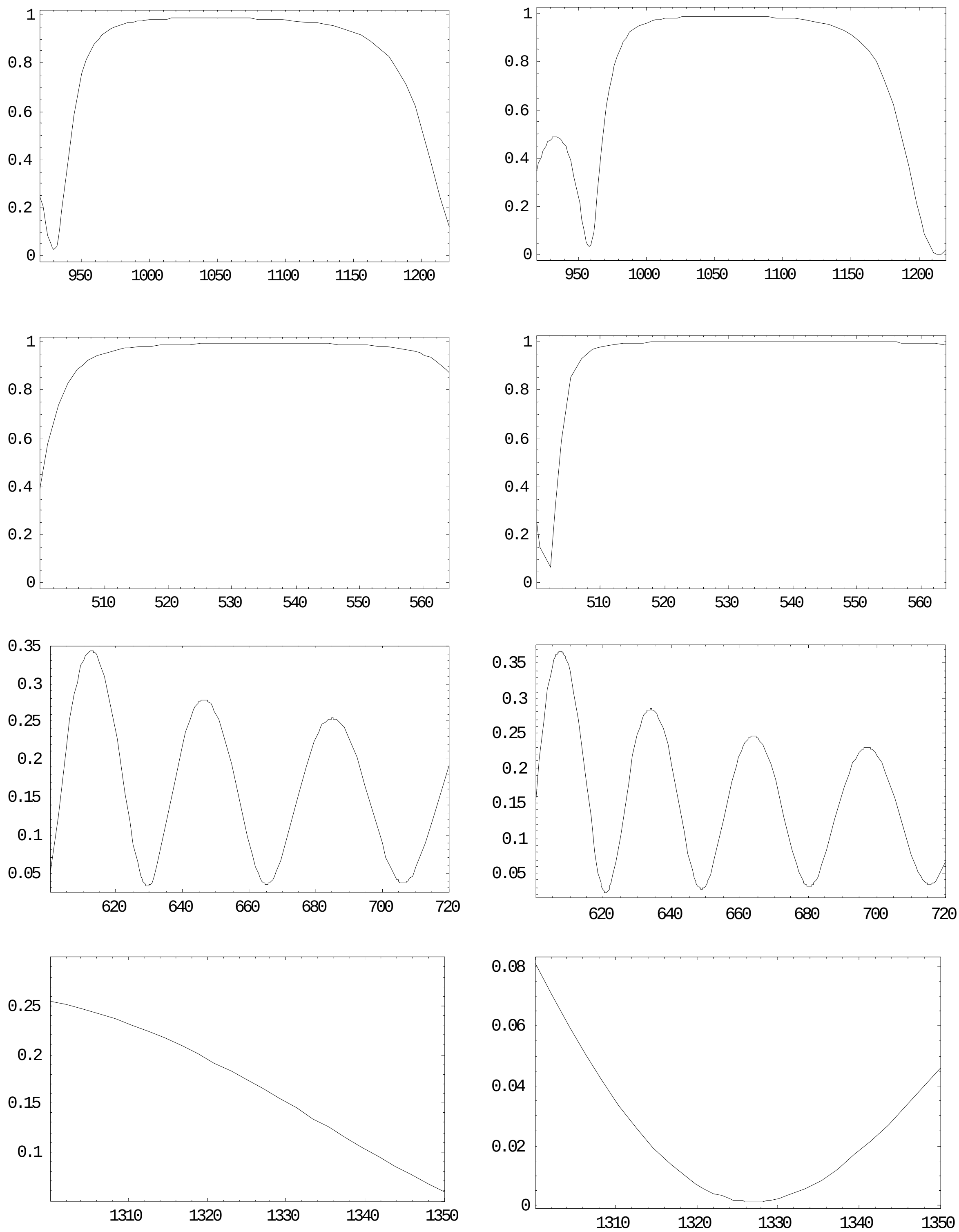


Figure 7. Power reflection coefficient vs wavelength for the optimized ETM coating design (left) and the reference coating design (right).

c) Sensitivity to Layer Thickness Errors

The thermal sensitivity of the coatings is related to the sensitivity to thickness layer deposition, a formulation resilient to thickness errors is automatically thermally insensitive.

The figures below displays the outcomes of the power transmission coefficient @1064nm [ppm] and the power reflection coefficient @532nm of 10^4 trials obtained by affecting each layer thickness by a Gaussian random error with standard deviation of 1%, for the ETM.

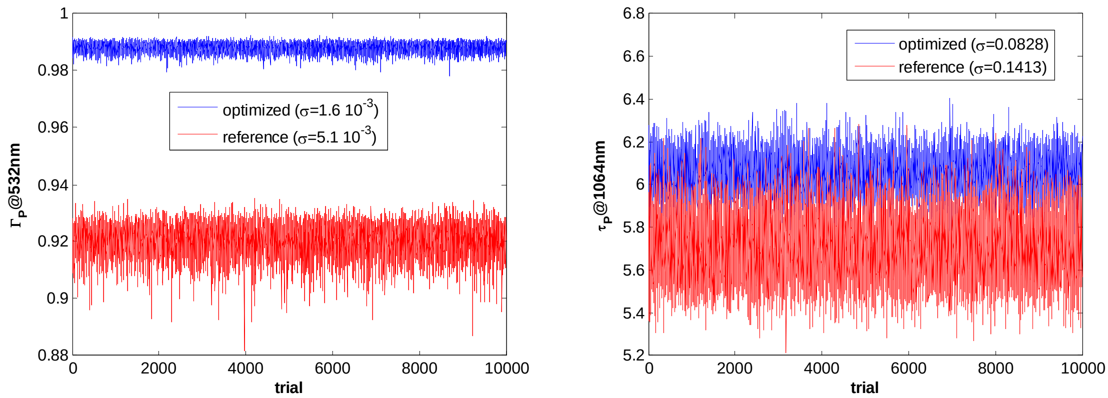


Figure 7. Effect of random (Gaussian std. dev. 1%) errors in the ETM layers' thicknesses. Power reflection @532nm (left) and power transmission @1064nm coefficients [ppm].

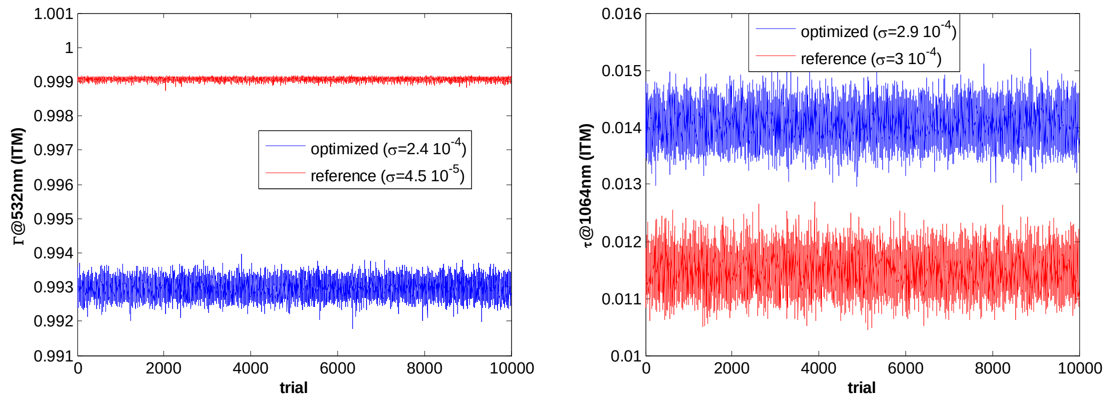


Figure 8. Effect of random (Gaussian std. dev. 1%) errors in the ITM layers' thicknesses. Power reflection @532nm (left) and power transmission @1064nm coefficients [ppm].

The optimized ETM design is more robust (lower variance, by a factor of ~2) against random deposition errors compared to the reference one, both at 1064 and 532 nm For the ITM, the situation is reversed at 532 nm, where the reference design (which is in fact QWL) is more robust. However, both the reference and the optimized design keep within acceptable distance from the nominal value.
It is more critical to see what the effects of systematic 1% errors in the layer's thicknesses may be.
As far as the noise reduction achievable using the optimized design, the results are shown below:

Systematic error	PSD_{opt}/PSD_{ref} ETM	PSD_{opt}/PSD_{ref} ITM	PSD_{opt}/PSD_{ref} & Event Rate Boost fully optimized design
+1%	0.927	0.93071	0.92854 (1.1176 e.r.b.)
-1%	0.909	0.91228	0.91015 (1.1517 e.r.b.)

Table 8. Effect of systematic thickness errors on PSD shrink

The effect of the above systematic thickness error on the reflection coefficients at the wavelengths of interest is summarized below.

	+1%			-1%		
	1064	5.85534	[ppm]	1064	6.73629	[ppm]
ETM, opt	532	0.964958		532	0.992116	
	670	0.0499758		670	0.140169	
	946	0.758419		946	0.988043	
	1319	0.0222319		1319	0.322941	
	1550	0.0927217		1550	0.0293523	
	1064	6.44755	[ppm]	1064	5.4766	[ppm]
ETM, ref	532	0.798266		532	0.832435	
	670	0.00825439		670	0.204874	
	946	0.319294		946	0.920107	
	1319	0.184616		1319	0.165386	
	1550	0.0253351		1550	0.114597	
	1064	13290.6	[ppm]	1064	15317.8	[ppm]
ITM, opt	532	0.991883		532	0.993194	
	670	0.0423102		670	0.185841	
	946	0.212009		946	0.838967	
	1319	0.243106		1319	0.140311	
	1550	0.0391836		1550	0.0134138	
	1064	11298.1	[ppm]	1064	12236.5	[ppm]
ITM, ref	532	0.998831		532	0.999167	
	670	0.245725		670	0.0611916	
	946	0.483949		946	0.07266	
	1319	0.0516078		1319	0.00426042	
	1550	0.0915831		1550	0.0981467	

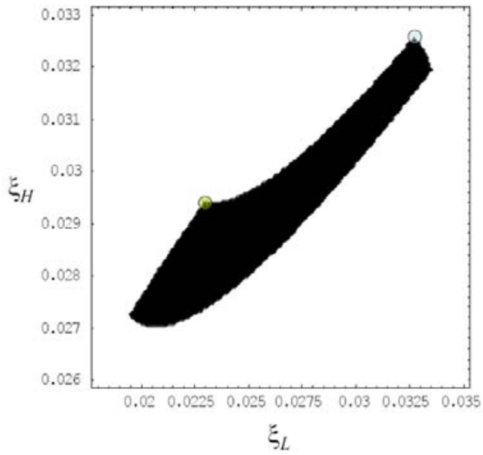
The 6ppm maximum transmittance at 1064nm requirement turns out to be slightly violated by both the reference and optimized design, respectively for 1% positive and negative systematic errors.

5. Tighter requirements

In order to probe the flexibility of the (Brownian) noise-optimized dichroic design, we show below the case where we enforce the following tighter inequalities:

$$\tau_p\ @1064nm \in [4.5, 5.5]\ ppm, \quad \Gamma_p\ @532nm \in [0.94, 0.96]$$

The domain where these constraints are fulfilled in the (ξ_L, ξ_H) plane is shown below (left), together with the corresponding noise contour plot (right)



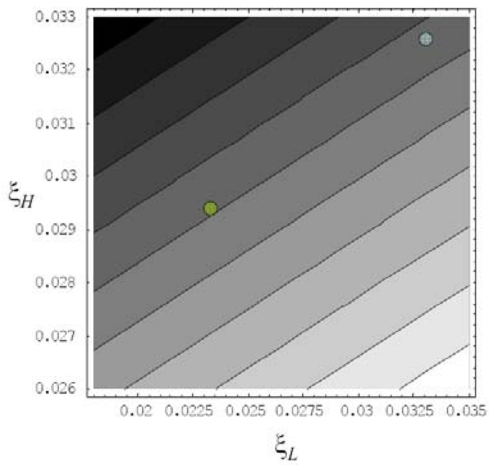


Figure 9. Left: region in the (ξ_L, ξ_H) space where the ETM transmittance/reflectance requirements (X) are fulfilled. N =20 doublets. Right: Brownian noise contour lines. The rightmost marker yields the minimal Brownian noise design.

The rightmost marker in the panels yields the minimum Brownian noise, but the leftmost one has only a slightly larger noise, viz.,

	ξ_L	ξ_H	PSD_{opt}/PSD_{ref}
Design A (Rightmost marker in Fig. X)	0.03271	0.03255	0.932
Design B (Leftmost marker in Fig. X)	0.03116	0.03169	0.933

Table 9. Dichroic 20-doublets ETM designs corresponding to the markers in Figure X.

On the other hand, the two designs exhibit somewhat different transmittances at other potentially useful wavelengths, as illustrated below:

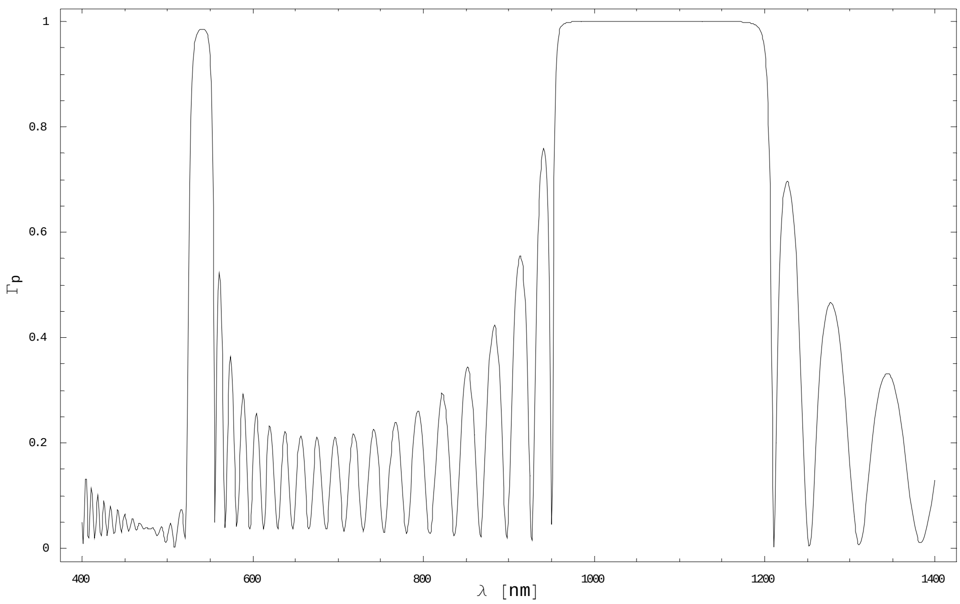


Figure 10. Spectral response of ETM design A

λ [nm]	(R)
1064	5.499 ppm
532	0.95999
670	0.115096
946	0.652854
980	0.999696
1319	0.083263
1550	0.062852

Table 10. Power transmission coefficient [ppm] @1064 nm and power reflection coefficients at several wavelengths for ETM design A.

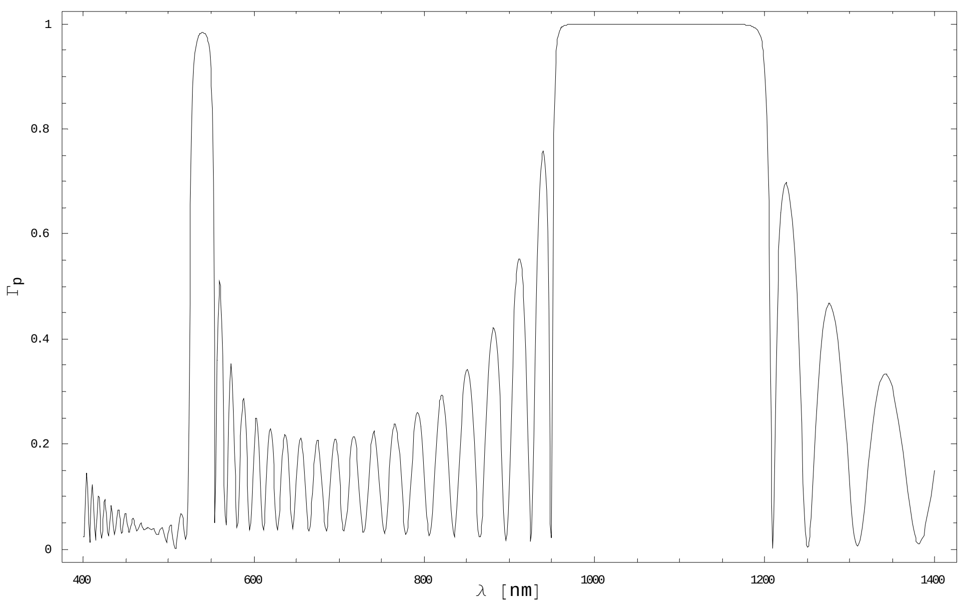


Figure 11. Spectral response of ETM design B

λ [nm]	(R)
1064	5.41 ppm
532	0.958
670	0.141141
946	0.518368
980	0.999752
1319	0.112414
1550	0.052687

Table 11. Power transmission coefficient [ppm] @1064 nm and power reflection coefficients at several wavelengths for ETM design B.

Spectral-response close-ups, in a neighbourhood of the potentially interesting wavelengths in Tables Z and Z1 are shown below side by side.

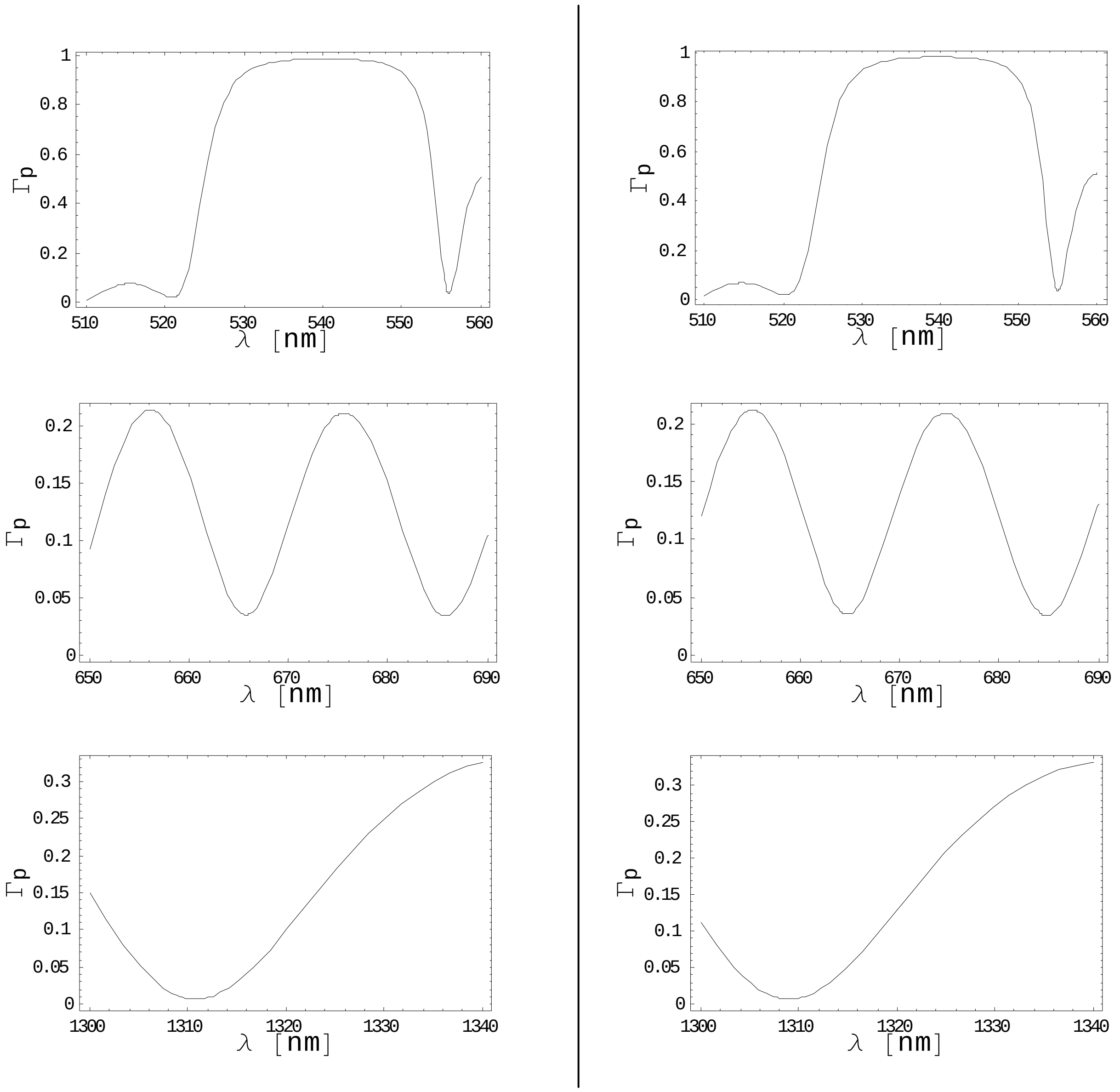


Figure 12. Spectral response close-ups of ETM design A (left) and B (right)

Conclusions

Many AdLIGO compliant dichroic syntheses possible. The “pure” SD design proposed here is better in terms of Brownian noise, structurally simpler (fewer doublets, only two thicknesses), no worse in terms of robustness against random and systematic deposition errors, and more flexible in terms of design constraints compared to the reference one.
Also, several quasi-optimal designs differing in response at other potentially useful wavelengths can be chosen.
While this optimization was done considering thermal noise only and may not be fully optimized for thermo-optic noise, inclusion of thermo-optic noise should not change this scenario (less Tantala expected to yield less noise, in any case [3]).
Optimization results with tighter constraints have been also shown.
Tighter designs (e.g., requiring ITM transmittance below 1000 ppm at 532 nm) are being investigated and will be reported soon.

References

[0] The AdLIGO Wiki.

[1] E. Gustaffson, personal communication [Riccardo, please check].

[2] J. Agresti, G. Castaldi, R. DeSalvo, V. Galdi, V. Pierro, I.M. Pinto, “Optimized multilayer dielectric mirror coatings for gravitational wave interferometers,” Proc. SPIE, Vol. 6286, 628608 (2006); DOI:10.1117/12.678977.

[3] I.M. Pinto, V. Galdi, M. Principe, LIGO-T070159-00-Z